Implementation into OpenSees of XFEM for Analysis of Crack Propagation in Brittle Materials

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Abstract. Fracture propagation simulations by means of the traditional Finite Element Method require progressive remeshing to match the geometry of the discontinuity, which heavily increases the computational effort. To overcome this limitation, methods like the eXtended Finite Element Method (XFEM), in which element nodes are enriched through the medium of Heaviside step function multiplied by nodal shape functions, may be used. The addition of a discontinuous field allows the full crack geometry to be modelled independently of the mesh, eliminating the need to remesh altogether. In this paper OpenSees framework has been used to evaluate crack propagation in brittle materials by means of the XFEM method. Two shell-type XFEM elements have been implemented into OpenSees: a three-node triangular element and a four-node quadrangular element. These elements are an improvement of the elements with drilling degrees of freedom lately suggested by the Authors [6]. The implementation of XFEM elements implied some major modifications directly into OpenSees code to take into account the rise of number of degrees of freedom in the enriched element nodes during the analysis. The developed XFEM elements have been used to evaluate crack propagation into a plane shell subject to monotonically increasing loads. Moreover, with due tuning, the modified XFEM OpenSees code can be used to study also other problems such as material discontinuities, complex geometries and contact problems.

Keywords: Finite Element Method, Extended Finite Element Method, Fracture Mechanics, Discontinuities, Enrichment.

1 Introduction

The eXtended Finite Element Method (XFEM) is a robust technique for analysing problems with discontinuities and singularities, such as material discontinuities, and fractures. It was initially proposed by Belytschko and Black [1], and was subsequently enhanced by Mos, Dolbow, and Belytschko [2].

In this formulation, the discontinuous displacement field along the surface of a fracture is modelled using extra nodal degrees of freedom and enrichment shape functions.

Unlike the traditional Finite Element Method, XFEM enables mesh definition independent of discontinuity position and does not require mesh refining close to discontinuities. This is a significant benefit. In addition, when utilising XFEM to solve problems involving crack propagation, remeshing is not required to follow the evolution of the fracture, which drastically reduces the computational cost.

Enrichment shape functions are non-differentiable and discontinuous, thus numerical issues might occur if the stiffness matrix of components with discontinuities is evaluated using a quadrature method (such as Gauss-Legendre quadrature). Partitioning these components into sub-elements will enable the integrands to be continuous and differentiable inside each subdomain, solving the issue. Ventura [3], [4], [5] has also provided an alternative method based on equivalent polynomials that does not require the integration domain to be partitioned.

This paper outlines the implementation of three-node triangular and four-node quadrilateral XFEM shell elements in OpenSees. These elements are an improvement over the recently presented finite elements with drilling degrees of freedom by the Authors [6]. The presented elements can describe fracture propagation in brittle and quasi-brittle materials and have been utilised to perform static analysis with incremental load on planar shells.

2 XFEM formulation overview

Based on the Finite Element Method (FEM), the Extended Finite Element Method (XFEM) is a numerical method built specifically for addressing discontinuities [1], [2]. In conventional FEM, the displacement field of a single element inside a domain Ω is given by:

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{i=1}^{n} N_i(\boldsymbol{x})\, \boldsymbol{u}_i = \boldsymbol{N}^T(\boldsymbol{x})\, \boldsymbol{u} \tag{1}$$

where $N_i(x)$ are the element shape functions, n is the number of nodes of the element and $\boldsymbol{u}_i$ are the components of the nodal displacement.

Eq. (1) is incapable of describing the displacement field behaviour when the element embeds discontinuities or singularities. To circumvent this restriction, one can enrich the interpolation on Eq. (1) using an enrichment function $\Psi(\boldsymbol{x})$ and a certain number of extra degrees of freedom $\boldsymbol{a}_i$:

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{i=1}^{n} N_i(\boldsymbol{x})\, \boldsymbol{u}_i + \sum_{i=1}^{n} N_i(\boldsymbol{x})\, \Psi(\boldsymbol{x})\, \boldsymbol{a}_i \tag{2}$$

The type of enrichment function $\Psi(\boldsymbol{x})$ being used is defined by the discontinuity being described. For instance, the most appropriate enrichment function to describe strong discontinuities (such as cracks), is the Heaviside step function:

$$\Psi(\boldsymbol{x}) = H(\varphi(\boldsymbol{x})) \tag{3}$$

$$H\big(\varphi(\boldsymbol{x})\big) = \begin{cases} -1 & \varphi(\boldsymbol{x}) < 0 \\ 1 & \varphi(\boldsymbol{x}) > 0 \end{cases} \tag{4}$$

where $\varphi(\boldsymbol{x})$ defines the discontinuity signed distance from the point of evaluation.

In the event of pronounced non-polynomial behaviour of the solution, XFEM formulation permits the representation of discontinuities or singularities in an appropriate manner and with high performance.

Due to the non-polynomial form of the enrichment function, the usual Gauss quadrature method cannot be employed if the element is intersected by a discontinuity. This issue may be resolved by dividing the integration domain Ω into two subdomains, Ω^+ and Ω^- , along the discontinuity, such that the enrichment function is continuous and differentiable inside each subdomain:

$$\int_{\Omega} \Psi(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega \Rightarrow \int_{\Omega^+} \Psi(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega + \int_{\Omega^-} \Psi(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega \tag{5}$$

($\wp(\boldsymbol{x})$ is a generic polynomial function, such as a stiffness matrix term).

Subdomain partitioning incorporates a 'mesh' requirement into the elegance of the XFEM formulation. Ventura proposed a technique to eliminate the necessity of sub-cell definition without adding any approximation in the quadrature using equivalent polynomials [3], [4]. It has been proven that there exists an equivalent polynomial function whose integral provides the precise values of the discontinuous/non-differentiable function integrated on sub-cells. The polynomial is defined over the whole element domain so that it may be simply integrated using Gauss quadrature without the need to create quadrature sub-domains. Eq. (5) then becomes:

$$\int_{\Omega^+} \Psi(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega + \int_{\Omega^-} \Psi(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega = \int_{\Omega} \widetilde{\Psi}(\boldsymbol{x})\, \wp_n(\boldsymbol{x})\, d\Omega \tag{6}$$

where $\widetilde{\Psi}(\boldsymbol{x})$ is the equivalent polynomial function. This approach avoids the splitting of the quadrature domain by doubling the polynomial degree of the integrand function.

3 OpenSees implementation

The flexibility of the OpenSees framework eased the implementation process. To dynamically vary the quantity of degrees of freedom of each node throughout the analysis, it was necessary to define a new *'node'* Class as an alternative to the original in the code.

This is a critical component for satisfying the displacement field solution in Eq. (2).

The introduction of a new Class of nodes enabled the further implementation of two new XFEM plane shell-type elements: a triangular element with three nodes and a quadrilateral element with four nodes. In this initial implementation, the in-plane behaviour of the proposed elements is indefinite linear elastic for compression and elastic-fragile for tension. Out-of-plane behaviour is considered linear elastic.

The proposed elements can embed a pre-existing fracture. A crossing point and the normal to the discontinuity itself can be used to define it. The elements might alternatively be described as initially undamaged; in this instance, however, they might fracture as a result of progressive loading.

The element will fracture when the main tensile stress exceeds the material tensile resistance. Utilizing the coordinates of the element point where this limit is surpassed, the fracture position is determined.

The presented elements can also be utilised to investigate how fractures originate and proceed to spread in brittle and quasi-brittle materials. The displacement field of modelled elements is defined by the interpolation law in Eq. (1) if they are not damaged and by the one in Eq. (2) if cracking occurs. As a result, the nodes will have more degrees of freedom as the analysis goes further since the enriched degrees of freedom $\boldsymbol{a}_i$ will be added to the standard degrees of freedom $\boldsymbol{u}_i$.

At the current implementation stage, each mesh element can only manage a single discontinuity. Elements capable of handling multiple discontinuities have been analysed and will be implemented in future versions. A non-linear compressive behaviour for the elements will also be included.

4 Numerical applications

Plane shells with discontinuities have been analysed using the presented XFEM elements.

4.1 Damaged cantilever beam

In the first case, the cantilever beam in Fig. (1) is examined. The beam embeds a fracture and two forces are applied to its free end: cross-sectional force F_y and axial force F_x.

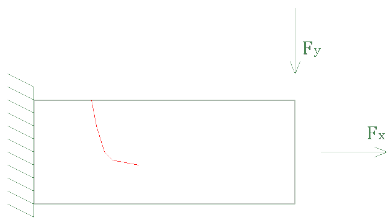

Fig. 1. Cracked cantilever beam subject to forces F_x and F_y. Crack is highlighted in red.

In Tab. (1) are illustrated the geometrical and mechanical properties of the beam.

Table 1. Beam geometrical and mechanical properties

Geometrical properties	$L = 50[cm]$	$b = 2.5[cm]$		$h = 20[cm]$
Mechanical properties	$E = 2796\left[\frac{kN}{cm^2}\right]$	$\nu = 0.2$	$F_x = 20[kN]$	$F_y = 20[kN]$

The beam has been modelled using both conventional FEM shell-type elements and the proposed XFEM shell-type quadrilateral elements.

Fig. (2) depicts the discretisation mesh for the structural element. In the case of conventional FEM (Fig. (2b)), the mesh had to be modified near the fracture to match the geometry of the discontinuity, necessitating the employment of triangular and deformed quadrilateral finite elements.

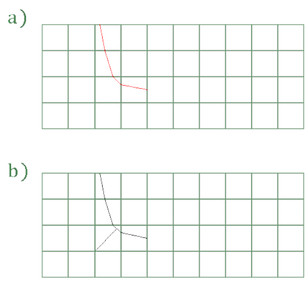

Fig. 2. Cantilever beam discretisation: a) XFEM discretisation; b) Standard FEM discretisation.

Fig. (3) illustrates the distorted configurations derived from the analysis. It is evident that both the standard FEM model and the XFEM model provide identical displacement values. Tab. (2) indicates that point A deflection is same in both models.

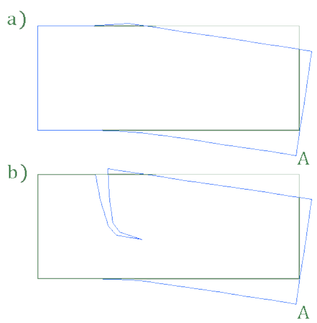

Fig. 3. Cantilever beam deformed configurations: a) XFEM discretisation; b) Standard FEM discretisation.

Table 2. Point A deflection

Standard FEM Model	Proposed XFEM Model
$u_x = 0.00037\ [cm]$	$u_x = 0.00037\ [cm]$

$u_y = 0.00177\ [cm]$	$u_y = 0.00177\ [cm]$

The proposed elements are validated by these outcomes.

4.2 Undamaged cantilever beam – progressive cracking

In the second example, an undamaged cantilever beam is examined. In the condition of tensile stress, the behaviour of the beam is considered to be linearly elastic, and in the condition of compression, it is assumed to be elastic-fragile. The tensile resistance of the material is defined as $f_t = 1.5\ \left[\frac{kN}{cm^2}\right]$.
In this scenario, an analysis has been conducted using a cross-sectional load F_y that increases monotonically. The outcomes up until the point of collapse are depicted in Fig. (4).

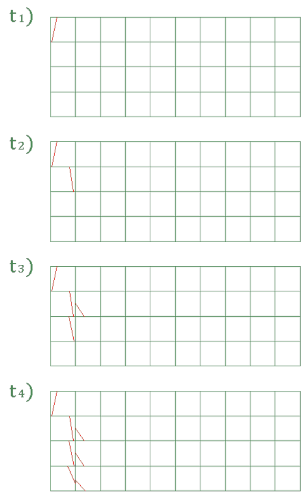

Fig. 4. Cantilever beam subjected to a cross-sectional load that increases monotonically. As the stress increases with each time-step until failure, progressive cracking occurs.

Notably, the objective of the XFEM model is not to precisely track the crack propagation path, but rather to evaluate the displacement field of a structural element susceptible to progressive cracking. Thus, the fracture pattern depicted in Fig. (4) is illustrative only.

4.3 Undamaged hinged beam – progressive cracking

The hinged beam in Fig. (5) has been examined.

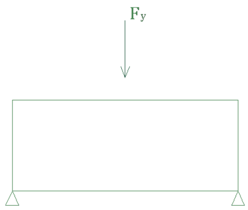

Fig. 5. Undamaged hinged beam subjected to a cross-sectional load F_y.

As in the last example, the beam is subject to a force that increases monotonically over its centreline. The outcomes of the analysis are illustrated in Fig. (6).

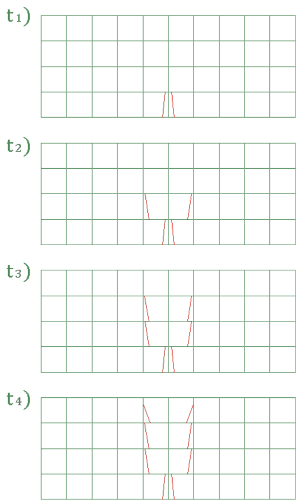

Fig. 6. Progressive cracks develop when the stress increases with each subsequent time step until failure occurs.

5 Conclusions

This paper presents two shell-type XFEM elements: a triangular element with three nodes and a quadrangular element with four nodes. These elements have been incorporated into OpenSees in order to assess the propagation of cracks in brittle and quasi-brittle materials. The presented XFEM elements are an improvement on the recently disclosed finite elements with drilling degrees of freedom by the Authors [6]. The implementation of the proposed XFEM elements implied some major modifications directly into the OpenSees code to take into account the rise of number of degrees of freedom in the enriched element nodes during the analysis. Utilizing the suggested XFEM elements, fracture propagation in a planar shell subject to monotonically rising stresses was evaluated. The numerical applications findings support the presented formulation and enable the usage of the Open-Sees framework to solve fracture mechanics problems. Future enhancements for the presented elements will include the ability for a single XFEM element to handle various discontinuities and a nonlinear compressive behaviour.

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