Nonlinear and Linearized Analysis of Vibrations of Loaded Anisotropic Beam/Plate/Shell Structures

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Abstract: *In the last decades, new challenges demanded by aerospace, automotive and other engineering fields require the adoption of sophisticated and eventually lightweight components. These highly flexible structures are employed extensively in various engineering fields. The great potential of these structures is to exhibit large displacements and rotations without showing plastic deformations. The static and dynamic behaviours of these structures play a crucial role in their function in many technical areas and their knowledge is important from the preliminary design phase with a significant impact on safety, performance and comfort. It is a matter of fact that stability considerations are a fundamental and inevitable topic in the design of many engineering structures employed in various fields. The concept of stability is intrinsically a dynamical one, and the formal analogy between buckling and vibration has stimulated the use of the vibration (dynamic) approach as a standard procedure in the design of structures to get important information to characterize buckling behaviours. The stability of structures subjected to compressive loads, essential for the design and validation of the safety of new structures, continues to be extensively verified through experimental tests. However, one of the researchers' goals over the years has been to reduce both the time and cost of operations of these complex analyses to determine the buckling load. One of the effective ways to do this is to adopt non-destructive experimental tests to evaluate the critical load of structures. The employment of non-destructive techniques in aerospace industries is rising thanks to advances in technologies and analysis. This part of the aerospace testing industry is essential to design and validate the new structures' methodology and safety. Therefore, robust and reliable non-destructive methods have been extensively studied for decades in order to reduce safety problems and maintenance costs. One of the most important and employed non-destructive methods to compute large-scale aerospace structures' critical buckling load is the Vibration Correlation Technique (VCT). This method computes the buckling load and the equivalent boundary conditions by interpolating the natural frequencies of the structures for progressively increasing applied loadings without reaching instabilities.*
In this context, the present work intends to propose an advance toward the efficient implementation of a novel numerical approach for carrying out nonlinear vibration-buckling investigations of isotropic, classical composite and variable-angle-tow (VAT) composite beam, plate and shell structures subjected to different mechanical and thermal loadings in order to predict buckling loads, to characterize the natural frequencies variation for progressively increasing loadings and to provide an efficient means for the verification of the experimental VCT results. In the case of large deformations, natural frequencies and mode shapes may be seriously affected by pre-stress states. As a consequence, important mode aberrations, e.g., veering, crossing and mode jumping, are observed. Moreover, the dynamic characteristics of any structure are inherently a property of the equilibrium condition. For this reason, the availability of accurate mathematical models able to deal with higher-order phenomena, which may occur within the structure, is of pivotal relevance. In addition, in many dynamic motions of engineering structures, nonlinear behaviours are always present. Consequently,

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in order to accurately investigate their structural dynamic behaviours is essential to consider the geometrical nonlinearities.
The proposed nonlinear methodology is based on the combination of the well-established Carrera Unified Formulation (CUF) and the Finite Element Method (FEM). CUF represents a hierarchical formulation in which the order of the structural model is considered as an input of the analysis. Thus, no ad-hoc formulations are needed to achieve any refined generic model. With this procedure, the nonlinear governing equations and the relative finite element arrays of the one-dimensional (1D) and two-dimensional (2D) theories are formulated in terms of Fundamental Nuclei (FNs). FNs represent the basic building blocks of the proposed formulation. According to CUF, the 3D displacement field can be expressed as an arbitrary expansion of the generalized displacements. Depending on the choice of the polynomials employed in the expansion, various classes of beam, plate and shell models can be implemented. In detail, both Taylor-like (TE) and Lagrange (LE) polynomials are considered for developing the kinematic expansion. However, it should be highlighted that when laminated composite structures are considered, the Layerwise (LW) models adopting LE and the Equivalent-single-layer (ESL) models using TE are employed and compared. All Green-Lagrange strain components are employed because far nonlinear regimes are investigated. Furthermore, the geometrical nonlinear equations are written in a total Lagrangian framework and solved with an opportune Newton-Raphson method along with a path-following approach based on the arc-length constraint.
The first part of the thesis is devoted to show the formulation of a displacement-based higher-order 1D and 2D theory for static and dynamic structural analyses. The importance of correctly choosing the structural theory, kinematic model and nonlinear strain measure model to perform accurate analyses is remarked through various numerical results. In detail, the need to adopt higher-order and full nonlinear strain models is emphasized in the analyses in order to accurately evaluate the 3D stress fields and undamped nonlinear transient responses in isotropic and composite structures.
The second part is primarily focused on the efficiency and reliability of the presented numerical approach when applied to vibration analysis of pre-stressed thin-walled structures in the highly nonlinear regime. In detail, the numerical results compared to the experimental one showed excellent accuracy and reliability of the presented approach. Results showed the potential of refined 1D and 2D theories to reduce the computational costs of real problems simulation without drastically compromising the accuracy. Various numerical examples for different equilibrium conditions in the moderate and large displacement fields are proposed. The validity of the proposed formulation is demonstrated and modal aberrations as a consequence of the loading, the nonlinear equilibrium state, and the material anisotropy are discussed. The virtual VCT approach presented can be used effectively during the preparation of experimental tests in order to appropriately investigate the boundary conditions to be applied or it can be a powerful method to be employed to investigate cases that are difficult to analyze experimentally, such as structures subjected to thermal or shear loads, among others.
Finally, possible future works include large strain analysis, accounting of material nonlinearities, hyper-elasticity and fluid-structure interaction applications.

Keywords: Finite Element Method; Carrera Unified Formulation; Higher-order beam/plate/shell models; Isotropic material; Composite material; Variable stiffness composites; Geometrical nonlinearity; Buckling; Large-deflection; Post-buckling; Vibration analysis; Vibration Correlation Technique; Mode aberration; Nonlinear dynamic response.