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Influence of Slenderness on the Evaluation of Epistemic Uncertainty Related to Non-Linear Numerical Analysis of RC Columns

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Abstract. This investigation is devoted to quantify the epistemic uncertainty related to the non-linear analysis of reinforced concrete columns characterized by high slenderness using numerical codes. The adoption of refined numerical tools, which are able to consider both mechanical and geometric non linearities, implies to perform assumptions and approximations with respect to reality. Whit reference to reliability analysis, these simplifications lead, inevitably, to additional uncertainties which are of epistemic nature. In fact, these uncertainties may be reduced by the engineers/analysts by increasing the level of refinement of the numerical model and/or increasing knowledge about parameters associated to material models. However, also numerical model established by expert engineers/analysts are affected by this kind of epistemic uncertainty. Accepting that the level of uncertainty associated to the experimental tests set are minimized, the epistemic uncertainty associated to non-linear numerical simulations can be quantified characterizing the model uncertainty random variable comparing the outcomes of numerical results to the associated experimental ones. The present investigation proposes the quantification of the model uncertainty related to non-linear numerical simulations of slender RC columns. A total number of 40 experimental results known from literature are herein selected in coherence with current Eurocodes specifications. The experiments are reproduced adopting non-linear numerical analysis differentiating between several modelling hypotheses (i.e., numerical code; materials models). The comparison between experimental and numerical results is adopted to characterize the most suitable probabilistic model for the model uncertainty random variable associated to non-linear numerical simulations of RC columns subjected to significant slenderness. The outcomes of the research are useful to provide background to the characterization of partial safety factor for model uncertainty in non-linear numerical analysis using the approach of the global resistance format for safety verifications.

1. Introduction

The adoption of non-linear analysis (NLA) in the civil engineering field it is becoming common practice for designers, analysts and researchers [1]-[3]. The effect of non-linearities related to both material and geometric characteristics can be easily accounted for within the assessment or design of reinforced concrete (RC) members and structures, also with reference to seismic safety issues [4]-[8]. Moreover, the contribution of materials of different nature can be included within the analysis with the aim to estimate the overall structural resistance [9]-[10]. For instance, the quantification of the uncertainties associated to NLAs of RC structures and components is a relevant topic. In this framework, the characterization of the uncertainties of both epistemic and aleatory nature related to NLAs of RC structures have been widely investigated in the last years [11]-[16]. Whit reference to reliability analysis [17], the simplifications adopted within the definition of numerical models lead, inevitably, to additional uncertainties which are of epistemic nature. In particular, these uncertainties may be reduced by the engineers/analysts by increasing the level of refinement of the numerical model and/or increasing knowledge about parameters associated to material models. However, also numerical models established by expert engineers/analysts are affected by this kind of epistemic uncertainty. With reference to the mentioned above issues, the characterization and quantification of the model uncertainties (which can be classified as epistemic ones) associated to NLAs of RC members subjected to significant slenderness have not yet been not carried out.

For instance, the present investigation proposes the quantification of the model uncertainty related to non-linear analysis of slender RC columns. A total number of 40 experimental results known from literature are herein selected in coherence with current Eurocodes specifications. The experiments are reproduced adopting non-linear numerical analysis differentiating between several modelling assumptions [13]-[14] (i.e., numerical code; materials models). Then, the comparison between experimental and numerical outcomes is adopted to characterize the probabilistic model for the model uncertainty random variable ϑ associated to non-linear numerical simulations of RC columns subjected to significant slenderness ratio. The outcomes of the research are useful to provide background to the characterization of partial safety factor for model uncertainty in non-linear numerical analysis using the approach of the global resistance format for safety verifications [18].

2. Test cases and numerical modelling

In order to perform the quantification of modelling uncertainty for NLAs of reinforced concrete columns, the results of 40 laboratory tests realized according to different experimental campaigns [19]-[27] has been considered for comparison with the outcomes of appropriate numerical simulations. The set of experimental results has been selected in line to the provision and limitation of [28] with reference to geometry, detailing (i.e., shear reinforcements, reinforcement ratio ϱ_l) and material strengths (i.e., concrete cylinder compressive strength f_c and reinforcement yielding strength f_y). The specifications of the tests set of slender RC columns selected for the investigation can be acknowledged in Table 1. The configuration of the laboratory tests [19]-[27] with reference to restraints and loading configuration is reported in Figure 1 differentiating between different types (i.e., A, B, C and D) of test set arrangement.

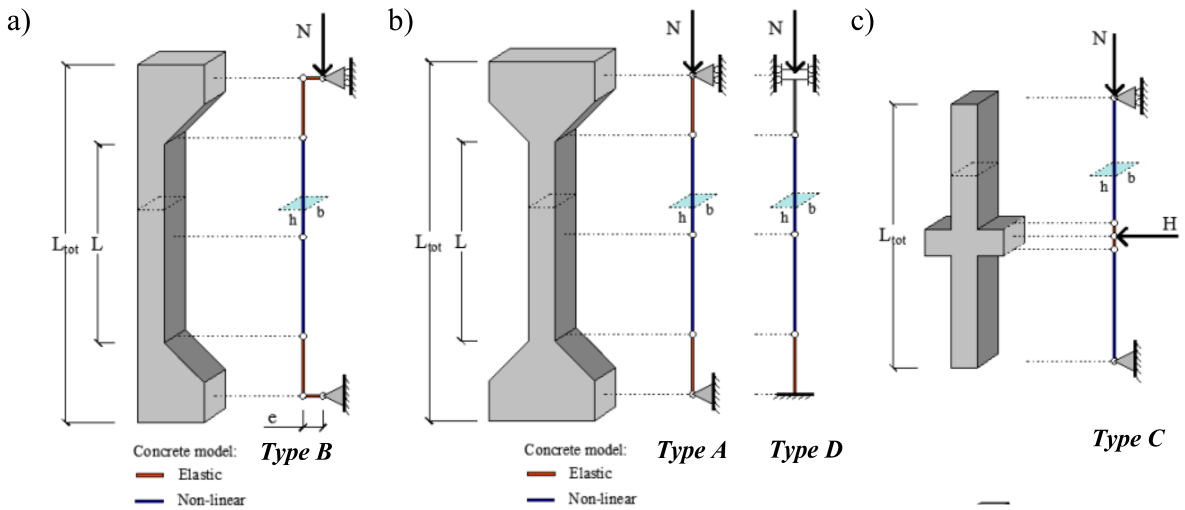

Figure 1. Test configuration for the different experimental investigations.

Table 1. Test cases considered for probabilistic calibration of model uncertainty with materials and geometrical properties, eccentricity of axial load *e* and experimental resistance R_{Test}.

Ref. [*]	Exp. test	Test set	L_{tot} [mm]	L [mm]	b [mm]	h [mm]	λ [-]	f_c [MPa]	f_y [MPa]	$\varrho_l = A_{sl}/A_c$ [%]	e/h [mm]	R_{Test} [kN]
[21]	**2L20-30**	*B*	1450	650	150	150	15	40.0	480.0	2.0	0.133	750.0
	2L20-60							43.0			0.133	700.0
	2L8-120R							56.0			0.053	1092.0
	4L8-30							43.0		4.1	0.053	1100.0
	4L20-120							40.0			0.133	900.0
	4L8-120R							56.0			0.053	1247.0
[24]	**C000**	*A*	680	600	120	120	17	27.0	347.2	4.0	-	559.6
	C020	*B*									0.200	327.3
	B020		1880	1800			52	27.6	355.0		0.200	271.5
	RL300		3000	2920	120	180	56	31.0	360.9	2.6	0.167	474.3
[25]	**A-17-0.25**	*B*	3400	2800	300	200	48	38.2	493.0	3.3	0.250	1181.4
	C-31.7-0.25		3800	3260	200	120	94	44.4	520.0	3.4		333.4
[19]	**3.3**	*B*	3400	2700	254	159	59	35.3	509.9	1.1	0.082	782.6
	5.1							40.6	426.8	3.1	0.165	735.5
	4.1		4500	3800	253	150	88	40.5	509.9	1.2	0.163	367.7
[26]	**N30-10.5-C0-3-30**	*C*	3300	2940	140	150	68	29.5	538.0	3.2	-	16.6 (280)*1
	H60-10.5-C0-1-30							58.5	531.0	1.4		17.2 (412)*1
[27]	**III**	*A*	3210	3000	140	140	74	16.1	294.2	1.4	-	343.2
	Va		3240		178	140		26.4	281.8	1.6		684.5
	2		3230	3010	250	125	83	33.5	304.0	0.6		235.4
	I		3210	3000	200	100	104	15.2	294.2	1.6		264.8
	VI		3000		198	98	106	24.9	294.2			392.3
	15		6510	6310	247	161	136	33.0	294.2	0.8		549.2
	3				250	160	137	33.5	294.2			666.9
	8	*B*	3230	3010	250	126	83	20.4	304.0	0.6	0.200	235.4
	9		6510	6310	250	162	135	24.5	294.2	0.8		205.9
	12							29.7	294.2		0.300	112.8
	6				250	160	137	32.2	294.2		0.200	225.6
[20]	**24D-2**	*D*	2697	2697	127	90	104	20.8	247.5	2.5	-	198.4
	15E-2	*A*	3597	3597			139	20.1	247.5			161.0
[23]	**S28**	*B*	5000	5000	104	104	167	24.4	304.0	4.2	0.144	44.0
	S30							25.7	300.0			48.0
	S25		6000	6000			200	24.7	282.0			36.0
[22]	**5**	*A*	6004	6004	100	100	208	33.1	278.5	4.5	-	72.7
	6							35.6				72.2
	17A		4940	4940	76	76	225	25.8	300.4	5.4		31.9
	20		5327	5327			243	38.1				37.9
	18							38.2				33.9
	8		6004	6004			274	36.5				31.9
	7							39.3				29.9

(-)*1 Compressive axial load constantly applied to the RC column for the entire duration of the test.

As shown by Table 1, a significant range of slenderness values λ have been accounted for in order to perform comprehensive quantification of the model uncertainty and understand their influence in reliability analysis of slender RC structural components.

The 40 laboratory tests of [19]-[27] have been reproduced by means appropriate NLAs adopting nine different modelling assumptions according to the approach of [13]-[14]. In particular, the assumptions and decisions performed by the analysis during the definition of a specific numerical model (i.e., related to constitutive models, solution methods for non-linear system of equations, kinematic compatibility of displacements, type of finite elements with related formulation, convergence criteria) are able to affect the global level of uncertainty.

For instance, in this paper, the software ADINA [29], TNO DIANA [30] and OpenSees [31] have been adopted to perform NLAs of the 40 RC columns (denoted generally as program I, II, III) . Moreover, 3 assumptions related to concrete tensile behavior have been adopted in line to [13]. Totally, 9 numerical models have been defined for each RC member selected from [19]-[27]. The models 1-3, 4-6 and 7-9 refers to the programs I, II and II, respectively, while, for each program, the first and the last models are related to elastic-brittle and plastic tensile behavior of concrete.

Table 2. Assumptions for numerical modelling of the tested RC slender components.

Program	I	II	III
Solution methods	- Method for solution of non-linear equations: full Newton-Raphson; - Equilibrium of forces evaluated in each numerical iteration with reference to the deformed configuration (2^{nd} order effects); maximum number of iterations: 100; - Displacements-based criteria for convergence of load steps (tolerance between iterations to reach convergence 1%); - Loading steps evaluated in line to laboratory loading sequence;		
Finite elements, Mesh	- 2 nodes 1D elements line to [29],[30] (force-based approach for fiber beams elements [31]); - Mesh dimension determined in order to achieve stability of numerical solution;		
Constitutive relationships for materials	*Concrete:* - Model for plane and confined concrete in compression defined with reference [32]; - Tensile behavior of concrete simulated with 3 hypotheses: 1) Elastic – Brittle; 2) Elastic - linear tension softening (i.e., LTS); 3) Elastic – plastic; *Reinforcements:* - Elastic – plastic with strength hardening; Other material properties (i.e., Young's modulus, resistances, strains at failure, etc.) have been adopted according to [19]-[27] and in compliance to [28] in case of lack of data.		

In each numerical program (I, II, III), both geometrical and material non linearities has been defined including also the influence of confinement provided by stirrups [32]. The numerical simulations have been performed according to the tests loading sequence specified by the original references [19]-[27]. The details of the numerical models with the related assumptions (i.e., modelling assumptions) are summarized in Table 2.

3. Outcomes from NLAs and quantification of model uncertainty

This section reports the outcomes from the 360 NLAs performed on the 40 columns introduced above. The Table 3 summarize the results in terms of failure axial load with reference to experiments and the 9 numerical modelling assumptions.

Table 3. Ultimate resistance for RC columns with reference to modelling hypotheses 1-9.

Ref. [*]	Exp. test	Type	λ [-]	R_{Test} [kN]	R_{NLA} [kN]								
					1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
[21]	**2L20-30**	*B*	15	750.0	910.9	910.9	916.2	728.3	742.5	742.5	691.6	694.3	694.6
	2L20-60			700.0	978.8	978.8	985.5	734.8	735.8	744.0	736.4	736.4	739.5
	2L8-120R			1092.0	1587.0	1587.0	1587.0	1087.3	1090.9	1090.1	1152.7	1152.7	1152.7
	4L8-30			1100.0	1351.0	1351.0	1353.0	1110.9	1110.9	1110.9	1032.9	1032.9	1032.9
	4L20-120			900.0	1037.0	1037.0	1046.0	787.5	787.5	787.5	826.1	830.7	830.7
	4L8-120R			1247.0	1608.0	1608.0	1608.0	1211.3	1211.3	1211.3	1319.5	1319.5	1319.5
[24]	**C000**	*A*	17	559.6	611.6	611.6	611.6	545.1	545.1	545.1	560.6	560.6	560.6
	C020	*B*		327.3	396.0	396.0	402.9	319.9	336.9	338.0	325.7	328.5	329.0
	B020		52	271.5	298.6	298.6	308.6	219.2	227.2	227.2	257.0	263.7	263.7
	RL300		56	474.3	334.0	351.0	351.0	381.4	395.5	395.5	414.9	423.3	423.3
[25]	**A-17-0.25**	*B*	48	1181.4	1273.0	1273.0	1307.0	1322.1	1322.1	1346.4	1367.4	1367.4	1393.9
	C-31.7-0.25		94	333.4	207.4	219.6	219.6	224.8	262.8	262.8	248.4	280.1	280.1
[19]	**3.3**	*B*	59	782.6	827.3	827.3	835.9	787.5	787.5	809.4	856.4	856.4	866.5
	5.1			735.5	745.6	745.6	793.5	725.4	725.4	762.1	810.8	810.8	853.8
	4.1		88	367.7	297.7	346.8	346.8	210.9	403.0	403.0	397.5	391.7	455.7
[26]	**N30-10.5-C0-3-30**	*C*	68	16.6 (280)^{*1}	11.6	11.9	11.9	23.5	23.5	24.6	16.2	16.6	17.6
	H60-10.5-C0-1-30			17.2 (412)^{*1}	13.9	13.9	13.9	17.1	17.1	16.4	17.9	17.9	20.5
[27]	**III**	*A*	74	343.2	341.8	341.8	341.8	332.4	332.4	332.4	347.3	347.3	347.3
	Va			684.5	603.6	662.3	744.4	608.6	608.6	608.6	680.7	680.7	680.7
	2		83	235.4	197.4	217.7	217.7	191.2	224.5	224.5	216.2	236.8	247.3
	I		104	264.8	259.9	259.9	259.9	252.4	252.4	251.4	258.0	258.0	258.0
	VI		106	392.3	361.0	361.8	361.8	327.3	327.9	327.9	363.2	363.2	363.2
	15		136	549.2	509.4	509.4	509.4	394.4	394.4	394.4	560.3	560.3	560.3
	3		137	666.9	511.4	511.4	511.4	456.6	456.6	393.6	563.4	563.4	563.4
	8	*B*	83	235.4	197.4	217.7	217.7	191.2	224.5	224.5	216.2	236.8	247.3
	9		135	205.9	163.7	184.5	184.5	205.3	205.3	205.3	161.1	205.9	208.7
	12			112.8	153.5	153.5	172.5	114.7	114.7	151.5	115.2	112.2	176.8
	6		137	225.6	185.7	204.1	204.1	153.4	223.2	223.2	187.4	227.6	244.0
[20]	**24D-2**	*D*	104	198.4	184.6	184.6	184.6	188.0	192.1	192.1	192.8	192.8	192.8
	15E-2	*A*	139	161.0	121.4	121.9	121.9	127.0	130.0	130.0	129.3	129.3	129.3
[23]	**S28**	*B*	167	44.0	53.9	53.9	58.6	55.4	54.9	54.9	49.9	49.9	55.8
	S30			48.0	54.9	54.9	59.8	56.1	56.1	60.6	58.5	53.4	66.7
	S25		200	36.0	39.5	39.5	43.6	31.2	37.8	42.0	42.3	42.3	49.0
[22]	**5**	*A*	208	72.7	67.6	67.6	67.8	53.9	53.9	53.9	78.7	78.7	78.7
	6			72.2	70.3	70.3	70.6	52.0	52.0	52.0	82.3	82.3	82.3
	17A		225	31.9	32.7	32.7	32.8	20.1	26.0	24.1	37.1	37.1	37.1
	20		243	37.9	29.1	30.2	30.2	27.4	27.4	27.4	39.8	39.8	39.8
	18			33.9	29.1	30.2	30.2	24.4	24.4	24.4	39.8	39.8	39.8
	8		274	31.9	26.6	26.6	26.6	23.3	23.3	23.3	31.0	31.0	31.0
	7			29.9	27.6	27.6	27.6	20.6	20.6	20.6	32.3	32.3	32.3

(-)^{*1} Compressive axial load constantly applied to the RC column for the entire duration of the test.

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The outcomes from NLAs are useful to quantify the model uncertainty related simulation of response of slender reinforced concrete columns by means of comparison with experimental results. In line to [11],[13]-[14], the model uncertainty can be characterized addressing the following ratio:

$$\vartheta = \frac{R_{Test}}{R_{NLA}} \tag{1}$$

where R_{Test} is the result from laboratory test and R_{NLA} is the related observed result from NLA performed with specific modelling assumption. The model uncertainty ϑ is able to quantify the level of epistemic uncertainty associated to definition of the NL numerical model and is useful to include such kind of uncertainty within reliability analysis of RC structures.

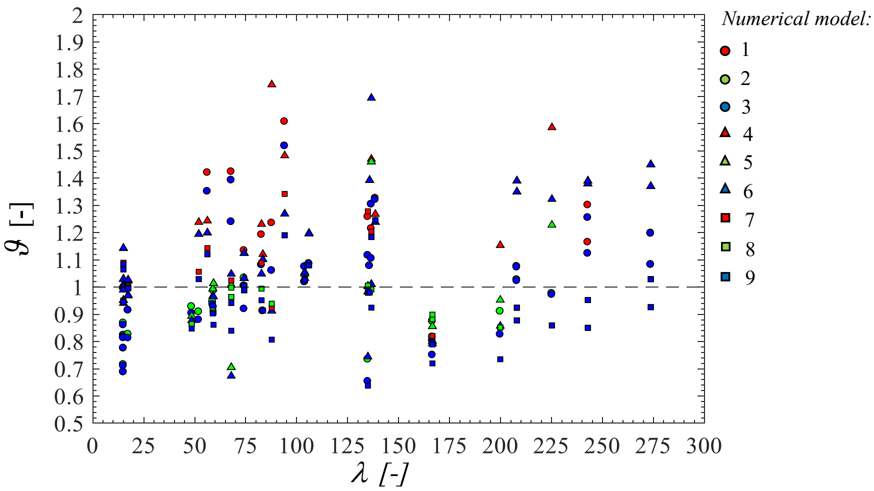

Figure 2. Summary of the ratios $\vartheta = R_{Test}R_{NLA}$.

4. Statistical assessment

The observed outcomes of model uncertainty ϑ presented in previous section should be appropriately evaluated from statistical point of view. In particular, the results associated to each modelling assumption has been evaluated by means Chi-Square goodness of fit test verifying that the lognormal probabilistic model is the most likely in order to reproduce the statistical variability of the random variable ϑ, also in line to [33]. For instance, see the lognormal fit and related frequency histogram reported in Figure 3 for the modelling assumption number 3.

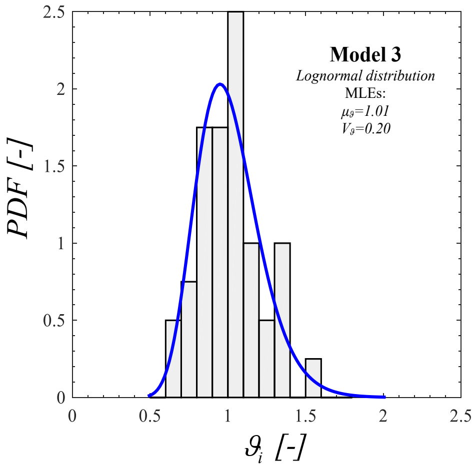

Figure 3. Relative frequency histogram and lognormal distribution PDF relate to the numerical model n°3.

The statistical parameters of lognormal probabilistic distribution for each modelling assumption has been evaluated using the maximum likelihood criteria [34] in order to minimize the statistical uncertainty in the estimates. The results in terms of mean value μ_ϑ and coefficient of variation V_ϑ are reported in Table 4 with reference to the nine modelling assumptions.

Table 4. Results in terms of mean value and coefficient of variation of model uncertainty for the different modelling assumptions.

Structural Model	*Statistical parameters*	
	μ_ϑ [-]	V_ϑ [-]
1	1.05	0.21
2	1.02	0.19
3	1.01	0.20
4	1.17	0.20
5	1.10	0.17
6	1.10	0.20
7	1.01	0.12
8	0.99	0.09
9	0.96	0.14

The results show that, in general, a slightly safe bias (i.e., mean value higher than 1.00) is present between the different assumptions while significant difference is present with respect to coefficient of variation that ranges between 0.09 and 0.21. In average, a value of 1.05 and 17% are recognised for the mean value μ_ϑ and the coefficient of variation V_ϑ, respectively.

5. Conclusions

The present study proposes the characterization of the model uncertainty related to non-linear numerical simulation of reinforced concrete columns that are realized with high slenderness ratio. 40 RC columns experimentally tested has been selected from literature and reproduced by means 9 modelling assumptions performing mechanical and geometrical non-linear analysis. The comparison between the experimental and numerical results allows to determine mean value and coefficient of variation of the model uncertainty random variable that, in general, can be represented by a lognormal probabilistic model. In particular, the mean value equal to 1.05 and coefficient of variation equal to 0.15 has been determined averaging the outcomes from the different modelling assumptions. The results are useful to allow to take into account the model uncertainty variable within the reliability analysis of slender reinforced concrete columns and also to determine appropriate partial safety factors for safety formats based on semi-probabilistic methods.

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