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A comprehensive framework for training stable and passive multivariate behavioral models

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***Abstract*—We present a theoretical framework and related algorithms for the construction of behavioral models of linear or linearized devices. Unlike competing approaches, the proposed method is robust and guarantees theoretically the uniform stability and passivity of the models in a multivariate setting, where the model behavior depends not only on time or frequency but also on a number of design/stochastic parameters. Various examples demonstrate the high accuracy and reliability of proposed framework.**

***Index Terms*—macromodeling, model order reduction, data-driven modeling, passivity, stability.**

I. Introduction

Surrogate models are valid and efficient alternatives to full-accuracy electromagnetic characterizations when dealing with the optimization of complex electrical and electronic components. Surrogates can be based on approximation, order reduction or data-driven identification methods, the latter including recently developed or improved machine learning techniques [1]–[3].

In many relevant applications, including system-level simulations for signal/power integrity verification and microwave component design, the optimization of individual components must necessarily be performed based on intensive transient simulations, in which non-ideal and parasitic phenomena must be properly taken into account. Surrogate models enable fast optimizations, provided that they are able to mimic the dynamic behavior of the underlying system in correspondence of a number of well-defined electrical ports, either as stand-alone units, or when they are inserted in a larger interconnected system. In this view, surrogate models are usually provided to the user in the form of reduced-order equivalent circuits, that must necessarily preserve the properties of causality, stability, or passivity, that characterize the structure under modeling. If these properties are not preserved by a reduced order circuit, the latter easily becomes the root cause of nonphysical or unstable transient simulation results. In this scenario, when dealing with linear or linearized devices, rational fitting approaches usually represent the modeling method of choice, as they are based on model structures that lend themselves naturally to the characterization and the enforcement of these fundamental properties.

Based on the recent developments of [4], in this contribution we present a comprehensive framework for the generation of guaranteed stable and passive parameterized macromodels. Differently from previously available methods [5], [6], the approach proposed in this work returns behavioral models that are at the same time of minimal order, accurate, and certified stable (passive) by construction, for all of the admissible parameter values. For completeness, we report a detailed introduction of the proposed model identification framework in Sec. II and III, although part of this material can be found in [4]. The main contribution of this work is a novel approach to enforce stability and passivity constraints with a tuneable trade-off between the computational requirements of the algorithm and the accuracy of the output model. This new formulation enables major accuracy improvements with respect to [4] at the same computational cost.

II. Background and Notation

In the following, s is the Laplace variable, $A^\top, A^\star$ denote the transpose and the Hermitian transpose of the matrix A respectively. The set of symmetric matrices of size n is denoted with $\mathbb{S}_n$. We define a multi-index as a d-dimensional collection of indices $\boldsymbol{i} = (i_1, \dots, i_d) \in \mathbb{N}^d$. Given two multi-indices $\boldsymbol{j}$ and $\boldsymbol{k}$, we write $\boldsymbol{j} \leq \boldsymbol{k}$ meaning $j_1 \leq k_1, \dots, j_d \leq k_d$. The functions $b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{x}) : \mathcal{X} \subset \mathbb{R}^d \to \mathbb{R}$ are d-variate Bernstein polynomials normalized over the hyper-rectangle $\mathcal{X} = [\overline{x}_1, \underline{x}_1] \times \dots \times [\overline{x}_d, \underline{x}_d]$. The degree of polynomials in each individual variable is $\bar{\boldsymbol{\ell}} = (\bar{\ell}_1, \dots, \bar{\ell}_d)$, and the multi-index $\boldsymbol{\ell}$ identifies each basis element. A polynomial matrix $F(\boldsymbol{x}) : \mathcal{X} \to \mathbb{R}^{n \times n}$, with entries of degree $\bar{\boldsymbol{\ell}}$ is written in the Bernstein basis as

$$F(\boldsymbol{x}) = \sum_{\boldsymbol{\ell} \in \mathcal{I}_{\bar{\boldsymbol{\ell}}}} F_{\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{x}), \quad \mathcal{I}_{\bar{\boldsymbol{\ell}}} = \{\boldsymbol{\ell} \in \mathbb{N}^d : \boldsymbol{\ell} \leq \bar{\boldsymbol{\ell}}\}. \quad (1)$$

The set of the matrix coefficients defining the expansion, to which we will refer to as *control points*, is compactly denoted as $\{F^{\boldsymbol{\ell}}\}$. When $F(\boldsymbol{x})$ is symmetric, with the notation $F(\boldsymbol{x}) \preceq 0$ we mean that $F(\boldsymbol{x})$ is negative-semi-definite over its domain. With $\{F^{\boldsymbol{\ell}}\} \preceq 0$ we mean that all its control points are negative semi-definite. Finally, we define the following block-matrix notation

$$\Omega(P, Q, R) = \begin{bmatrix} P^\top R + RP & RQ \\ Q^\top R & 0 \end{bmatrix} \quad (2)$$

A. Problem Statement

We consider a complex linear time invariant (LTI) electrical system $\mathbf{S}$, accessible from a set of P well defined electrical ports, and depending on d physical or design parameters, collected in a vector $\boldsymbol{\vartheta} \in \Theta = [\overline{\vartheta}_1, \underline{\vartheta}_1] \times \dots \times [\overline{\vartheta}_d, \underline{\vartheta}_d]$. For all of the admissible parameter configurations, $\mathbf{S}$ is known to be stable and/or passive. It is assumed that $\mathbf{S}$ is characterized only in terms of samples of its scattering matrix $S(s, \boldsymbol{\vartheta}) \in \mathbb{C}^{\mathsf{P}\times\mathsf{P}}$, provided in correspondence of a finite number of frequency-parameter configurations

$$S_{k,m} = S(j\omega_k, \boldsymbol{\vartheta}_m), \quad 1 \le k \le \bar{k},\ 1 \le m \le \bar{m}. \tag{3}$$

Our objective is to build a parameterized low complexity LTI model $\mathbf{H}$, with scattering matrix $\mathsf{H}(s, \boldsymbol{\vartheta})$, that

1) mimics the behavior of $\mathbf{S}$ at its electrical ports $\forall \vartheta \in \Theta$, by verifying the approximation
$$\mathsf{H}(j\omega_k, \boldsymbol{\vartheta}_m) \approx S_{k,m},\ \forall k,\ \forall m \tag{4}$$
2) inherits the stability (passivity) properties of $\mathbf{S}$ for all the admissible $\boldsymbol{\vartheta}$.

B. Model Structure and Passivity Characterization

In the proposed modeling framework, the scattering parameters of the model $\mathbf{H}$ obey the following structure

$$\mathsf{H}(s, \boldsymbol{\vartheta}) = \frac{\mathsf{N}(s, \boldsymbol{\vartheta})}{\mathsf{D}(s, \boldsymbol{\vartheta})} = \frac{\sum_{i=0}^{n} R_i(\boldsymbol{\vartheta})\varphi_i(s)}{\sum_{i=0}^{n} r_i(\boldsymbol{\vartheta})\varphi_i(s)}, \tag{5}$$

where $\varphi_0(s) = 1$, and $\varphi_i(s) = (s - q_i)^{-1}$, $i > 0$ are partial fractions with fixed and known stable poles. The dependency on $\boldsymbol{\vartheta}$ is induced by the parameterized residues

$$R_i(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathcal{I}_{\overline{\boldsymbol{\ell}}}} R_{i,\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}), \quad r_i(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathcal{I}_{\overline{\boldsymbol{\ell}}}} r_{i,\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}), \tag{6}$$

where $r_{i,\boldsymbol{\ell}} \in \mathbb{R}$ and $R_{i,\boldsymbol{\ell}} \in \mathbb{R}^{\mathsf{P}\times\mathsf{P}}$ are unknown model coefficients. From these definitions, it follows that $\mathsf{N}(s, \boldsymbol{\vartheta}) \in \mathbb{C}^{\mathsf{P}\times\mathsf{P}}$ and $\mathsf{D}(s, \boldsymbol{\vartheta}) \in \mathbb{C}$ are both rational functions of s with parameterized zeros and the same fixed common poles. This implies that H is a proper scattering matrix of order n, with parameterized poles and zeros: the former are the zeros of D, the latter are the zeros of N.

To provide a numerically exploitable passivity characterization for $\mathbf{H}$, we will make use of the following state space realizations associated with D and N[1]

$$\mathsf{D}(s, \boldsymbol{\vartheta}) \leftrightarrow \Sigma_{\mathsf{D}} = \left(\begin{array}{c|c} A & B \\ \hline C_1(\boldsymbol{\vartheta}) & D_1(\boldsymbol{\vartheta}) \end{array}\right), \tag{7}$$

$$\mathsf{N}(s, \boldsymbol{\vartheta}) \leftrightarrow \Sigma_{\mathsf{N}} = \left(\begin{array}{c|c} A & B \\ \hline C_2(\boldsymbol{\vartheta}) & D_2(\boldsymbol{\vartheta}) \end{array}\right), \tag{8}$$

where A and B constant matrices depending on the basis functions $\varphi_i(s)$, A asymptotically stable, and

$$C_1(\boldsymbol{\vartheta}) = [r_1(\boldsymbol{\vartheta}), \ldots, r_n(\boldsymbol{\vartheta})], \qquad D_1(\boldsymbol{\vartheta}) = r_0(\boldsymbol{\vartheta}), \tag{9}$$

$$C_2(\boldsymbol{\vartheta}) = [R_1(\boldsymbol{\vartheta}), \ldots, R_n(\boldsymbol{\vartheta})], \qquad D_2(\boldsymbol{\vartheta}) = R_0(\boldsymbol{\vartheta}). \tag{10}$$

Also, setting $\overline{\boldsymbol{m}} = 2\overline{\boldsymbol{\ell}}$ we define for readability

$$X(\boldsymbol{\vartheta}) = \begin{bmatrix} C_1^\top(\boldsymbol{\vartheta}) \\ D_1^\top(\boldsymbol{\vartheta}) \end{bmatrix} \begin{bmatrix} C_1(\boldsymbol{\vartheta}) & D_1(\boldsymbol{\vartheta}) \end{bmatrix} = \sum_{\boldsymbol{m}\in\mathcal{I}_{\overline{\boldsymbol{m}}}} X_{\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}), \tag{11}$$

[1]Without loss of generality, the presentation in this paper assumes the scalar (one-port) case P = 1. Generalizations to the case P > 1 are straightforward and thoroughly treated in [4]

and an $\overline{\boldsymbol{m}}$-degree expression for the output matrices of Σ_{N}

$$Y(\boldsymbol{\vartheta}) = \begin{bmatrix} C_2^\top(\boldsymbol{\vartheta}) \\ D_2^\top(\boldsymbol{\vartheta}) \end{bmatrix} = \sum_{\boldsymbol{\ell}\in\mathcal{I}_{\overline{\boldsymbol{\ell}}}} \begin{bmatrix} R_{\boldsymbol{\ell}}^\top \\ R_{0,\boldsymbol{\ell}} \end{bmatrix} b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{m}\in\mathcal{I}_{\overline{\boldsymbol{m}}}} Y_{\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}), \tag{12}$$

where, the coefficients $Y_{\boldsymbol{m}}$ are obtained as linear combinations of $\begin{bmatrix} R_{\boldsymbol{\ell}}, & R_{0,\boldsymbol{\ell}} \end{bmatrix}^\top = \begin{bmatrix} R_{1,\boldsymbol{\ell}}, \ldots, R_{n,\boldsymbol{\ell}}, R_{0,\boldsymbol{\ell}} \end{bmatrix}^\top$, by applying the degree elevation property of the Bernstein polynomials [7]. Now, considering the functions

$$\begin{aligned} &L(\boldsymbol{\vartheta}) : \Theta \to \mathbb{S}_n, \quad G(\boldsymbol{\vartheta}) : \Theta \to \mathbb{S}_{n+1}, \\ &G(\boldsymbol{\vartheta}) = \Omega(A, B, L(\boldsymbol{\vartheta})) - \begin{bmatrix} 0 & C_1(\boldsymbol{\vartheta})^\top \\ C_1(\boldsymbol{\vartheta}) & 2D_1(\boldsymbol{\vartheta}) \end{bmatrix}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} &P(\boldsymbol{\vartheta}) : \Theta \to \mathbb{S}_{n\mathsf{P}}, \quad F(\boldsymbol{\vartheta}) : \Theta \to \mathbb{S}_{\mathsf{P}(n+2)} \\ &F(\boldsymbol{\vartheta}) = \begin{bmatrix} \Omega(A, B, P(\boldsymbol{\vartheta})) - X(\boldsymbol{\vartheta}) & Y(\boldsymbol{\vartheta}) \\ Y^\top(\boldsymbol{\vartheta}) & -\mathbb{I}_{\mathsf{P}} \end{bmatrix}, \end{aligned} \tag{14}$$

we have the following theorem, proved in [4]

Theorem 1: The model $\mathbf{H}$ is stable $\forall \boldsymbol{\vartheta} \in \Theta$ if

$$\exists L(\boldsymbol{\vartheta}) : G(\boldsymbol{\vartheta}) \preceq 0. \tag{15}$$

When (15) holds, $\mathbf{H}$ is passive $\forall \boldsymbol{\vartheta} \in \Theta$ if and only if, additionally

$$\exists P(\boldsymbol{\vartheta}) : F(\boldsymbol{\vartheta}) \preceq 0. \tag{16}$$

Notice that in the above theorem, $G(\boldsymbol{\vartheta})$ depends linearly on $r_{i,\boldsymbol{\ell}}$, while $F(\boldsymbol{\vartheta})$ depends linearly on $R_{i,\boldsymbol{\ell}}$. Matrices $L(\boldsymbol{\vartheta})$ and $P(\boldsymbol{\vartheta})$ are generalized energy storage functions.

III. Modeling Algorithm Outline

The model generation is performed applying the Parameterized Sanathanan Koerner Iteration (PSK) [8], augmented with numerically tractable stability and passivity constraints. Condition (4) is met by iteratively enforcing the linearized approximation

$$\frac{\mathsf{N}^\mu(j\omega_k, \boldsymbol{\vartheta}_m) - \mathsf{D}^\mu(j\omega_k, \boldsymbol{\vartheta}_m) S_{k,m}}{\mathsf{D}^{\mu-1}(j\omega_k, \boldsymbol{\vartheta}_m)} \approx 0, \forall k, \forall m \tag{17}$$

in a least squares-sense, being $\mu = 1, 2, \ldots$ an iteration index. By setting $\mathsf{D}^0(j\omega, \boldsymbol{\vartheta}) = 1$, the function $\mathsf{D}^{\mu-1}$ is numerically available at each iteration μ, and (17) can be enforced repeatedly until convergence, that is reached when $\mathsf{D}^\mu(j\omega, \boldsymbol{\vartheta}) \simeq \mathsf{D}^{\mu-1}(j\omega, \boldsymbol{\vartheta})$. Since only the knowledge of the denominator variables $r_{i,\boldsymbol{\ell}}^{\mu-1}$ is required to setup the approximation problem (17), the PSK iteration admits a fast implementation based on the elimination of the numerator unknowns $R_{i,\ell}^\mu$, that are computed only once, after convergence is met [9]. By collecting the variables $r_{i,\boldsymbol{\ell}}^\mu$ in the vector x^μ and denoting with Γ^μ a known regression matrix, the fast iteration is performed by solving until convergence the problem

$$\min_{x^\mu} \|\Gamma^\mu x^\mu\|. \tag{18}$$

We now augment this optimization problem with the model stability constraints. To verify condition (15), while solving for x^μ we look for a function $L(\boldsymbol{\vartheta})$ such that $G(\boldsymbol{\vartheta}) \preceq 0$. We force $L(\boldsymbol{\vartheta})$ to be structured as

$$L(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell} \in \mathcal{I}_{\overline{\boldsymbol{\ell}}}} L_{\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}), \quad L_{\boldsymbol{\ell}} \in \mathbb{S}_n, \tag{19}$$

being $\{L_{\boldsymbol{\ell}}\}$ unknown matrix coefficients. This choice returns

$$G(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell} \in \mathcal{I}_{\overline{\boldsymbol{\ell}}}} G_{\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) \tag{20}$$

where $\{G_{\boldsymbol{\ell}}\}$ depend linearly on the unknowns $L_{\boldsymbol{\ell}}$ and x^μ. Due to the non-negativity of the Bernstein polynomials, the condition $G(\boldsymbol{\vartheta}) \preceq 0$ is implied by $\{G_{\boldsymbol{\ell}}\} \preceq 0$. Therefore, in place of (18) we solve the semi-definite program

$$\min_{x^\mu, L_{\boldsymbol{\ell}}} \|\Gamma^\mu x^\mu\|, \quad \text{s.t.} \quad \{G_{\boldsymbol{\ell}}\} \preceq 0, \tag{21}$$

which guarantees model stability by construction.

Assuming that the denominator convergence is attained at iteration $\bar{\mu}$, collecting the unknowns $R_{i,\boldsymbol{\ell}}^{\bar{\mu}}$ in a vector y, the standard unconstrained model generation would be completed by solving

$$\min_{y} \left\|\Phi y + \Gamma^{\bar{\mu}} x^{\bar{\mu}}\right\| \tag{22}$$

where again Φ is a known regression matrix. We constrain (22) in such a way that the optimal model verifies also condition (16), so that it is certified passive. We restrict the admissible $P(\boldsymbol{\vartheta})$ to be structured as

$$P(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{m} \in \mathcal{I}_{\overline{\boldsymbol{m}}}} P_{\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}), \quad P_{\boldsymbol{m}} \in \mathbb{S}_{n\mathrm{P}}, \tag{23}$$

with unknown $\{P_{\boldsymbol{m}}\}$. With this choice we obtain

$$F(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{m} \in \mathcal{I}_{\overline{\boldsymbol{m}}}} F_{\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}). \tag{24}$$

As in (20), also in this case the control points $\{F_{\boldsymbol{m}}\}$ depend linearly on the unknowns $P_{\boldsymbol{m}}$ and y. Therefore in place of (22) we solve the semi-definite program

$$\min_{y, P_{\boldsymbol{m}}} \left\|\Phi y + \Gamma^{\bar{\mu}} x^{\bar{\mu}}\right\|, \quad \text{s.t.} \quad \{F_{\boldsymbol{m}}\} \preceq 0. \tag{25}$$

Since $\{F_{\boldsymbol{m}}\} \preceq 0$ implies $F(\boldsymbol{\vartheta}) \preceq 0$, $\mathbf{H}$ verifies both conditions (15) and (16), and is stable and passive by construction.

IV. Conservativity Reduction

The proposed stability and passivity constraints are sufficient but not necessary to guarantee that $\mathbf{H}$ fulfils conditions (15) and (16). This may introduce some conservativity in the estimate of the model coefficients, thus degrading the achievable model accuracy. One source of conservativity stems from the fact that we are considering as admissible functions $L(\boldsymbol{\vartheta})$ and $P(\boldsymbol{\vartheta})$ only those that are structured as in (19) and (23), thus restricting the space of the admissible solutions. To relieve this conservativity, we propose a numerically viable strategy that allows to admit also piece-wise polynomial storage functions. In the following derivations, we focus on the enforcement of condition (16), but similar techniques can be applied to enforce (15).

We start by considering a subdivision of the domain Θ in j smaller hyper-rectangles $\Theta_j = [a_1^j, b_1^j] \times ... \times [a_d^j, b_d^j]$ such that

$$\Theta = \bigcup_j \Theta_j, \quad \mathrm{Vol}(\Theta_z \cap \Theta_l) = 0 \; \forall z \neq l. \tag{26}$$

Over each subdomain we can define the functions

$$P_j(\boldsymbol{\vartheta}_j) : \Theta_j \to \mathbb{S}_{\mathrm{P}n}, \quad P_j(\boldsymbol{\vartheta}_j) = \sum_{\boldsymbol{m} \in \mathcal{I}_{\overline{\boldsymbol{m}}}} P_{j,\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}_j) \tag{27}$$

where $P_{j,\boldsymbol{m}}$ are unknowns matrix coefficients. With this choice, we can consider the restriction

$$F_j(\boldsymbol{\vartheta}_j) : \Theta_j \to \mathbb{S}_{\mathrm{P}(n+2)}, \quad F_j(\boldsymbol{\vartheta}_j) = F(\boldsymbol{\vartheta}_j) \;\; \forall \boldsymbol{\vartheta}_j \in \Theta_j. \tag{28}$$

Applying the Bernstein subdivision property [10] to the blocks $X(\boldsymbol{\vartheta})$ and $Y(\boldsymbol{\vartheta})$ in (14) , each restriction F_j can still be written as a Bernstein polynomial expansion of degree $\overline{\boldsymbol{m}}$ over Θ_j

$$F_j(\boldsymbol{\vartheta}_j) = \sum_{\boldsymbol{m} \in \mathcal{I}_{\overline{\boldsymbol{m}}}} F_{j,\boldsymbol{m}}\, b_{\boldsymbol{m}}^{\overline{\boldsymbol{m}}}(\boldsymbol{\vartheta}_j) \tag{29}$$

where the control points $\{F_{j,\boldsymbol{m}}\}$ depend linearly on $P_{j,\boldsymbol{m}}$ and the numerator unknowns $R_{i,\boldsymbol{\ell}}$. Clearly, $F_j(\boldsymbol{\vartheta}_j) \preceq 0 \;\forall j$ is equivalent to $F(\boldsymbol{\vartheta}) \preceq 0$, and we can replace problem (22) with the less conservative semidefinite program

$$\min_{y} \left\|\Phi y + \Gamma^{\bar{\mu}} x^{\bar{\mu}}\right\|, \quad \text{s.t.} \quad \{F_{j,\boldsymbol{m}}\} \preceq 0 \; \forall j. \tag{30}$$

Although the size of problem (30) is larger then (22), the conservativity reduction drastically improves the model accuracy, also for small j, as will be shown in the following example.

V. Modeling a PCB link

We consider a 2-port high-speed stripline running through two PCBs linked by a connector, as described in [11]. The permittivity of the PCB substrate is $\epsilon_r = 3$ and $\tan\delta = 0.002$. The vias are parameterized by the pad radius, $\vartheta_1 \in [100, 300]$ μm. The scattering matrix samples $S_{k.m}$ are obtained from a field solver for 9 different parameter configurations, in the bandwidth $[0.5, 5]$ Ghz. Exploiting these data, we build a parameterized macromodel of order $n = 28$, parameterizing the residues of the numerator and of the denominator with Bernstein polynomials of degree 3 and 2 respectively.

We build two different models, one obtained by solving (21) and (25), the second by solving (30) with $j = 4$ in place of (25). The subdomains Θ_j are obtained by subdividing Θ in four adjacent intervals of equal length. Using a standard workstation, the models generation took 18 s and 30 s respectively for the former and the latter case.

In Figure 1 we report the trend of the real and imaginary parts of the transmission coefficient $S(2,1)$ of the PCB, compared with those of the resulting models, for 3 selected parameter values, $\vartheta = 200, 225, 250$ μm. As expected, the model obtained via the domain subdivision strategy is more

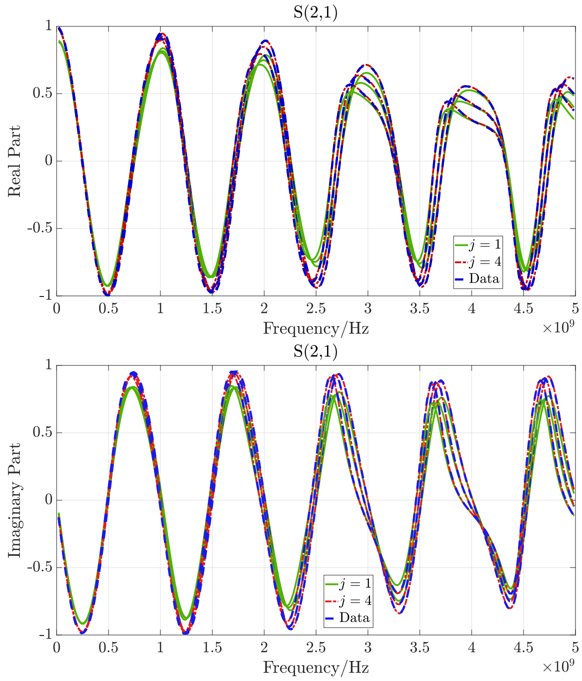

Fig. 1: The real (top panel) and the imaginary (bottom panel) components of the PCB transmission coefficient, compared with those of two macromodels obtained by applying the proposed modeling approach. The results show that the accuracy of the resulting models is effectively improved by applying the conservativity reduction strategy presented in Section IV (case $j = 4$).

accurate than the one obtained by solving (25). In particular, for the case $\vartheta = 200\ \mu$m, the constraint refinement strategy improves the average model error on the real part of $S(2,1)$ from 6.7×10^{-2} to 5.3×10^{-3}. The same metric on the imaginary part is improved from 6.8×10^{-2} to 4.4×10^{-3}. Similar results are obtained for $\vartheta = 225, 250\ \mu$m.

In order to test the robustness of the proposed approach, using the same strategy for the domain subdivision, me model the same structure in the bandwidth $[0.2, 10]$ GHz, using a order $n = 46$ for this broader band macromodel. In this case, the model generation took 210 s. The extremely accurate results of the fitting are shown in Figure 2, for all of the parameter configurations for which data are available.

VI. Conclusions

We presented a comprehensive framework for the data-driven generation of guaranteed stable and passive parameterized macromodels. Starting from a collection of samples of the reference system scattering matrix, we train the model applying a multivariate rational fitting scheme, augmented with suitable convex constraints. The latter are purely algebraic and guarantee the stability and/or passivity of the models for all the admissible parameters configurations, thanks to the particular adopted model structure and parameterization. This formulation allows to

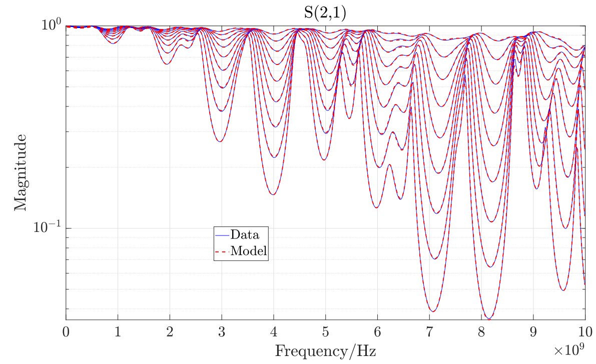

Fig. 2: Comparison between the magnitude of the PCB reflection coefficient and that of a passive broadband macromodel of order $n = 46$.

cast the constrained fitting problem into a sequence of semi-definite programs, that can be solved deterministically using off-the-shelf convex optimization solvers. Finally, we have shown that these constraints allow a user-defined trade-off between accuracy and efficiency, thanks to a conservativity reduction strategy based on a partition of the parameter space.

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