

Structural and seismic monitoring of historical and contemporary buildings: general principles and applications

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## Structural and seismic monitoring of historical and contemporary buildings: general principles and applications

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**Abstract.** *Structural Health Monitoring (SHM) indicates the continuous or periodic assessment of the conditions of a structure or a set of structures using information from sensor systems, integrated or autonomous, and from any further operation that is aimed at preserving structural integrity. SHM is a broad and multidisciplinary field, both for the spectrum of sciences and technologies involved and for the variety of applications. The technological developments that have made the advancement of this discipline possible come from many fields, including physics, chemistry, materials science, biology, but above all aerospace, civil, electronic and mechanical engineering. The first applications, at the turn of the sixties and seventies, concerned the integrity control of remote structural elements, such as foundation piles and submerged parts of off-shore platforms, but nowadays this type of monitoring is practiced on airplanes, vehicles spacecraft, ships, helicopters, automobiles, bridges, buildings, civil infrastructure, power plants, pipelines, electronic systems, manufacturing and processing facilities, and biological systems. This paper carries out an extensive examination of the theoretical and applicative foundations of structural and seismic monitoring, focusing in particular on methods that exploit natural vibrations and their use both in the diagnosis and in the prediction of the seismic response of civil structures, infrastructure networks, and traditional and modern architectural heritage.*

**KEYWORDS:** Structural Health Monitoring, System Identification, Structural Dynamics, Seismic Structural Health Monitoring, Architectural Heritage.

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**Riassunto.** *Nell'accezione comune, l'espressione inglese «Structural Health Monitoring» indica la valutazione continua o periodica delle condizioni di una struttura o di un insieme di strutture utilizzando le informazioni provenienti da sistemi di sensori, integrati o autonomi, e da qualsiasi ulteriore operazione che sia finalizzata a preservare l'integrità strutturale. Si tratta di un ambito ampio e multidisciplinare, sia per lo spettro di scienze e tecnologie coinvolte che per la varietà delle applicazioni. Gli sviluppi tecnologici che hanno reso possibile l'avanzamento di questa disciplina provengono da molti campi, tra cui la fisica, la chimica, la scienza dei materiali, la biologia, ma soprattutto l'ingegneria aerospaziale, civile, elettronica e meccanica. Le prime applicazioni, a cavallo tra gli anni sessanta e settanta, riguardavano il controllo dell'integrità di elementi strutturali inaccessibili, quali i pali di fondazione e parti immerse delle piattaforme off-shore, ma oggi questo tipo di monitoraggio viene praticato su aerei, veicoli spaziali, navi, elicotteri, automobili, ponti, edifici, infrastrutture civili, centrali elettriche, condutture, sistemi elettronici, impianti di produzione e lavorazione e sistemi biologici. La presente memoria svolge un'ampia disamina dei fondamenti teorici e applicativi del monitoraggio strutturale e sismico, soffermandosi in particolare sui metodi che sfruttano le vibrazioni naturali e sul loro impiego nella diagnosi e nello studio della risposta sismica di strutture civili, reti di infrastrutture, beni architettonici tradizionali e moderni.*

**PAROLE CHIAVE:** dinamica delle strutture, identificazione strutturale, monitoraggio strutturale, monitoraggio sismico, patrimonio architettonico.

Structural Health Monitoring (SHM) indicates the continuous or periodic assessment of the conditions of a structure or a set of structures using information from sensor systems, integrated or autonomous, and from any further operation that is aimed at preserving structural integrity. SHM is a broad and multidisciplinary field, both for the spectrum of sciences and technologies involved and for the variety of applications. The technological developments that have made the advancement of this discipline possible come from many fields, including physics, chemistry, materials science, biology, but above all aerospace, civil, electronic and mechanical engineering. The first applications, at the turn of the sixties and seventies, concerned the integrity control of remote structural elements, such as foundation piles and submerged parts of off-shore platforms, but nowadays this type of monitoring is practiced on airplanes, vehicles spacecraft, ships, helicopters, automobiles, bridges, buildings, civil infrastructure, power plants, pipelines, electronic systems, manufacturing and processing facilities, and biological systems. This paper carries out an extensive examination of the theoretical and applicative foundations

of structural and seismic monitoring, focusing in particular on methods that exploit natural vibrations and their use both in the diagnosis and in the prediction of the seismic response of civil structures, infrastructure networks, and traditional and modern architectural heritage.

## 1. Introduction

Over the past three decades, the interest for structural diagnosis has seen an unprecedented growth in many engineering fields (Doebeling *et al.* 1996, Boller 2009). Unfortunately, theoretical and experimental studies have seldom translated into established techniques for real-world applications.

A monitoring system is generally conceived to record the variations of parameters that are deemed to be enough sensitive to the presence of damage, in order to support diagnostic or even prognostic evaluations. In this sense, the first critical issue is the definition itself of structural damage, which may include very different phenomena, e.g. cracks, corrosion, delamination and debonding, fiber pullout, fiber breakage and matrix cracking, fretting in crevices, slips, loose joints and fasteners, creep, buckling, penetration and plastic deformation, weld defects, residual stresses, etc.

The traditional diagnostic evaluation methods are known to be affected by a large series of technical drawbacks. Visual inspections are generally not performed frequently enough, which risks affecting their predictive nature. Moreover, they are neither exhaustive, because they do not allow the hidden defects or the invisible effects of an on-going damage process to be detected, nor are they objective, because the estimation is related to the subjective judgment of an expert who can be fallible. More objective evaluation techniques are often of a destructive kind, not being suitable for many applications, such as those for Cultural Heritage (CH).

Non-destructive evaluation (NDE), however, can count on several established techniques, such as: radiography, ultrasonic testing, acoustic emission, vibration-based methods, optical methods, thermography, electromagnetic testing, magnetic particle inspection, etc. Within the NDE family, often Non-Destructive Testing (NDT) refers specifically to techniques that are performed off-line and only after the damage has been located (Shull 2002). This means that, in the meanwhile, an excessive level of deterioration could have been reached. Moreover, diagnostic evaluations in this case are performed in a local manner and so provide information that only refers to a limited portion of the structure.

Modern NDE, including Structural Health Monitoring (SHM) systems, has the prerogative of overcoming these limitations by providing an exhaustive

depiction of the structural health state and easing the plan of maintenance and repair interventions.

SHM is a combination of words that has emerged around the late 1980s (Boller 2009) in homology with medicine. Although it is based on innovative measures, analyses, algorithms and communication techniques, SHM shares the same objectives as traditional approaches. The distinctive features of the new approach is rather the following:

- (i) differently from NDT, global quantities are basically monitored, the guiding concept being that of non-local (or remote) monitoring (it is no coincidence that the first applications refer to foundation piles and remote parts of offshore structures);
- (ii) online implementation is possible, at least in principle.

Farrar and Worden (2012) defined SHM as a process which involves the periodic monitoring of a structure through measurements, the extraction of features symptomatic to the phenomena under investigation and their statistical analysis to determine the actual state of the system. In real practice, a SHM system is the result of the integration of several sensors, devices and auxiliary tools, such as: a measurement system; an acquisition system; a data processing system; a communication/warning system; an identification/modeling system; a decision making system. Even if it is based on innovative measuring, analyzing, modeling and communication techniques, SHM can be considered as an extension of the well-established investigation practices since it integrates these novel technologies into a unique smart system.

Vibration-based SHM techniques, in particular, have long been used for damage identification in existing structures (Briard 1970, Loland & Mackenzie 1975, Ceravolo & De Stefano 1996). However, many issues require further investigations and still represent challenges that have to be undertaken. Above all, a new philosophy should be pursued, which comprises the importance of a rational design of the monitoring system. It must integrate a sensors network that is capable of carrying out a continuous or periodic surveillance and providing reliable analyses based on different information sources. The environmental and operating condition variability must also be taken into account.

To take a prominent example, monitoring practices play a crucial role in CH conservation (Ceravolo *et al.* 2019), providing first-hand data for decision making. The continuous and global structural knowledge, as well as the widespread and accurate information about the structural performance and integrity that only a monitoring procedure can achieve, favors the implementation of preventive conservation and the realization of ready and targeted interventions, limiting cost, invasiveness and reducing the risk of incurring irreparable damage.

## 2. Different approaches to SHM

Whatever the monitoring approach and technique, when designing a SHM system it is necessary to perform preventively an accurate analysis of the structural behaviour, in order to monitor the most expressive and sensitive parameters. The approaches to tackling an SHM procedure can be divided into two main classes: *model-driven* methods and *data-driven* methods.

Data-driven methods exploit monitoring data and adopt Pattern Recognition (PR), Machine Learning (ML), or other heuristic techniques, to create a statistical representation of the system from them (Worden & Manson 2006). Usually, data-driven approaches necessitate a huge amount of information coming from permanent monitoring systems, or from simulations when the structure's dynamic behaviour can be easily identified and reproduced. Moreover, statistical models of the system are easily defined, and noise levels and environmental variations are established naturally.

The model-driven methods, instead, apply an inverse approach to a law-based model, commonly referring to the updating of a Finite Element (FE) model (Friswell 2007). This process involves adjusting some parameters of the model to reduce the residual between experimental measurements and model predictions; then simulations and tests on the updated model help to deduce the damage in the structure. Approaches that are driven by high-fidelity models of the structure can potentially work without a validated damage model, but noise and other environmental effects are difficult to incorporate.

Model-driven methods correspond to axiomatic thus more general formulations, so much so that they constitute the traditional approach to engineering problems. However, models are often characterized by a large number of parameters, and their settings must be carefully evaluated, thoroughly understanding the underlying physics, e.g. checking that the values of the “healthy parameters” always maintain a physical meaning, as well as those set to simulate damage. The latter are very difficult to validate. The inevitable presence of errors, since the model by definition is a simplification of reality, is another issue that plagues this approach. Model-driven methods are also computationally heavy in that require multiple runs of a FE model to make predictions. Choosing the parameters to be calibrated will always imply that those not subjected to the same process are characterized by uncertainty. It must also be considered that even the best model may not reflect reality due to variations on the latter, compared to the data used in the calibration, for example due to environmental effects. These issues become even more serious when a historical structure is to be modelled. The uncertainties about the materials and their characteristics, the unusual geometry, the lack of knowledge on the

connections, on the interventions undergone and on the present crack pattern, create difficulties in defining laws for a generalized application.

System identification is the core of any model-driven SHM approach. Indeed, identification techniques and algorithms are indispensable in order to produce a realistic model of a structure, especially in the case of uncertain material properties and ill-defined structural schemes. In permanent monitoring systems, varied or anomalous parameters can be associated to damage, and reliability can be defined as a function of identified quantities that reflect the damage, referred to as symptoms.

Alternatively, especially when the analysed buildings present a complex structural scheme, a numerical model can be updated on the grounds of the identified parameters in order to simulate the real behaviour of the structure and to overcome uncertainties. Since in typical structural problems safety assessment relies on mechanical models, the engineer is prone to basing any final evaluation, prognosis or decision on results coming from an updated model, rather than on symptoms (Ceravolo *et al.* 2019).

### **3. System Identification**

System identification refers to the development of structural models from input and output measurements performed on a real structure using sensing devices. Dynamic system identification is a major tool for monitoring and diagnosis of structures: experimental results from dynamic testing give knowledge about global structural behaviour and can be used in calibrating numerical models, in forecasting the response to dynamic and earthquake loading and can help in evaluating safety conditions (Natke *et al.* 1993, Ghanem & Shinozuka 1995, Maia & Silva 1998).

Even if the age of virtual prototyping has already started (Van Der Auwerker 2002), experimental testing and system identification still play a key role because they help the structural dynamicist to reconcile numerical predictions with experimental investigations. The term “system identification” is sometimes used in a broader context in the technical literature and may also refer to the extraction of information about the structural behaviour directly from experimental data, i.e., without necessarily referring to a model (e.g., identification of the number of active modes or the presence of natural frequencies within a certain frequency range).

#### *3.1. Linear system identification*

Linear system identification in its current formulation is a discipline that ideally was born within the control community (Ho & Kalman 1965, Aström &

Bohlin 1965) but has evolved considerably over the last 40 years (Soderstrom and Stoica 1989). Experimental modal analysis is by all means the most popular approach to performing linear system identification in structural dynamics. The modal model of the system is expressed in the form of modal parameters, namely natural frequencies, mode shapes and damping ratios. The popularity of modal analysis stems from its great generality; modal parameters can describe the behaviour of a system for any input type and any range of the input.

The field of linear system identification now offers a vast range of effective techniques. Over the years, time domain techniques have been used rather successfully, thanks to the great spectral resolution offered and to their modal uncoupling capability (Masri *et al.* 1982, Shinozuka *et al.* 1982, Natke & Yao 1986, Safak 1991, Safak & Celebi 1991, Peeters & DeRoek 1999, Loh *et al.* 2000, Ceravolo & Abbiati 2013). One of the basic shortcomings of these methods is that they often produce spurious modes, whose true nature, however, can usually be identified by means of simple modal form correlation indicators (Ewins 2000), or, as an alternative, with the aid of numerical models.

An important family of time domain methods makes use of time series autoregressive models and exploits the theoretical results coming from research in the field of system control (Ljung 1999). These techniques provide a very general and attractive formulation, and are frequently applied to civil structures. The most critical aspect resides in the computational complexity associated with applications to multi-degree-of-freedom (M-DOF) systems. The extension of the parameter estimation techniques to stochastic multi-variate models, in fact, is far from being trivial, and additional difficulties arise from local minimum points and algorithmic instabilities (Fassois & Lee 1993).

Among the deterministic methods, in addition to the historic Ibrahim Time Domain (Ibrahim & Mikulcik 1977), we should mention the Eigensystem Realisation Algorithm (ERA) (Juang & Pappa 1984), which, based on a Single Value Decomposition (SVD) of Hankel's matrix, has been closely studied in the literature (e.g. Lew *et al.* 1993), and the Polyreference Time Domain (PRTD) stemming from a generalisation of Prony's method (Vold 1982, Deblauwe & Allemang 1985).

Since the beginning of the nineties, there has been an increasing interest in so-called Stochastic Subspace Identification methods, in which statistical, algebraic and numerical concepts and algorithms cooperate, leading to user-friendly software for linear system identification (Zeiger & McEwen 1974, James *et al.* 1995, Peeters & DeRoek 1999). Contrary to classical algorithms, subspace algorithms do not suffer from the problems caused by a-priori parametrizations and non-linear optimizations. Van Overschee & DeMoor (1996) studied three different subspace algorithms for the identification of combined



deterministic-stochastic systems by stating a unifying theorem, of which the three algorithms are special cases.

For a description and classification of various input-output modal analysis techniques the reader may consult specialized texts (Heylen *et al.* 1997, Maia & Silva 1997, Ewins 2000). Unification of the theoretical development of modal identification algorithms has also been attempted, e.g. in Allemang & Brown (1998) and Allemang & Phillips (2004), this being a sign of the maturity of this research field.

Different is the situation with modal analysis algorithms that, being conceived to work with output data (output-only or input-unknown techniques), are of special interest for structures exposed to natural vibration (bridges, towers, buildings etc.). These issues in the nineties gave rise to a new research area, officially inaugurated in a special session of IMAC-XIV organised by Felber & Ventura (1996), which today is often referred to as “operational” modal analysis (e.g. Cunha & Caetano 2007, Brincker & Moller 2007, Reynder *et al.* 2008, Giraldo *et al.* 2009). In ambient vibration conditions, there is still a need to determine to what extent the use of these techniques in non-ideal conditions, as is in the typical case, is deemed acceptable, or whether it proves necessary to resort to techniques specially conceived for dealing with non-stationarity. Inherently non-stationary techniques include stochastic approaches (e.g. Yuen *et al.* 2002), time-frequency instantaneous estimators (e.g. Ceravolo 2004) or time-varying estimators (e.g. Poulimenos & Fassois 2006, Du & Wang 2009).

### 3.2. *Non-linear system identification*

Though the word non-linearity has a tautological character, a classification of possible sources of non-linearity is of practical interest in structural identification. A drawback in using this term in a survey is rather due to the vast range of problems and techniques that deserve a coverage (e.g. Adams & Allemang 1998, Doebling 2001). A first category includes identification methods using various strategies to by-pass non-linearity. Other methods can be framed respectively in the parametric and the non-parametric approach: in the former case, a priori selection of a specific model for the dynamic behaviour of the system is needed and the identification process consists of determining the coefficients for such model. Non-parametric methods, instead, do not require any assumption on the type and localisation of structural non-linearities but, generally, the quantities identified cannot be directly correlated to the system equation of motion.

### 3.2.1. By-passing non-linearity

Traditional techniques for analysing the dynamics of non-linear structures are based on the assumptions of weak non-linearities and of a non-linear modal structure that is similar or a small perturbation of the underlying linearised system.

Caughey (1959) proposed replacing a nonlinear oscillator with external Gaussian excitation with a linear one with the same excitation so that the mean square error between the actual nonlinear and linearized systems was statistically minimized. The procedure, known as equivalent linearization, works directly on the equations of motion. Many developments have been proposed after Caughey's work (Roberts & Spanos 1990).

This widespread approach has proved useful in most applications, particularly for the random vibration analysis of systems where the nonlinear restoring force is hysteretic. For experimental applications, the extraction of a linear model requires the knowledge of the functional form of the restoring force, which is generally not the case. Hagedorn & Wallaschek (1987) have developed an effective experimental procedure for doing precisely this. This work triggered the development of the concept of equivalent linear systems with random coefficients which has enjoyed some success for system identification of nonlinear systems (Soize & Le Fur 1997, Bellizzi & Defilippi 2003).

The harmonic balance method described by Nayfeh & Mook (1995) can be also employed for linearising nonlinear equations of motion with harmonic forcing. This method has been the basis of several nonlinear system identification techniques (Yasuda *et al.* 1988, Benhafsi *et al.* 1992, Meyer *et al.* 2003, Ozer *et al.* 2005, among others).

For multi-degree-of-freedom (MDOF) systems, a suggestive way to make a transition between linear and non-linear dynamics is through the extension of the normal mode concept of classical linear vibration theory to non-linear systems. Under particular conditions, the concept of nonlinear normal mode (NNM) was introduced by Rosenberg (1966) and developed by Vakakis (1997). The identification of individual NNMs may represent a limitation when considering the arbitrary motion of a non-linear system; in this case, the NNMs are bound to interact. Several authors have used other types of non-linear modes for the identification of non-linear systems from free vibration (Bellizzi *et al.* 2001, Hasselman *et al.* 1998, Hemez & Doebling 2001).

### 3.2.2. Parametric approaches

Apart from linearization techniques, which are usually parametric, a very straightforward strategy to obtain a typically parametric identification algo-

rithm is to extend the use of time series models (Ljung 1999) to non-linear systems. A suggestive extension is represented by NARMAX (Nonlinear ARMA with eXogeneous input) model proposed by Leontaritis & Billings (1985). The NARMAX structure is general enough to admit many forms of model including neural networks, although the estimation problem becomes non-linear and the orthogonal estimator will not work (Billings *et al.* 1991). In fact, the application of NARMAX to structures is extremely complex and no relevant applications to real structures are reported to date.

### 3.2.3. *Non-parametric approaches*

The Volterra series representation of the input/output relationship is one of the main tools for the study of weakly non-linear systems. In this theoretical framework the problem of identification boils down to the determination of higher order frequency response functions in the frequency domain, or higher order impulse response functions in the time domain, from experimental data. Usually, the methods based on the Volterra series representation are classified as non-parametric, as are all those that make use of non-linear functionals.

The structures can be tested applying loads deterministic (i.e. stepped-sine test) or stochastic in nature. In the latter case, there is quite an extensive literature about the techniques for identifying Volterra systems: one of the first attempts to determine the linear and quadratic frequency response functions of a quadratic system was performed by Tick (1961), under the assumption that system excitation is white Gaussian noise. The hypothesis of Gaussianity greatly simplifies the problem of identification but can lead to unrealistic results. This difficulty has been overcome with the formulation of identification methods which can be applied in conditions characterised by excitation with arbitrary spectral properties, defined both in the time domain (e.g. Koukoulakis & Kalouptsidis 2000) and in the frequency domain (e.g. Kim & Powers 1993). All these methods require the calculation of higher order statistical moments: in structural engineering applications it is not possible to obtain a number of experimental measurements large enough as necessary to get a consistent estimate of the statistical moments of interest. The availability of a limited number of experimental data can be obviated through the time-frequency representation of the signals and the definition of instantaneous estimators of the mechanical properties to be identified (Demarie *et al.* 2005).

It is worth underlining that the vast majority of identification techniques, especially non-parametric ones, admit heuristic extensions. In this regard, it

is necessary to mention neural networks, for their characteristics of universal approximation, and neuro-fuzzy models, for their semantic transparency (e.g. Juditsky *et al.* 1995, Sjöberg *et al.* 1995, Chassiakos & Masri 1996, Kosmatopoulos *et al.* 2001, Le Riche *et al.* 2001, Song *et al.* 2004, Liang *et al.* 2001, Fan & Li 2002).

### 3.2.4. Approaches based on instantaneous estimation

This class of methods was already considered in the 1960s for problems in acoustics and vibrations (Priestley 1967), but it is only from the 1990s that it gained widespread popularity within the structural dynamics community. A survey of the analysis of non-stationary signals using time-frequency methods is available in Hammond and & (1996), Hammond and Waters (2001), and Ceravolo (2009).

Feldman showed how to use the traditional definition of the analytic signal and the time-domain Hilbert transform in order to identify nonlinear models of SDOF systems. The FREEVIB approach proposed in (Feldman 1994a) is based on free vibration whereas the FORCEVIB approach proposed in (Feldman 1994b) deals with forced vibration. These approaches can be used to construct the instantaneous damping and stiffness curves for a large class of nonlinear systems, but are only suitable for monocomponent signals (Feldman 2007). A popular method for the decomposition of signals with multiple components into a collection of monocomponents signals, termed intrinsic mode functions (IMFs), is due to Huang *et al.* (1998) and is now referred to as Huang-Hilbert transform in the time-frequency literature. The IMFs are constructed such that they have the same number of extrema and zero-crossings, and only one extremum between successive zero-crossings. As a result, they admit a well-behaved Hilbert transform. The method has seen several applications to structural dynamics including linear system identification (Yang *et al.* 2003) and damage detection (Yang & Lin 2004).

The time-frequency representation is also suitable for the analysis of non-linear oscillations. Linear representations have been used for instance by Spina *et al.* (1996), Abbiati *et al.* (2013), Miraglia *et al.* (2021). An overview of the use of the wavelet transform in nonlinear dynamics can be found in Staszewski (2000), while interesting applications are reported by Newland (1999) and Erlicher & Argoul (2007), among others. Quadratic representations which include the Wigner-Ville distribution and the Cohen-class of distributions have also received some attention (Feldman & Braun 1995, Bonato *et al.* 1997, Wang *et al.* 2003a).

### 3.2.5. Identification of hysteretic and time-varying systems

Several contributions in the last decades were concerned with the identification of hysteretic models, in particular the Bouc-Wen (BW) model. Due to its great simplicity, related to the absence of an elastic domain, the BW model has been extensively used in the seismic analysis, both deterministic, e.g. Follente (1995), and stochastic, e.g. Casciati (1989). Chassiakos *et al.* (1995) and Smyth *et al.* (2002) proposed a parametric method in which the parameters of the BW model are identified through an adaptive procedure, based on the application of least square techniques of estimation. An alternative approach is due to Kyprianou *et al.* (2001), who introduced a differential evolutive method for the identification of the parameters, whose formulation in many respects comes close to that of genetic algorithms.

Classical non-parametric methods are based on the extension of the restoring force surface approach: Benedettini *et al.* (1995) approximated the surface of the time derivative of the internal restoring force on a polynomial basis, by assuming as state variables the force itself and velocity; Masri *et al.* (2004) extended this approach by proposing a polynomial base approximation of the restoring force as a function of velocity, displacement and the excitation. A similar approach may be also used to define instantaneous estimators of the system dynamic properties (Bursi *et al.* 2012, Ceravolo *et al.* 2013).

In the frame of non-parametric approaches, Pei *et al.* (2004) used a special type of neural network, which showed good performances in the identification of hysteretic systems. Saadat *et al.* (2004) formulated a hybrid approach that combines the potentials of both the parametric and non parametric approaches in a single identification procedure. Wu & Smyth (2006), among others, have successfully applied the Unscented Kalman Filter (UKF) to identify the parameters of hysteretic systems.

## 4. Corroboration of engineering models

Be it mathematical or physical, a model should always contain the essential aspects of the system it seeks to emulate, to understand the system in a comprehensive, but also exploitable way (Hesse 1963). To this end, modeling must be undertaken in distinct phases. The first is analysis, in which all possible information about the system is collected and analyzed. In the second, called synthesis, the information is critically analyzed to minimize unnecessary data. Then, in the modeling phase, the information that has passed the synthesis phase contributes to the construction of the model. In the final step, the model is used to extract new information about the system itself that could not be gathered without the use of the model.

Figure 1 shows the physical model of the Norfolk Scope Arena, Virginia, U.S.A. that, in 1969, was experimented in the wind tunnel of the Politecnico di Torino, and used to predict the behaviour of the building in strong storms.

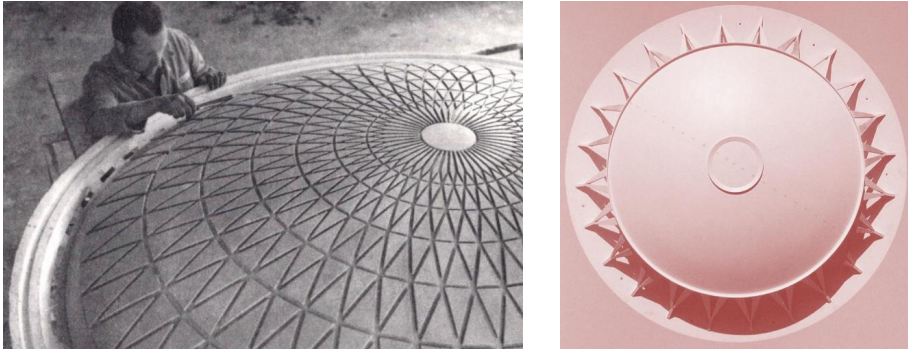


Fig. 1. Model of the Norfolk Scope designed by P.L. Nervi (Marchis 1988).

The correct reproduction of the occupied space is the basis of any model (physical or mathematical) of a system that admits a spatial representation. Nowadays, geometric modeling can rely on advanced techniques, such as the 3D laser scanner or others. All the information collected by surveys provides not only the geometric and dimensional characteristics of the structure, but also the classification of the elements and materials used. After the synthesis phase, a simplified geometric model can be used to build a mechanical model, such as a finite element (FE) model (e.g. Lenticchia *et al.* 2021).

As regards models, it is worth reporting an extract of a paper by Guido Oberti about testing physical models to investigate complex problems: «new methods, rather than obeying pre-conceived schematizations, prefer to approach the reality of the single case by reproducing their peculiar characteristics; thus, one does not hesitate to introduce into the model anisotropic materials for the foundations, constraints, joints and executive modalities in general. This, while making it harder to provide analytical interpretations and causing a certain dispersion of results in repeated tests (especially around discontinuities), allows for a more realistic and synthetic vision of the problem, and therefore more in keeping with the true aims of the experimental test» (Oberti 1966).

The relevance of these concepts is discovered every time it is necessary to corroborate numerical models that really reflect what was observed in the experiments conducted on real structures. After all, reality itself is constantly evolving, like the results of experimental tests, since both the system and the



surrounding environment undergo continuous changes. Therefore, a model should be designed to be updated and to incorporate new information.

As said, monitoring, whether continuous or periodic, can detect changes in system properties or the surrounding environment. For example, a change in ambient temperature is commonly reflected in deviations of the system's vibration modes. If this change is not only relatively small, but also periodic (e.g. seasonal) and persistent, it is said to be physiological. When using a mathematical or numerical model to predict a specific response, all physiological phenomena that bring a certain variability, and therefore uncertainty, should be incorporated in it, in order to allow a reliable comparison between the prediction of the model and the actual response of the system. If it is important to monitor the physiological behaviours of the system in the model, it is essential to incorporate the pathological ones. Pathological behaviour is due to a temporary or permanent change in an environmental condition that produces a permanent change in structural properties. An example of pathological behaviour is the permanent reduction of the natural frequencies of a system after an earthquake. When they occur, such pathological behaviours should be considered in models by updating the constitutive laws of materials, or even geometric and topological properties.

Physical models, while not suitable for representing structural systems when the scale of the reproduction becomes too small, are very effective in simulating extremely complex behaviours, especially if reproduced on a scale as close as possible to the original. On the contrary, numerical models prove effective in solving problems in which the theory corresponds well to the actual observation, e.g. linear elasticity, within the limits of instrumental or computational approximations. A synergy between numerical and physical models is obtained by the so-called hybrid modeling (Hakuno *et al.* 1969, Miraglia 2019). It consists of detecting the causes of the complex behaviour and splits the modeling into two parts: (i) a mathematical/numerical part; and (ii) a physical part. The latter is commonly a physical, life-size reproduction of a sub-part of the system, used to predict the complex non-linear behavior of the analysed system.

#### *4.1. Corroboration of numerical models with experimental data*

The corroboration of numerical models entails the acquisition of adequate knowledge on a structure, this being especially true for historic constructions.

Indeed, for historic and monumental structures, the concept of *path of knowledge* is introduced as: [...] *a series of standardised actions and aspects, which have to interact in order to achieve the desired level of knowledge of*

*the building* [...] (ICOMOS 2003). More specifically, the procedure to produce a predictive model must start with a historic analysis from a social, environmental, and structural point of view (first step). The second step is to collect the main data for the building and surrounding territory. This data regards the geometry, details, materials and other types of data, such as the dynamic characteristics of both building and soil.

Having in mind the scope of the mathematical model, it is then possible to select the type of model (e.g., scale, level of accuracy, etc.). The idea is to predict one or more desired outputs for decision making. Moreover, the model should be able to incorporate new data, e.g. from periodic or continuous monitoring.

#### 4.2. Dynamic characterization

Vibration measurements allow to go back to the structural behaviour of the building system with reduced costs and minimally invasive actions, which is important when the necessity to preserve the material integrity becomes truly significant; moreover, in comparison with other investigation techniques, they provide information on the global behaviour of a structure (e.g. Ceravolo *et al.* 2016).

The test design should be supported by the preliminary numerical models, possibly after translating into the FE at least the information gathered during the archival research and the geometric survey.

In permanent monitoring systems, varied or anomalous parameters are directly associated to damage, and structural reliability can be defined as a function of identified quantities that reflects the damage, referred to as symptoms. In alternative, a numerical model (e.g. a FE model) can be updated on the grounds of the identified parameters.

##### 4.2.1. Sensitivity analysis

Once the numerical model is set up, a sensitivity analysis can be performed on the numerical model of a structure to determine which parameters are the most important and most likely to influence the behaviour of the system. Following a local (Mottershead & Friswell 1993) or global sensitivity analysis (Santelli *et al.* 2008, Boscato *et al.* 2015), the parameters with negligible effects can be ignored or simplified. The selection of the parameters must still be performed with a critical sense, as sometimes some parameters are not very sensitive but nonetheless directly related to the damage.

Sensitivity analysis results in reducing the computational burden of the calibration procedure, which in many cases would be unmanageable. In fact, the most important class of “penalty” model updating methods is sometimes



also referred to as “sensitivity-based”.

#### *4.2.2. Optimal sensor placement*

Optimal Sensor Placement (OSP) methods consist of selecting the optimal position and directions in which to place the sensors on the structure (e.g. Lenticchia *et al.* 2018, Murugan *et al.* 2019, Civera *et al.* 2021). This is necessary since vibration-based techniques must implement sensors (mainly accelerometers) to record environmental vibrations. Factors such as cost, accessibility, energy availability, etc. actually limit the number of sensors that can be used for this purpose. Most conventional sensor positioning algorithms aim to make the numeric modal forms captured by the monitoring system linearly independent.

In relation to a possible automation of the positioning procedures, it must be said that the OSP procedures are affected by the presence of errors and uncertainties, and this is even more true when the structure in question presents great complexity, for example geometric or mechanical (e.g. CH structures). In these situations, the OSP must be verified in the field, in order to validate or correct the numerical prediction based on the experimental evidence that the models are unable or difficult to simulate.

#### *4.2.3. Experimental modal analysis*

After the OSP phase, the experimental setup can be designed and the sensors installed on the structure. The records (e.g. acceleration signals) are processed by specific algorithms that eliminate experimental errors/artifacts and components that are not relevant for monitoring (data cleansing). Other effects are also considered during this preliminary phase, such as the effects of Environmental and Operational Variations (EOVs), which can influence the estimate of the modal parameter of the system. The estimation is performed with vibration-based techniques, such as Experimental Modal Analysis (EMA) or more often (given the difficulty of exciting a full-scale structure) with Operational Modal Analysis (OMA), therefore with unmeasured natural excitation (output-only). The modal parameters reflect the mass, the stiffness and the dissipative capacities of the structure. Significant deviations in modal parameters identify changes in the structure, with the possibility of detecting and/or quantifying the damage through Vibration-Based Damage Detection (VBDD) techniques (De Stefano & Ceravolo 2007, Ramos *et al.* 2010, Russo 2012, Ceravolo *et al.* 2016).

#### 4.2.4. Model updating

Experimental information on the modal properties of a structure (i.e., natural frequency, damping, mode shapes) makes it possible to enhance the predictive capability of a numerical model. This activity is referred to as *model updating*. It consists of updating the significant mechanical parameters of a numerical model, in order to make the predicted structural response consistent with the modal response experimentally identified in a previous phase.

Model updating methods can generally be subdivided into two categories based on direct and iterative approaches. Both methods are discussed in the literature (Mottershead & Friswell 1993). Direct methods, which are commonly used for simple systems where a closed formulation is available, update all the model parameters in one step. On the contrary, iterative methods are currently exploring the use of optimization algorithms to resolve the inverse problem of parameter calibration.

In order to distinguish between different types of iterative calibration, Modal Model Updating (MMU) is often used when the global system mass and stiffness matrices are updated from experimentally identified modal quantities, typically limited to frequencies and modes, as modal damping data show higher dispersion. However, more generally speaking, the FE model calibration process may consider parameters belonging to different physical fields, such as Thermo-MEchanical Model Updating (T-MEMU), where the thermal and mechanical parameters are updated in a holistic framework (Cervolo *et al.* 2020).

The possibility that a numerical model is updated continuously and indefinitely, using experimental and multiphysics monitoring data, may in principle lead to the definition of a digital twin with predictive capabilities, at least in the mechanical sense of this term.

#### 4.2.5. Predictive models and realization theory

The simple observation that a model has been updated or calibrated with respect to some parameters, possibly minimizing a cost function, does not imply that it has also acquired predictive capabilities. The predictive capabilities must be demonstrated on the basis of physical interpretations and experiments, in which case the updated model will be said to have been verified. A predictive numerical model can be used for high levels of SHM, i.e. in the prognosis and estimation of the residual life of a structure.

Speaking about prediction, a suggestive example is the on-line correction of models used in control engineering, from measurements of the input and/or of the response of the system. In this case, state observer models are intro-

duced that are fed with on-line measurements and updated accordingly (Inman 1989, Juang 1994). Determining all the matrices of the state observer model is referred to as system realization (Ho & Kalman 1965).

In linear time-invariant systems, the state observer model provides, among other things, on-line estimates of natural frequencies, damping ratios, and mode shapes of a vibrating structure. A more challenging case is when parameters to be identified are inherently time-varying (e.g. an instantaneous frequency, degradation in stiffness or strength etc.) without a predefined evolution law. In this situation appropriate methodologies apply, such as time-frequency estimators (Ceravolo 2004, Ceravolo 2009), or extended or unscented Kalman filters, etc. (e.g. Wu & Smith 2006).

## **5. Machine learning approaches to Structural Health Monitoring**

In the data-driven SHM approach, a structure is often conceived as an empirical system associated with an input-output relationship. Not taking into account the existence of a state space, such an approach is unable to exploit some fundamental properties of mechanical systems (causality, linearity, etc.) or to solve the problem of realization. However, the system realization is not always required in diagnostics, as the problem reduces to the evaluation of symptoms revealing the presence and nature of a fault, so that one can define it as “model recognition” rather than “identification”. This entails the need to select a suitable set of input and output parameters and to determine the number of tests to be performed for a univocal identification of damage. In practical applications, the tendency is to use a large number of input and output parameters and the largest possible series of tests. Syntactic, statistical, and especially neural PR can naturally meet these requirements and, in fact, these strategies began to be applied to SHM since the early nineties (Ceravolo *et al.* 1996).

Recently, in SHM, as in other engineering sectors, ML techniques have begun to spread, aimed at synthesizing and generalizing data by extracting information to base a decision-making process (Farrar & Worden 2012, Figueiredo *et al.* 2011, Flah *et al.* 2020, Smarsly *et al.* 2016). ML emulates the learning ability of human by using computers to acquire knowledge and skills automatically and learning to refine continuously its performance, achieving self-improvement. The goal is to find intrinsic relationships in the available data, thereby predicting unknown data or judging its characteristics. For example, in the monitoring of civil structures ML aims to associate certain diagnostic characteristics measured on a building with a structural condition, both phys-

iological and pathological. However, generalization remains the most critical aspect.

From the perspective of the theory of knowledge, ML is very similar to human learning (Mitchell 2000). Both machine and human learning are knowledge-increasing processes and both algorithms and humans aim to become intelligent. However, the following differences can be highlighted:

- Human learning is a long-term process; ML is generally fast and short.
- Human beings are forgetful and can only remember some knowledge while machines can remember all knowledge it has learned.
- Knowledge is not transportable for human learning, i.e. a person's knowledge cannot be copied directly to another. ML can copy the knowledge learned into any other system.
- A distinct characteristic of human learning is generating ideas in a best way, while the ideas obtained by ML are usually not the best.
- The connection and inspiration of humans are complex to be simulated by a machine. Human learning can be of jumping style, while ML always follows rules docilely. This is caused by the different logics followed by human learning and ML, respectively.

In SHM, the basic idea is to learn from the training dataset a relationship between these characteristics and the presence/type of damage, and then re-apply it thereafter on new, unknown data. Integrating these methodologies into a procedure for monitoring would allow the process to be automated by minimizing the intervention of an expert operator. A well-calibrated algorithm should even exceed the performance of an operator, having the speed and computing power of a machine able to easily handle large amounts of data, even high dimensional ones, and not being subject to human error.

Some key concepts of ML from SHM's point of view are recalled below. For more detailed information on the general theory of ML and PR, reference can be made to Bishop (2006).

### *5.1. Supervised vs. unsupervised Machine Learning approaches*

ML falls into two macro-categories: those of supervised learning and those of unsupervised learning. The difference between them lies in the availability of the output, or *labels*, in the training phase of an algorithm.

In supervised ML, labels are available and the algorithm (Coletta 2022) can learn a relationship between them and the measured data, with the intent of applying it to new, unknown data to predict their labels. Depending on the type of label, the algorithm can address two tasks: classification or regression. A supervised

classification problem is addressed when, given a single or multidimensional data set, the goal is to assign a discrete class (marked by a label) to each data; for example, in SHM classes are generally represented by a diagnosis, for example “normal condition” or “damaged condition” (in the best case each class indicates a type of damage). In regression problems labels are one or more continuous variables but the basic idea is analogous to that of classification. Regression apply to SHM, for example, to formulate a diagnosis based on specific predictor variables, which can be other diagnostic parameters (homogeneous or inhomogeneous with those predicted) environmental or operational variables. The prediction of the behaviour of a structure under normal conditions offers the opportunity to make a comparison with the one actually measured, from which important differences should emerge if the structure deviates from its usual behaviour.

In an unsupervised problem there are no training labels available, so the ultimate goal cannot be, as in the previous case, to associate a concrete meaning to the data, but to draw relevant information from them. Among the possible strategies, when there is no labelled data, there are clustering analysis and anomaly detection analysis. In clustering, the data is observed by the algorithm and a logical grouping is derived from it rather than forcing the grouping, as in the previous cases, in accordance with the categories that are attributed to it externally. The other task, namely anomaly detection, also called novelty detection, or outlier analysis, is of great interest for SHM. If data from the normal conditions of a structure are available, it is possible to make a statistic. At that point, all the data subsequently collected can be tested to see if they conform in some way to the normality model; otherwise it can be said that the non-conformity will correspond to a damage.

## *5.2. Training data*

In ML three phases can be defined: training, validation and test. Training datasets are supplied to the algorithm with or without the corresponding outputs, depending on the type of problem to be addressed. The model evaluates the data and uses them to define its parameters, with the aim of building a model that also generalizes well the new data, unknown to the algorithm.

In some supervised problems validation is expected on a small set of labelled data, which however is not yet used for training. This is mainly done to avoid the phenomenon of overfitting, i.e. that the algorithm adapts too much to the training data and therefore fails to generalize. This validation dataset is used to adjust some parameters of the algorithm (feedback), before it becomes definitive. The test dataset, on the other hand, is the one on which the algorithm, already trained and validated, will be applied, in order to provide

labels, discrete or continuous, define the cluster to which it belongs and detect a certain novelty.

### 5.3. Transfer Learning

The generalization of data complex and rich in information of various kinds should be addressed recurring to strategies that can handle high-dimensional data and the presence of outliers, and provide satisfactory performance even in the case of relatively few examples for training. Yet the application of current ML techniques alone would not be able to manage the differences that inevitably exist between real structures.

The Transfer Learning (TL) theory addresses precisely the problem of the limitation of labelled data, but in this case the data used for training and testing can belong to different “domains” and do not have the same distributions and tasks (Dai *et al.* 2007, Pan & Yang 2009, Taylor & Stone 2009, Weiss *et al.* 2016). The concept behind TL is very simple and unconsciously applied in many daily practices. For example, gaining experience to ride a motorcycle simulator can definitely help you ride a real motorcycle. TL is a convenient way to deal with the problems in which one wants to investigate a little-known system, jointly using a lot of information available from another system, which is somehow related to the first.

The use of TL in SHM is mostly motivated by the lack of labelled data belonging to damaged conditions or to particular operational conditions of that structure, which may not be present in the training set. For many structures, in fact, measurements relating to damage cannot be obtained concretely. In other cases, although obtainable, the labelled data relating to damage are expensive and can take a long time; for example, the labelling of data belonging to a damaged condition requires advanced and specific professional knowledge in structural engineering. The sources for TL include any built heritage, settlement and installation whose structural reliability is affected by degradation or natural and manmade hazards.

In this approach, two domains are defined: a source and a target domain. The first  $\mathcal{D}_S = \{\mathcal{X}_S, P_S(X)\}$  contains the labelled data, i.e. the information we intend to transfer. The target domain,  $\mathcal{D}_T = \{\mathcal{X}_T, P_T(X)\}$  instead, contains data that come from the system to be investigated. Each domain is associated to a task, defined as a function of a label space and an objective predictive function that can be used to predict the corresponding label. In general, if two domains are different, either their feature spaces or their marginal probability distributions differ, and for this reason the classification algorithm may fail to classify moving from one domain to another. TL aims at improving the

learning of the target predictive function using the knowledge acquired in the source domain (Coletta *et al.* 2020, Coletta 2022).

## **6. The role of SHM in the conservation of cultural heritage structures**

Historical constructions consist of a large variety of building technologies, stylistic canons, materials, and *rules of art*, which may differ according to the time period and geographical areas. Inspection and diagnosis of structures have been practised for years, and have been now established as important means for the safety assessment of CH. Indeed, the international deontological guidelines, such as the ones from International Council on Monuments (ICOMOS-ISCARSAH 2003), define the rehabilitation process of heritage structures similarly to the treatment of a human disease: «the heritage structures require anamnesis, diagnosis, therapy and controls, corresponding respectively to the search for significant data and information, identification of the causes of damage and decay, choice of the remedial measures and control of the efficiency of the interventions», an operation that calls for a multi-disciplinary approach.

In more detail, a CH structure requires a path of knowledge articulated in phases: general identification of the structure within its environment factors; collection of geometric and structural information; identification of the materials and their state of conservation, historical documentation; mechanical characterization of the materials by means of different investigation techniques; soil and foundation analysis, and relevant monitoring activities. The documentation process, in particular, will cover aspects such as construction defects, deterioration, irregularities, damage produced by previous events (anamnesis), and in general any factor that makes each of these structures unique and involve a greater degree of complexity when interpreting the structural behaviour (Ceravolo *et al.* 2016). ICOMOS standards also emphasize the importance of periodic building inspections as a primary tool for conservation.

In the light of the above mentioned concepts, inspection and monitoring activities play a fundamental role in both assessment and conservation processes of CH (Ceravolo *et al.* 2019, Lorenzoni *et al.* 2016).

While accurate evaluations in terms of bearing capacity normally entail destructive testing, very seldom sampling is allowed in the case of CH structures. Inspections, by means of endoscopes, thermographs, radar, metal detectors; physical measures, via sonic tomography; or geometric measures by photogrammetry, or other technologies, can be executed periodically to improve the knowledge level and to reduce the uncertainties. Spectrometry and



false colour images can reveal chemical degradation. Unfortunately, all these surveys and measures, while increasing the knowledge level, provide only local information.

The introduction of modern SHM techniques, on the other hand, has led to the growing diffusion of permanent monitoring systems. The latter make it possible to measure and record the dynamic response of structures daily and during seismic events, and transmit these records to seismic network databases for the purposes of damage detection and emergency management. As an example, the Seismic Observatory for Structures (OSS), set up at the end of the 1990s within the Italian Department of Civil Protection, is a nation-wide network for the permanent monitoring of the seismic response of strategic public buildings in Italy (Dolce *et al.* 2019). More generally, a seismic SHM network aims at providing, in the aftermath of an earthquake, a rapid estimation of the seismic damage suffered by the monitored buildings and, plausibly, by similar neighbouring constructions, helping in planning and managing emergency activities.

The availability of simple and direct relationships between modal parameter deviations and presumed damage levels, as determined for different building types, is of the utmost importance for the practical usability of the data collected by seismic SHM networks during seismic events. Yet, it is well known that even undamaged structures may exhibit significant variations, or wanderings (Ceravolo *et al.* 2017), of their dynamic characteristics as a consequence of response nonlinearities and/or time-varying environmental conditions, which makes the problem of reliably inferring damage from modal deviations still an open research issue.

### 6.1. Environmental effects

Structural diagnosis features are known to be influenced by some Environmental and Operational Variations (EOVs), which cause fluctuations that can be confused with the appearance of damage, or worse, hide it (Deraemaeker *et al.* 2008, Ubertini *et al.* 2017, Coletta *et al.* 2021). Among all fields of SHM, the issue of environmental variations mainly concerns civil structures, which by their nature are totally exposed to climatic conditions and, in fact, many studies on these effects have been carried out using signals from historical structures and bridges (Barsocchi *et al.* 2020, Cabboi *et al.* 2017, Catbas *et al.* 2008, Gentile *et al.* 2019, Ni *et al.* 2005).

The analysis of environmental time series, treated with mathematical and statistical tools, can lead to mechanical interpretations of the observed struc-



tural behaviour, especially in terms of correlations between different factors affecting measurements. Therefore, these data are deemed relevant in the practice of long-term monitoring of CH.

In data-driven SHM, environmental effects are an important issue for the choice of training data (Coletta 2022). In fact, the more experience the algorithm accumulates with respect to the variations given by environmental conditions, the more easily it will be able to recognize them and distinguish them from damage. In fact, generally the best choice when selecting a training set within an available dataset is to include important climatic variations, such as snowfall, wide temperature and humidity ranges and other phenomena that will affect the behaviour of that specific structure.

## *6.2. Perspectives in the use of satellite data for SHM*

Differently from other engineering fields (e.g. aeronautical, mechanical, aerospace, etc.), the civil structures are in most cases connected directly to the ground, being strongly affected by the foundation soil, its characteristics and evolution. This is especially true for the diagnostic parameters, including vibration modes or other quantities that are typically monitored in CH buildings.

Some recent research works (Cavalagli *et al.* 2019, Delo *et al.* 2022, among others) have focused on exploiting interferometric satellite data obtained from radar instruments, e.g., Synthetic Aperture Radar (SAR), to detect displacements on the Earth's surface in the order of centimetres and millimetres. Also hyperspectral and multispectral sensors, mainly employed for environmental monitoring (i.e. fires, glaciers, droughts, etc), were recently proposed to integrate SHM operations (Coccimiglio *et al.* 2022).

As said in the previous section, the behaviour of an exposed structure depends on EOVs. These factors act directly on the structure, but also affect its foundation soil, altering and modifying its properties. Decoupling the two effects is very difficult as many causes (rain, snow, ice, temperature variation, humidity, etc.) could act in combination and, in some cases, the relationships between variables are complicated. In this sense, satellite data constitute a source of information regarding the state of the soil, as they are available on almost the entire surface of the planet continuously from several online platforms.

## 7. Examples of structural and seismic monitoring of historical and contemporary buildings

### 7.1. Static and dynamic continuous monitoring of the world's largest oval-shaped masonry dome

The construction of the Sanctuary of Vicoforte (Fig. 2) began at the end of the 16<sup>th</sup> century on a project by Ascanio Vitozzi. For decades the construction stopped at the level of the shutter of the dome. Only at the beginning of the 18<sup>th</sup> century, the project was taken up again by the architect engineer Francesco Gallo, who saw the construction of the high drum and the majestic dome, the largest oval-shaped masonry dome in the world with internal axes of 37.15 and 24.80 meters. An iron ring system was also put in place, consisting of three rings of bars with a total section of about 140 cm<sup>2</sup>, which was intended to absorb a component of the horizontal thrust. The dome was disarmed in 1732 (Cozzo *et al.* 2017).

After the exhaustive surveys and reports by engineer Martino Garro (1962), the structural criticalities of the building led to the decision to undertake investigations and research. However, it was only in the late 1970s that the scientific community began to systematically investigate the structural health of the Sanctuary.

At the beginning of the 1980s a hooping system was installed to avoid the widening of the crack pattern localized above all in the dome-drum system. The system consists of four high-strength steel bars in each of the 14 tangential directions. Contrasting steel frames connect the heads of the bars of two adjacent stretches. The tie-bars, slightly tensioned at 50 kN by jacks, were re-tensioned in 1997 to compensate for the physiological load losses (Chiorino *et al.* 2008).

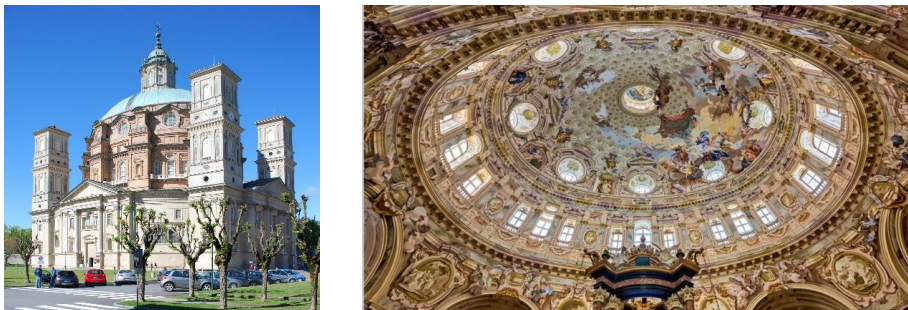


Fig. 2. The Sanctuary of Vicoforte: external and internal view of the dome.

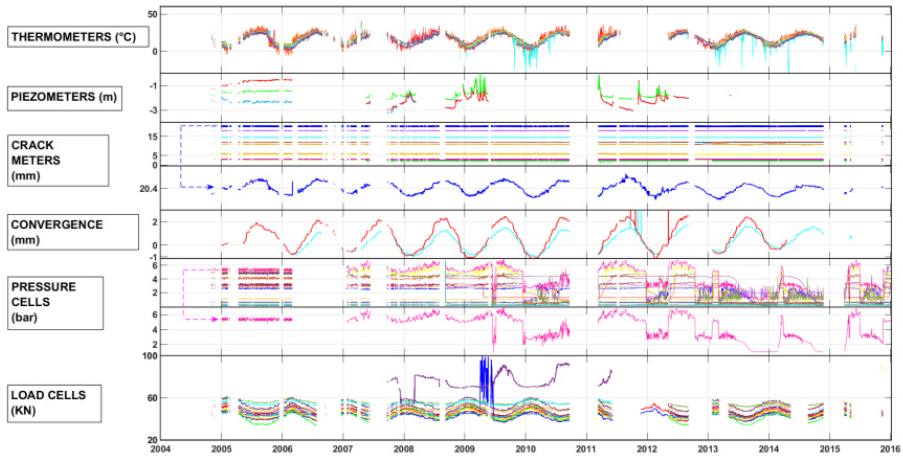


Fig. 3. Static monitoring data. From the top to the bottom: thermometers, piezometers, crack meters (all and a selected time series), convergence, pressure cells (all and a selected time series), load cells.

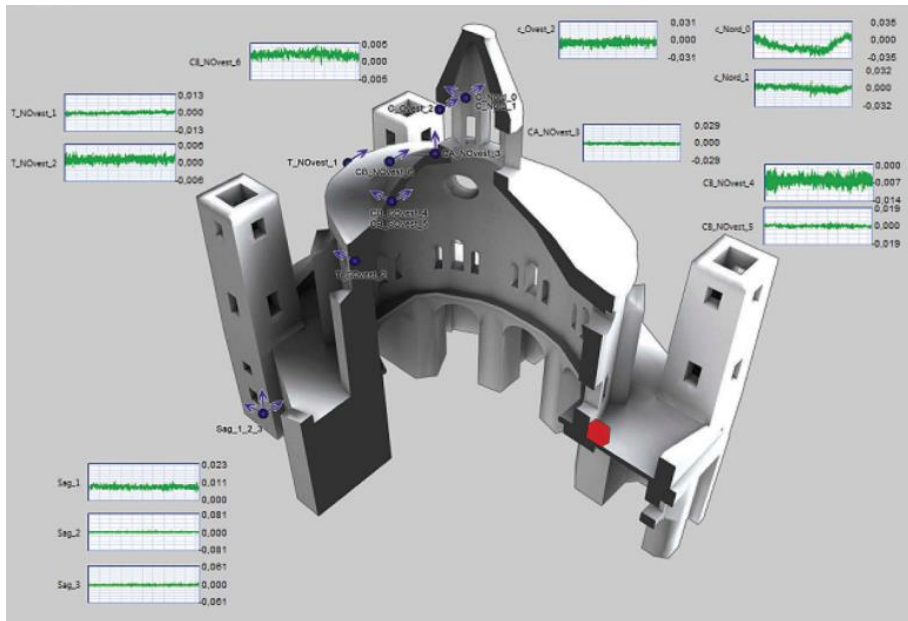


Fig. 4. Layout of the dynamic monitoring system installed in the Sanctuary.

Geological and geophysical investigations, conducted between 1976 and 2008, have confirmed that different materials make up the subsoil of the Sanctuary (Scandella *et al.* 2011). A layer of marl slopes downwards from north-east to southwest, while a layer of clay is present under the rest of the building, causing serious crack patterns.

The monitoring activity on the Vicoforte Sanctuary began in 1983 with the installation of instruments to investigate the evolution of the crack pattern. Since then, the static monitoring system has undergone several updates, until the last one in 2004 when the acquisition procedure was automated. This monitoring system consists of 133 instruments which are specially placed on the dome-drum system. The sensors can be divided into two main groups.

The first group, for the measurement of strains, stresses, and cracks, includes: 12 crack-meters to check the evolution of the cracks; 20 horizontal pressure cells to define the stress in the dome and in the eight pillars; 1 vertical pressure cell near the top of the dome above the main meridian crack (northern side) to measure the circumferential compression stress; 56 load cells installed on each tie-bars to control its load condition; 2 orthogonal wire gauges measuring the main axes of the dome at its impost to assess the building overall geometry; 12 nails for additional manual measurements of convergence. The second one, for the measurement of the environmental parameters, includes: 25 temperature sensors; 3 piezometric electric cells; 1 hydrometer.

As it can be seen from Figure 3, many of the recorded static parameters show seasonal fluctuations. Some interruptions are also visible, since the static system has undergone periodic malfunctions over time, due to several environmental (thunderstorms and lightning) and technical factors (problems with the electrical box). Finally, over the years, some devices have started to show anomalies caused by local phenomena or sensor failure. For instance, the pressure cells seem to have collected reliable data up to about 2009.

Data acquired from ten years of monitoring activities (November 2004 to November 2014) have been analysed by Ceravolo *et al.* (2017). The aim of these analyses was to check the damage state of the building and to verify the effectiveness of the 1987 strengthening system. From the analysis of the crack openings, ten years of monitoring data show the seasonal influence of the temperature on the structural behaviour of the Sanctuary and the substantial stability of the monitored parameters, demonstrating the efficacy of the tie-bar system.

However, the static monitoring system only provides local information about the health state of the Sanctuary. Consequently, a permanent dynamic monitoring system, designed through a model-based optimal sensor placement procedure (Pecorelli *et al.* 2020), was installed in December 2015 to in-

investigate the global phenomena affecting the structure (Fig. 4). The positions of the 12 mono-axial piezoelectric accelerometers (PCB Piezotronic, model 393B12, seismic, high sensitivity, ceramic shear ICP® accel., 10 V/g, 0.15 to 1k Hz, Resonant Frequency  $\geq 10,000$  Hz, Overload Limit  $\pm 5000$  g pk, Temperature Range  $-50$  to  $+180$  °F) were defined through optimal sensor placement techniques. As shown in Fig. 4, three orthogonal accelerometers are located at the base of the crypt to record the ground accelerations (Sag\_1, Sag\_2, Sag\_3).

A set of nine accelerometers are located at different levels of the lantern-dome-drum area, along longitudinal and transverse directions. In more details: (i) two accelerometers are at the base of the dome at 30 m height (T\_NOvest\_1; T\_SOvest\_2); (ii) three sensors are on the dome at 45 m height (CB\_SOvest\_4, CB\_SOvest\_5, CB\_NOvest\_6); (iii) one vertical accelerometer is located at the base of the lantern at 50 m height (CA\_NOvest\_3); and (iv) the last three accelerometers are located at about middle height of the lantern (CB\_Ovest\_2, C\_Nord\_0, C\_Nord\_1).

The acquisition system is designed as a master/slave scheme to limit the distortion due to cable length that becomes significant over 50 m length. The data acquired by the accelerometers in the crypt are transmitted to the slave unit, and then to the master unit; the data recorded by all the other instruments converge directly to the master unit. A GPS receiver is used to synchronize the time of all the instruments. The acquired data are transmitted to the Earth-

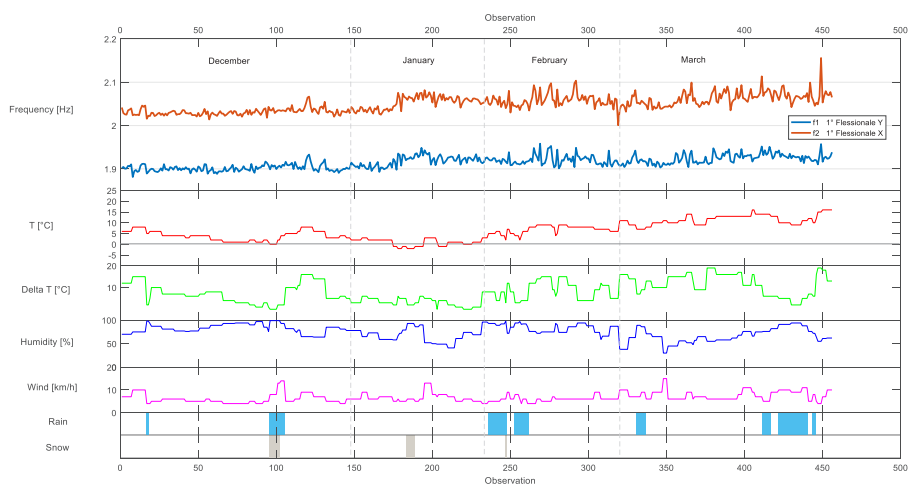


Fig. 5. First two frequencies, temperature,  $\Delta T$ , humidity, wind, rain and snow (from the top to the bottom).

quake Engineering and Dynamics lab of the Politecnico di Torino and to OSS (Dolce *et al.* 2019).

The acquisition system was set to record data according to two criteria: (i) a time criterion and (ii) a threshold criterion. In details, the first criterion implies to record data for 20 min every hour, also in order to limit data storage, whereas the second criterion entails to record the accelerations when the ground horizontal acceleration exceeds a pre-set value (acceleration measured by sensors Sag\_2 and Sag\_3). This value is set in accordance with the seismic hazard of the area defined by the Italian regulations. More specifically, the pre-set value is 0.042 g, that equals the PGA related with the Damage Limit State and a return period of 50 years. The aim of this last criterion is to record the dynamic response of the Sanctuary during seismic and other dynamic events.

The data recorded by the dynamic monitoring system are automatically processed to estimate the main frequencies and modal shapes of the Sanctuary. The code implemented in Matlab® was updated to give the same results of the identification procedure that is performed manually by an expert operator. In automatic identification procedures, a cluster analysis is used to group the possible physical modes into homogeneous sets representing the same physical mode.

The systematic dynamic and seismic monitoring of the lantern-dome-drum system of the Sanctuary started in December 2016. Some sample data limited to the first two translational modes are reported in Figure 5, together with environmental data (Pecorelli *et al.* 2020).

### 7.1.1. Monitoring data analysis

A systematic analysis of the static and dynamic monitoring data, with correlations among different measurements, including environmental time series, has been recently published (Ceravolo *et al.* 2021).

The results of the correlation analysis confirmed that both the static and dynamic behaviour of the Sanctuary are greatly influenced by variations in the ambient temperature, while no significant correlation is observed with other environmental phenomena taken into consideration, such as humidity and rain. The highest coefficients among the static data relate to the temperature of the masonry, of the load in the tie-bars and of the crack gauges. The coefficients indicate that the increase in the external temperature corresponds to an increase in that of the internal masonry and a delay is observed in the time series that varies from 10 to 30 days depending on the position of the sensor, due to the thermal inertia of the material.



The increase in temperature causes the opening of the cracks at the level of the balcony, which is accompanied by a decrease in the load in the bars (Fig. 6). This can be motivated by the fact that the steel of the bars expands more than the masonry, as indicated by the difference in their coefficients of thermal expansion. In this situation, the bars tend to compress and consequently the tension decreases. Strain gauges records suggest that the masonry of the dome expands in the summer months and closes in cold periods, i.e. the measurement of the elongation of the axes shows a trend directly proportional to the external temperature.

On the other hand, as shown on the right side of Figure 6, the first five frequencies tend to increase with increasing external temperature, except for very low values: a bilinear behaviour with slope inversion is observed for negative temperature values, as also observed in other case studies (e.g. Gentile *et al.* 2019, Kasimzade 2018, Peeters & DeRoek 2001). A plausible interpretation, concerns the effect of ice, which is known to significantly increase structural rigidity (Peeters & DeRoek 2001).

Again with reference to Figure 6, a counter-intuitive observation emerges from the comparison between dynamic and static data. It can be noted that the increase in the vibration frequencies for high temperatures corresponds to the increase observed in the opening of the crack and the decrease in the load in the post-tensioned bars. This calls into question different and not modelled phenomena, including micro-cracks, or possibly seasonal cycles of the soil, also because the observation refers in particular to the first modes.

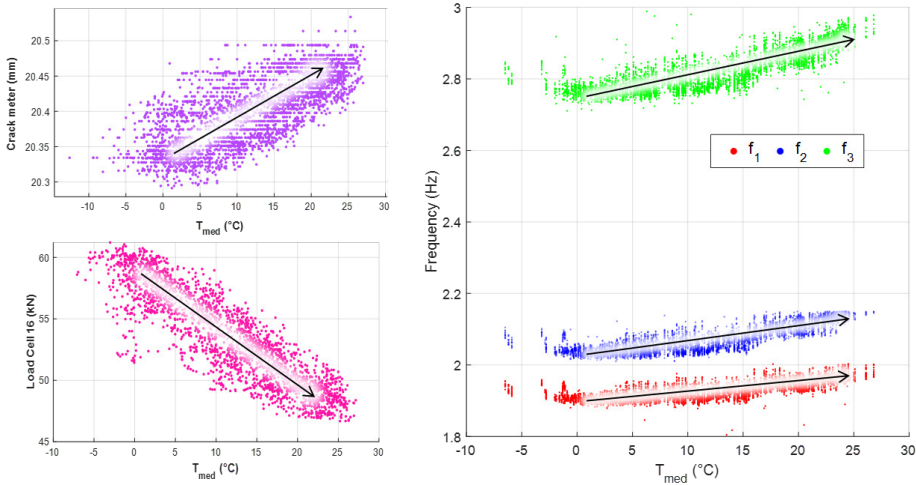


Fig. 6. Comparison of the results coming from the static and the dynamic monitoring systems.

### 7.1.2. Model-driven SHM

The first model updating of the Sanctuary was based on the results of a dynamic test campaign conducted in 2008 (Chiorino *et al.* 2011). Successively, in order to consider the structural, typological and historical peculiarities of each component of the structure, a more accurate FE model was built that consisted of 9 homogeneous macro-elements: 7 for the building (lantern, dome, drum, basement, buttresses, bell-towers and tie-bars) and 2 for the soil (marl and clay), as shown in Figure 7.

Then a thermo-elastic updating was performed using multiphysics data, including the thermal analysis to obtain the temperature distribution of the drum-dome system as related to the forces acting in the tie-bars (Ceravolo *et al.* 2000). This distribution was determined by applying local temperature measurements to the thermal FE model. The temperature distribution obtained for the drum-dome system was then used as the input for the successive T-MEMU. The thermal analysis was performed on the partial FE model of the Sanctuary. In detail, this model was limited to the upper macro-elements: lantern, dome, drum, buttresses, tie-bars (see Fig. 7). A 4-node shell element

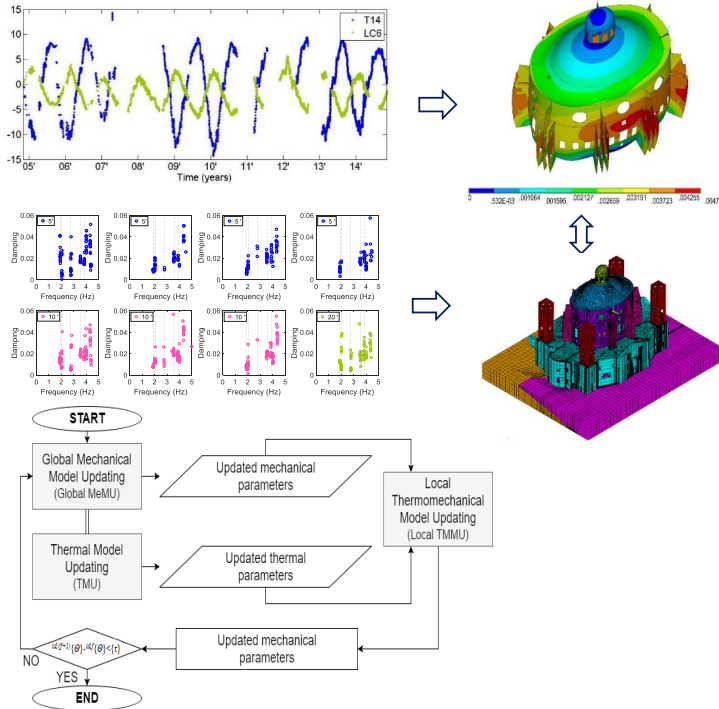


Fig. 7. The FE model of the Sanctuary and updating flowchart.



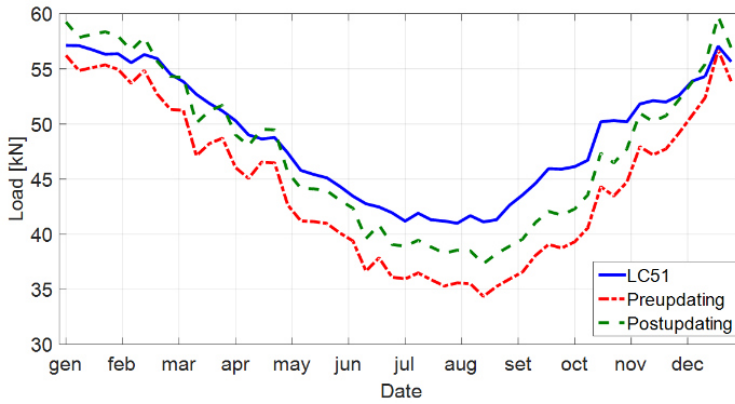


Fig. 8. Pre- and post-updating load trends associated with data acquired by a load cell LC51.

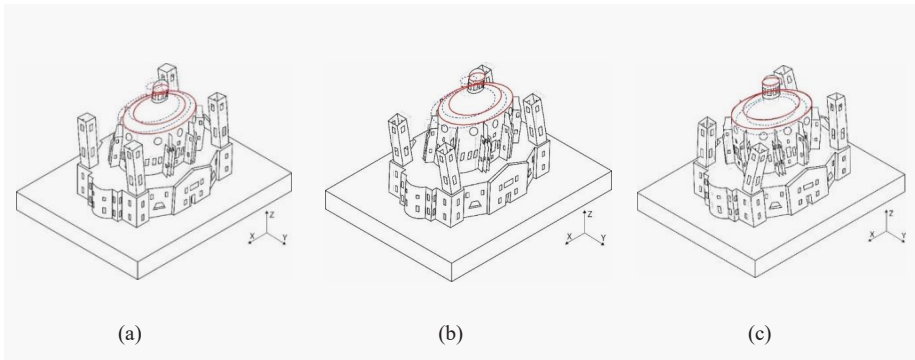


Fig. 9. First three FE modal shapes of the Sanctuary of Vicoforte: (a) 1-Y, (b) 1-X, (c) 1-T. Dotted lines identify the undeformed configuration at the dome level.

was used to model the lantern, the dome, the drum and the buttresses. This is a layered shell element having in-plane and through-thickness thermal conduction capability, suitable for conducting static and transient thermal analysis. The tie-bars of the strengthening system were modelled using 2-node beam elements. Both materials, masonry and steel, were assumed to have isotropic thermal conductivity. For the conductivity parameter, typical values were  $K_c$  assumed, whilst the mass densities were the same ones used in the preliminary model calibration.

Macro-element	E (GPa)		$\nu$ (-)		$\rho$ (Kg/m <sup>3</sup> )	
	pre.	fin.	pre.	fin.	pre.	fin.
Bell-towers	2.00	<b>4.50</b>	0.35	0.35	1800	1800
Basement	2.90	<b>2.00</b>	0.35	0.35	1800	1800
Buttresses	2.70	<b>5.50</b>	0.30	0.30	1700	1700
Clay	0.55	<b>0.75</b>	0.35	0.35	1900	1900
Dome	5.90	<b>5.50</b>	0.35	0.35	1800	1800
Drum	2.60	<b>2.30</b>	0.30	<b>0.35</b>	1700	1700
Lantern	1.80	<b>5.60</b>	0.35	0.35	1800	1800
Marlstone	4.15	<b>5.60</b>	0.35	0.35	2100	2100
Steel	210	210	0.30	0.30	7800	7800

Table 1. Comparison between preliminary (*pre.*) and final (*fin.*) elastic parameters.

Table 1 reports a comparison between the preliminary elastic parameters and those resulting from the updating process described in the flowchart of Figure 7. From this table can be observed that there are significant changes in the mechanical properties of many of the macro-elements relative to the preliminary model updating. This is visible also by looking at the Young's modulus of the buttresses before and after the calibration. Its value changed about of the 100% after the updating, only thanks to the use of local thermo-elastic data, i.e. temperatures and internal forces in the tie-bars. Finally, it is worth noting that also the bell-towers, that are not instrumented, underwent important changes after calibration.

Figure 8 reports pre- and post-updating load trends associated with data acquired by a single load cell, while Figure 9 depicts the first three modes of the updated FE model. The diagnosis of the Sanctuary can therefore be performed on a model-driven-approach.

### 7.1.3. Data-driven SHM

As said, SHM of heritage buildings is influenced by environmental conditions, especially temperature. To avoid false-positive or false-negative alarms of damage, Cross et al. (2012) proposed a ML approach based on the concept of linear cointegration. Another interesting class of machine learners based on statistical learning theory and Bayesian variants to perform nonlinear cointegration are the Support Vector Machines (SVMs) and Relevance Vector Machines (RVMs), which have the advantage of working well with sparse data sets. The Augmented Dickey-Fuller test confirmed that the model residuals  $\varepsilon$  are stationary and remain so throughout the monitoring

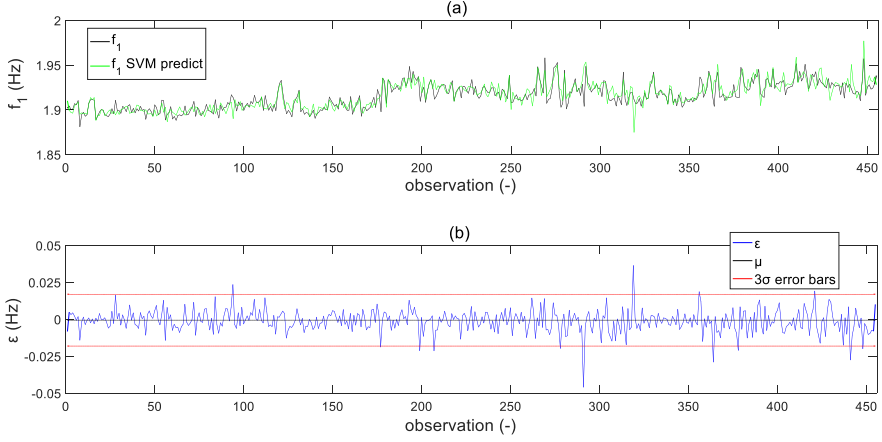


Fig. 10. SVM model of  $f_1$  using  $f_1, f_2, f_3, f_4$  and  $f_5$  (experimental modal frequencies).

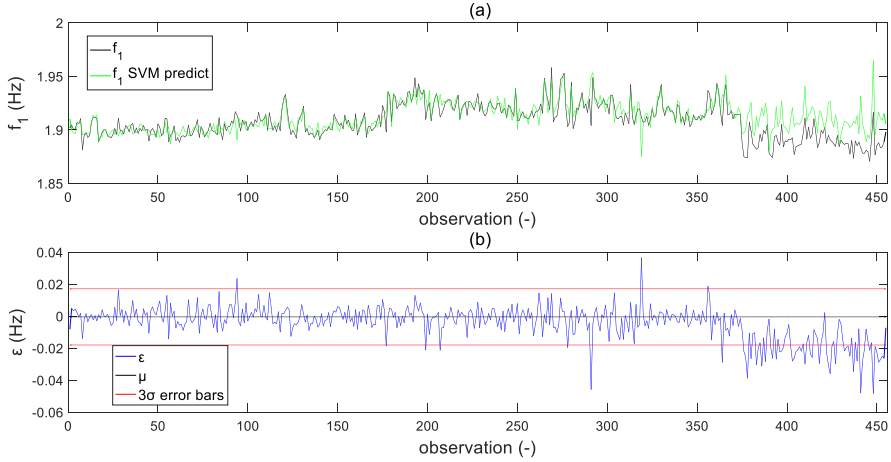


Fig. 11. SVM model of  $f_1$  using  $f_1$  and  $f_2$  (numerical model - damage in the buttresses).

period (no damage progression), as reported in Figure 10 for the SVM model (Coletta *et al.* 2019).

In order to validate the effectiveness of the damage detection method with labelled data, damage scenarios were simulated using the calibrated FE model of the Sanctuary that was available from the model-driven SHM. The damage in the FE model was simulated as a reduction of the Young's modulus of the

zones characterised by higher stresses under the self-weight load. The higher stresses occur at the base of the buttresses, therefore the Young's modulus of these macro-elements is set equal to 3.3 GPa, being reduced to 40% of the initial modulus reported in Table 1. A sample results for the damaged numerical case, reported in Figure 11, shows the difference between false-positive damage and true occurrence of damage in terms of regression residual.

As mentioned above, the lack of labelled data related to structural damage conditions represents a very significant problem in the field of SHM. With this in mind, recently a first application of a TL algorithm was applied to the Sanctuary's data (Coletta 2020). The Transfer Component Analysis (TCA) is a domain adaptation algorithm introduced by Pan *et al.* (2010) that tries to learn some transfer components across domains in a Reproducing Kernel Hilbert Space using the Maximum Mean Discrepancy as an embedding criterion. As a result, the data distributions from different domains are brought closer together in the subspace spanned by these transfer components. In this new subspace, ML algorithms can be trained for classification or regression problems on data from the source domain and tested on the (unlabelled or partially-labelled) target domain.

Also in this case it is the calibrated model that provides the missing information, acting as a source domain (Fig. 12). Two systems are thus considered to belong to a homogeneous population since they are intended to be identical in topology, geometry and materials (Gardner *et al.* 2020), aiming to improve the recognition of different environmental conditions, expressed by a temperature variation, within the distribution of dynamic parameters.

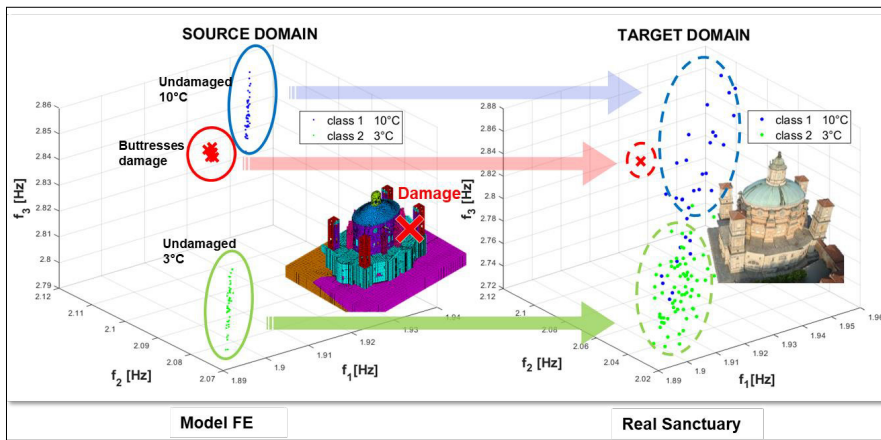


Fig. 12. Source and target domain features.

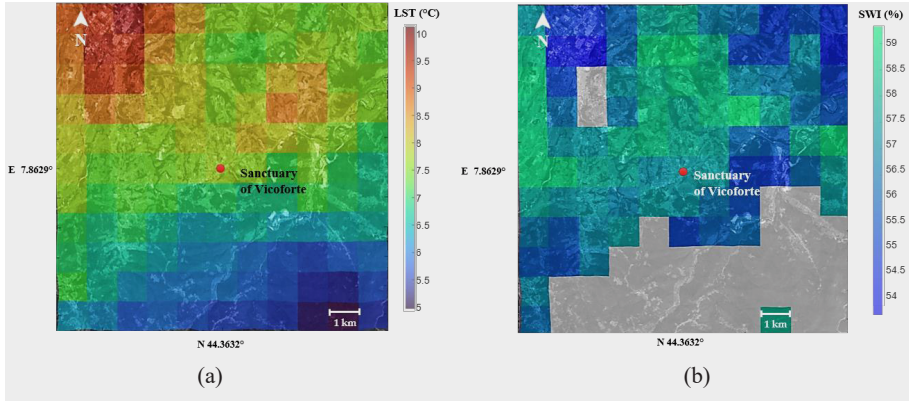


Fig. 13. Two-year (2018-19) average data interpolated on 121 points in the 11 km x 11 km grid around the Sanctuary: LST<sub>original</sub> (a) and SWI (b) data (Coccimiglio *et al.* 2022).

An RVM classifier (Tipping 2001) has been implemented on the Sanctuary data, before and after the use of the domain adaptation technique. TCA led to a marked improvement in the classification of the experimental data of the Sanctuary, subjected to two different environmental temperatures. The good results obtained in the recognition of different environmental conditions encourage the application of transfer learning for the purpose of damage detection, in which the FE model would be used to produce data related to virtual damages that would otherwise be difficult or impossible to obtain. Insights will be needed to recreate the optimal composition of this virtual data domain and to define to what extent the simplifications of a model can be crucial in improving SHM methods.

The need for greater control over the built environment has progressively led to the development of monitoring technologies that work at the territorial scale. However, when going on a larger scale the information available, e.g. from satellite multispectral data (Fig. 13) and aerial systems, turns out to be much less specific and effective. In this sense, TL algorithms can prove efficient in transferring knowledge from already investigated data sets in which the data labels are known, i.e. the health state of the structure on which those data were measured, to another set for which less specific information is available. Bringing together the different scales of monitoring, and putting them to interact in a multi-scale system would lead to significant progress in the SHM field.

## 7.2. Condition assessment of an iconic pre-stressed concrete building

The underground pavilion was designed by Morandi in 1958 as an expansion of the Exhibition Center dedicated to hosting the industrial vehicle section of the Turin Automobile Show. Pavilion V consists of a single wide space, 69 m in width and 151 m in length, located 8 m below ground level (Fig. 14).

The decision to build a hypogeum pavilion was made following various compromises and hypotheses elaborated by the engineer Bonadè Bottino, director of the Società Torino Esposizioni, and the Superintendence of Monuments of Piedmont. Bottino involved Morandi in the elaboration of the final project and for the structural calculations. The project was an opportunity for Morandi to concretize the long years of experimentation on pre-stressed reinforced concrete (Bruno 2013).

The general static scheme corresponds to the one frequently used by Morandi in bridges and overpasses, consisting of post-tensioned beams on two inclined supports, with two cantilevering side spans subsequently anchored by post-tensioning tendons at their ends, exerting a balancing effect on the bending moments in the main span. Different from these usual schemes, in the Pavilion V the main post-tensioned ribs are not parallel beams, but are diagonally directed and multiply reciprocally interconnected in order to obtain a spatial structure offering an high overall rigidity and lateral stability, and to contrast the instability of the very thin webs (16 cm) of the main ribs. In addition, the post-tensioned ties at the ends of the side spans are not inclined tendons anchored on the foundations of the main inclined supports, as in the bridges by Morandi, but are short ties embedded in prestressed concrete prismatic elements (shorter strut-beams), whose tension forces are balanced by the lateral retaining walls and by the load of the soil acting on their foundations.

The main supports for the entire structure are the internal inclined strut beams, which have a hexagonal shape, tapering from the center towards the two ends to perform the hinge constraint at the extremity points. At the top of these elements, the steel plates provide the connection with the ribs. The steel plate allows the rotation with respect to the vertical plan, ideally creating an element capable of supporting axial actions but unable to absorb bending moments (as reported in the static scheme by Morandi). The shorter strut beams were conceived with the aim of transforming the static scheme from determinate to indeterminate. The role of these latter elements is no less important, not only for the balanced beam pattern, but above all because they actually represent the most rigid restraint with respect to the longitudinal seismic action.

As shown in Figure 15 with a symmetrical half section from the original drawing, in each balanced beam, four long cables have been positioned lon-

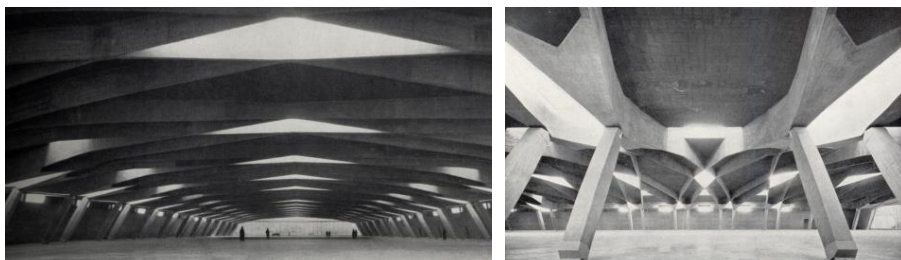


Fig. 14. Turin Exhibition Center, underground Pavilion by Riccardo Morandi: general views.

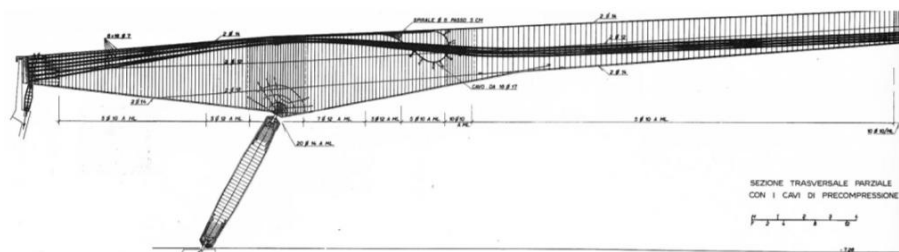


Fig. 15. Post-tensioning cables of the Pavilion V balanced beam (half section) from a drawing in Morandi (1959).

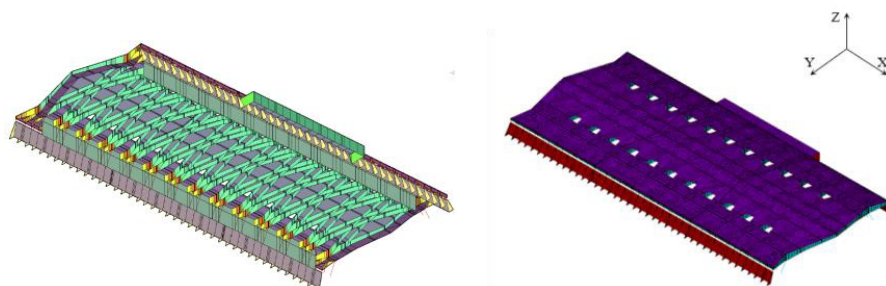


Fig. 16. Pavilion V: geometric model (left) and FE model (right).



gitudinally that cross the entire rib and two short cables for each of the two lateral cantilevers. Moreover, vertical cables have been placed into the shorter strut beams to apply a concentrated downward load at the edges. The tensioning of these elements was based on a series of consecutive operations.

The diagnostic activity began with the creation of a 3D geometric model of the Pavilion V on the base of geometric information obtained from the existing documentation, as well as from additional data collected during surveys. Figure 16 (left) reports the geometric model of the pavilion with detailed geometric information, which provided a proper reading of the structural typology of the pavilion and the recognition of possible design and construction principles. The structure is divided into three main bodies by means of two expansion joints, which cross the roof and the external walls, and whose behaviour was uncertain. The resulting FE model (Fig. 16 right) was corroborated with the data acquired after the experimental tests.

### 7.2.1. *Experimental campaign*

A test campaign was executed in 2019 in order to assess the condition of the structure. Both destructive and non-destructive tests were performed (e.g. see Fig. 17) in order to evaluate the health state of the various structural elements (MASTRLAB 2019, Ceravolo *et al.* 2022, Oliva *et al.* 2022). In details, the campaign started with inspection wells at the foundations, thermographic analysis of the structure, moisture content tests, geognostic surveys (SPT), state of possible subsoil contamination, ground penetrating radar (GPR) analysis and installation of new piezometers. Inspections and checks on the structures were then carried out, namely: determination of physical-chemical properties of the materials, tests on concrete, ordinary and pre-stressing steel, concrete cover, layout and characteristics of post-tensioning cables, possible grouting defects, steel corrosion and other chemical attacks, geometric characteristics in terms of position and diameters of reinforcing bars. Finally, mechanical tests have been executed in order to evaluate the compressive strength of the different elements of the structure: samples were extracted from foundations, retaining walls, ribs and longer strut-beams (see Fig. 17 left). Each sample was analysed with phenolphthalein to determine the progression of the carbonation front (Fig. 17 centre) and then subjected to a compression test.



Table 2 reports the average, minimum and maximum compressive strength values obtained for different elements, as well as Young's moduli.

ELEMENT	$f_{cMEAN}$ [MPa]	$f_{cMIN}$ [MPa]	$f_{cMAX}$ [MPa]	$E_{MEAN}$ [GPa]
Foundations	34.4	20.0	57.7	-
Retaining walls	56.9	46.6	63.0	37.0
Longer strut-beams	48.7	35.6	66.6	32.0
Ribs	41.0	25.4	61.9	33.0

Table 2. Pavilion V: results of compression tests on concrete samples.

The results show that the structure is made of a concrete with fairly high compressive strength. Different strength values, e.g. higher in the retaining walls, can be ascribed to different consolidation conditions, e.g. humidity. Furthermore, the carbonation levels of most samples are relatively low. The investigations have also verified the position of the reinforcement through metal detector tests, as well as the state of the post-tensioning system through the scarification of some ribs and shorter strut-beams. These investigations were useful to diagnose the important state of corrosion of the cable and the grouting defects. The program also included static tests on two different ribs, in order to assess the bearing capacity of the structure under characteristic variable actions.

The investigation campaign also included static tests on two different ribs to test the bearing capacity of the structure under the characteristic combination of the actions. The maximum displacement measured during the static test was 4.36 mm. These results showed a relatively high stiffness of the struc-



Fig. 17. Pavilion V: Extraction of concrete sample from a strut-beam (left); carbonation tests on sample of the ribs (centre); view of the static tests with trucks connected to the ribs through jacks (right).

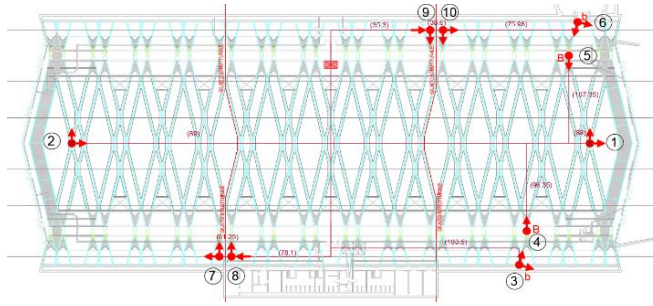


Fig. 18. Dynamic tests on Pavilion V: Setup 1.

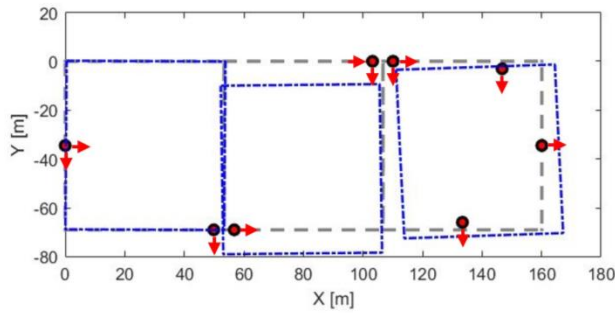


Fig. 19. Identified mode shape at 2.57 Hz (horizontal components).

ture under the imposed loads, despite a span of approximately 48 m between the inner supports.

Given the presence of uncertainties in the global dynamic behaviour of the pavilion, ambient vibration measurements were carried out to identify the modal characteristics and highlight possible criticalities for the preservation with respect to seismic actions. In the analysis of Pavilion V, dynamic tests represented an effective tool for increasing the knowledge level of the structure. These tests allow overall dynamic behaviour characterization of the structure and identify the possible losses of elasticity not immediately visible. The dynamic tests were carried out with natural excitation, which represents non-invasive investigation technique to avoid damaging the structure.

The preliminary FE model of the pavilion provided useful data to design the acquisition setups in order to maximize the content of extractable information and the spatial visualization of the modes. In fact, the structural complexity of Pavilion V directly affects the dynamic behaviour of the building

and the design of a successful design of the dynamic tests setup. The sources of complexity are the great rigidity of the system, the uncertainties related to behaviour of the joints, as well as the interaction with soil and non-structural inner walls in cellular concrete. In these conditions, the proper design of the dynamic tests plays a decisive role in the characterization process. The designed acquisition system was composed of 20 monoaxial piezoelectric accelerometers. In particular, accelerometers were placed in correspondence with the joints linking the blocks, to investigate how the interaction affects the dynamic behaviour of the three distinct bodies (Ceravolo *et al.* 2022, Scussolini *et al.* 2023). Overall, two setups were designed, paying close attention to favoring the modal decoupling. The first configuration was designed to obtain information in the horizontal (x-y) plane, while the second one mainly focuses on the vertical direction. Figure 18 reports with red arrows only the sensor measuring horizontal components.

The acquisitions lasted between 18 and 98 minutes. The signals, in terms of accelerations, were acquired by adopting two different sampling frequencies (128 Hz and 256 Hz), whilst the main modes were confined in the first 20 Hz. The results of the dynamic tests, the used identification algorithm and the estimation of the modal parameters can be found in Ceravolo *et al.* (2022). From the analysis of the mode shapes, the blocks are not appreciably affected by mutual interaction, this being indicative of the full effectiveness of the joints. In other words, the three blocks are likely to behave as fairly separated dynamic systems. As an example Figure 19 reports the representation of the horizontal components of the first mode (undeformed configuration in dashed lines, with considered sensor positions) as obtained exploiting a simplified model.

The results of the experimental tests were used to corroborate the model, in terms of elastic moduli and distributed mass on the roof as detailed in Laboratorio di Dinamica e Sismica (2019). Subsequently, the experimentally corroborated model can be used as a strategy to accomplish the condition assessment of the pavilion to preserve the structure, as described in the following section.

### 7.2.2. *Structural safety assessment*

For Morandi's pavilion one of the main static problems is represented by the conditions concerning the corrosion of the post-tensioning cables (Oliva 2022). In fact, although static and mechanical tests have shown an appropriate stiffness and concrete strength of the building, the structural safety in static configuration depends above all on the condition of the tendons. The dura-

bility of the elements is affected by poor grouting inside the tendons, which aggravate the problem of corrosion. In Pavilion V, direct inspections on the tendons of two ribs have revealed the presence of corrosion and poor grouting (Fig. 20 left). The poor grouting allowed to carry out an endoscopy inspection inside the duct (Fig. 20 right). In particular, hardened gray grout was found along the lower portion of the ducts, while poor segregated grout was found at the upper portion of the tendon. Generally, regions of varied grout quality and presence of strand corrosion products were visually assessed.

Since air voids and segregation defects are the major issues for post-tensioned structures and accelerate corrosion, sensitivity analysis were performed to investigate the safety level trend of balanced beam under changing corrosion scenario. In particular, ultimate load multiplier  $\alpha = q_{Rd}/q_d$  was defined as the ration between the ultimate crowd load on the roof,  $q_{Rd}$ , and the imposed load,  $q_d$ . It is worth highlighting that Morandi assumed a crowd load on the roof,  $q_d$ , equal to 4 kN/m<sup>2</sup>. Exploiting the calibrated FE model, the ultimate load multipliers  $\alpha$  were evaluated with the current European standards as a function of the percent increase of the corroded steel area  $A_{r,corr}$  (Fig. 21). This analysis can be a useful tool to accomplish the condition assessment of early post-tensioned systems, especially when it is not available exhaustive experimental checks on the cables for the entire building.



Fig. 20. Visual inspection of post-tensioning tendons in a rib (left), and endoscopy inspection inside a duct of the rib (right) (MASTRLAB 2019).

Furthermore, as expected, Pavilion V was conceived to withstand essentially vertical loads. For this reason, the calibrated model was used also for the seismic assessment with the current Italian national standards. A standard multimodal analysis with elastic response spectra was executed to evaluate the main criticalities and vulnerabilities of the pavilion. The seismic actions were applied along the two horizontal and vertical directions, according to

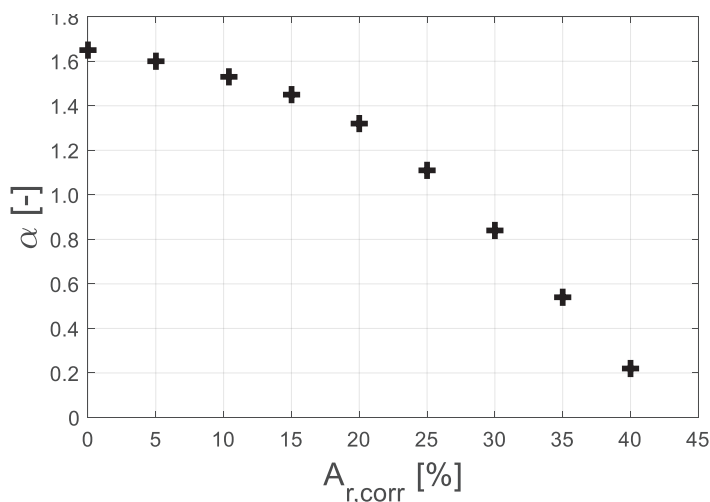


Fig. 21. Sensitivity analysis on the rib with the calibrated FE model: the ultimate load multiplier as a function of corroded post-tensioning steel area in the rib (Oliva 2022).

italian seismic standard. For the seismic action a return period  $T_R = 949$  years was assumed (strategic structure use).

As a result of the seismic assessment, both shorter and longer strut beams of the pavilion were recognized as critical elements. The assessment of the selected elements provided the following results along the longitudinal axis: i) verifications of the longer strut beams not satisfied with respect to axial and bending forces; ii) insufficient shear reinforcement in the shorter strut beams (Fig. 15).

Moreover, based on the results coming from the vibration tests, the presence of effective joints could constitute a factor of vulnerability due to the possible pounding between the three distinct bodies. Pounding could be aggravated by the lack of edge beams at some sections of the joints. Therefore, the corrosion of steel in the tendons and the above-mentioned seismic vulnerabilities are confirmed as the main obstacles in the preservation of the pavilion. Possible solutions include an external exoskeleton to suspend the structure.

## Conclusions

Structural Health Monitoring in its modern meaning was born in the early seventies for the monitoring of inaccessible structural parts and has developed up to the present day, both with very refined axiomatic formulations, often

coming from the control sector, and thanks to heuristic applications, including those relying on Machine Learning.

The lecture discussed the lines of development of this distinctly multidisciplinary research field. The intrinsic difficulty in correctly formulating the great variety of pathologies and inverse problems that characterize structural diagnostics has been highlighted, as well as the difficulty of finding labelled information, in the sense of Machine Learning. On the other hand, some successful applications have been presented for the permanent and periodic monitoring of complex objects such as cultural heritage, which bodes well for the outcome of the great research effort expended in recent years.

The two case studies, which concerned traditional and contemporary architectural heritage, evidenced some technical limitations, namely (i) the influence of environmental and operational factors, (ii) the lack training labelled data relating to damaged conditions and (iii) the influence of soil conditions. In the context of the individual applications, some possible solutions to the problems raised were also presented. The proposed strategies are not always generalizable, but this is still a peculiarity of any approach to diagnostics.

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