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Dynamic crack initiation by Finite Fracture Mechanics

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Abstract

The development of robust failure criteria suitable for predicting crack initiation under both quasi-static and dynamic loading conditions has always garnered attention in the scientific community due to its positive impact in the trustworthiness of the failure predictions for critical structural applications. In this sense, the present work applies the well-established Finite Fracture Mechanics framework to yield failure predictions depending on the loading rate. To achieve so yet keep the analysis simple, the analogy existent between the Incubation Time dynamic failure criterion and the static formulation of the Theory of Critical Distances is herein exploited to enable the application of Finite Fracture Mechanics under certain high loading rate conditions. In particular, this is achieved by coupling the concept of incubation time to its conventional formulation.

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Keywords: Finite Fracture Mechanics; Dynamic Fracture; Loading rate

1. Introduction

Despite the important advances that the field of Fracture Mechanics has undergone in the last decades, the prediction of failure for complex geometries and/or loading regimes still remains an open topic. In this regard, a groundbreaking contribution was made by Leguillon (2002) and Cornetti et al. (2006) with the introduction of the Finite Fracture Mechanics failure criterion, which relied on assuming that failure initiation occurs in a finite manner once stress and

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energy conditions are simultaneously met. Indeed, the coupling of various necessary non-local conditions for fracture resulted in overcoming many of the limitations present in the conventional Linear Elastic Fracture Mechanics. Subsequently, the criterion has been ever since successfully applied to a wide range of geometrical setups under both quasi-static and fatigue loading regimes (see e.g. Ferrian et al. 2022, Sapora et al. 2021).

Nonetheless, the prediction of failure under dynamic loading conditions results noticeably more arduous due to the complexity in coping with varying-in-time magnitudes. As of this, the pieces of research focusing on the applicability of Finite Fracture Mechanics to dynamic loading conditions are still scarce, and only the previous works by Laschuetza et al. (2021) and Doitrand et al. (2022) contain relevant discussions. On the other hand, several different failure criteria were proposed in the past, among which the following can be highlighted: the "Classical dynamics approach" (see Petrov et al. 2003), the Dynamic reformulation of the Theory of Critical Distances (Yin et al. 2015), the Dynamic Quantized Fracture Mechanics (Pugno 2006) and the Incubation Time failure criterion (Petrov and Morozov 1994). In particular, the latter approach resulted the most promising and was reported to yield good accuracy in comparison with experimental data, although its application to small-sized specimens being precluded. Consequently, the present work will exploit the concept of incubation time and merge it to the Finite Fracture Mechanics formulation in an aim to benefit from the strengths of Finite Fracture Mechanics and improve the performance of the previously proposed dynamic failure criteria (in the sense of criteria suitable for rapidly exerted loadings).

2. Application of Finite Fracture Mechanics to dynamic crack initiation

Let us consider a generic loading case as that one described in Fig. 1: a structural domain Ω filled with a homogeneous, isotropic, linear elastic and brittle material is subjected to a dynamic loading $\Sigma^{dyn}(t)$ and to certain boundary conditions, so that the prospective failure is expected to stem from the stress raiser Γ in pure Mode I. Then, for said setup, failure initiation according to the Incubation Time (IT) failure criterion (Petrov and Morozov 1994) is governed by:

$$\frac{1}{\tau}\left\{\int_{t_f}^{t_f-\tau}\left[\frac{1}{d}\int_0^d \sigma_I^{dyn}\left(r,t\right)\mathrm{d}r\right]\mathrm{d}t\right\}=\sigma_c\,, \tag{1}$$

where $\sigma_I^{dyn}(r,t)$ is the dynamic crack-opening stress component, τ is the incubation time, and d is a fixed characteristic length defined in terms of the fracture toughness K_{Ic} and the quasi-static strength σ_c as:

$$d=\frac{2}{\pi}\left(\frac{K_{Ic}}{\sigma_c}\right)^2. \tag{2}$$

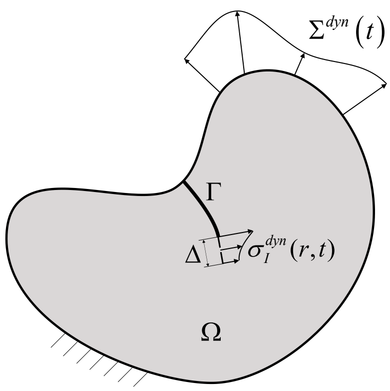


Fig. 1. Schematic representation of a generic dynamic loading scenario.

Clearly, should the variability in time of $\Sigma^{dyn}(t)$ be negligible and thus the loading be quasi-static, the expression in Eq. (1) particularizes as:

$$\frac{1}{d}\int_0^d \sigma_I\left(r\right)\,\mathrm{d}r = \sigma_c\,, \tag{3}$$

which exactly coincides with the expression of the Line Method of the Theory of Critical Distances (TCD-LM) proposed by Taylor (2007). Consequently, it is seen that both the IT and the TCD-LM criteria are mutually related in the sense that the latter is the quasi-static particular case of the former. In turn, for the cases in which there exists a one-to-one relation between the Σ^{dyn} and σ_I^{dyn} so that the all the temporal dependence of the stress arises from the external load, i.e. it is possible to write $\sigma_I^{dyn}\left(r,t\right) = \sigma_I^{dyn}\left(r,\Sigma^{dyn}\right)$, the IT failure criterion can be rewritten as:

$$\frac{1}{\tau}\left[\int_{t_f}^{t_f-\tau} \Sigma^{dyn}\left(t\right)\mathrm{d}t\right] = \Sigma_f^{TCD-LM}\,, \tag{4}$$

meaning that the instant of failure t_f –and the corresponding dynamic failure load $\Sigma^{dyn}(t_f)$ – can be directly determined from the quasi-static failure prediction yielded by the TCD-LM Σ_f^{TCD-LM}. Nonetheless, it is also seen that the existence of the univocal $\sigma_I^{dyn} - \Sigma^{dyn}$ relation also leads to the uncoupling of the spatial and temporal aspects of failure. In turn, this implies that Σ_f^{TCD-LM} in Eq. (4) can be substituted by the quasistatic failure prediction provided by any criterion, thus allowing them to incorporate the incubation time concept and yield dynamic failure predictions.

In particular, the average stress formulation of Finite Fracture Mechanics (FFM) proposed by Cornetti et al. (2006) and defined in Eqs. (5) will be utilized in the next section to provide the quasistatic failure prediction on which to base the dynamic ones. In said equation, Δ is a measure of the finite crack advance, $A(\Delta)$ the associated crack surface, and Δ_f its critical value for failure, whereas $G(a,\Sigma)$ stands for the energy release rate and G_c for the fracture energy.

$$\begin{gathered}\Sigma_f^{FFM} = \min\left(\Sigma \in S\right);\quad \Delta_f = \Delta(\Sigma_f^{FFM});\\ \text{where } S := \left\{\Sigma \,\middle|\, \Sigma : \begin{bmatrix}\int_{A(\Delta)} \sigma_I(r,\Sigma)\,\mathrm{d}r \geq \sigma_c\, A\left(\Delta\right)\\ \int_{A(\Delta)} G(a,\Sigma)\,\mathrm{d}a \geq G_c\, A\left(\Delta\right)\end{bmatrix}\right\}.\end{gathered} \tag{5}$$

Likewise, for failure scenarios under pure Mode I conditions as what herein hypothesized, it is possible to write the energy balance in terms of the Stress Intensity Factor $K_I\left(a\right)$ and the fracture toughness K_{Ic} by virtue of the Irwin's relation.

3. Comparison with experiments

The correctness of the approach presented in the previous section will now be tested against the relevant experiments performed by Dai et al. (2010a) on Semi-Circular Bend (SCB) specimens made out of Laurentian Granite (see Fig. 2 (a)). The tested specimens were reported to have a radius $R = 20$ mm, an out-of-plane thickness $B = 16$ mm, and a distance between the supporting rollers $S = 21.8$ mm.

Furthermore, one must keep in mind that the validity of Eq. (4) is the keystone of the procedure herein used for coupling the incubation time concept and the FFM formulation. Therefore, there must be ensured that, for the experimental setup selected for comparison, the dynamic stress is an explicit function of just the spatial coordinates and the dynamic external loading, i.e. $\sigma_I^{dyn} = \sigma_I^{dyn}\left(r,\Sigma^{dyn}\right)$. In this regard, the work by Dai et al. (2008) proved that, for specimens as the one under consideration, dynamic equilibrium conditions take place when tested with a Split Hopkinson Bar using the pulse shaping technique (Frew et al. 2002). Moreover, these conditions were shown to further imply that the expression relating the static stress field and the static external load still holds for the dynamic case, thus meaning that Eq. (4) remains valid for the case at hand.

In particular, the pulse shaping technique utilized by Dai et al. (2010) was reported to exert a dynamic external load $P(t)$ onto the SCB specimen (see Fig. 2(a)) following a ramp of constant slope, which mathematically can be expressed as:

$$P(t)=\begin{cases} 0 & \text{for } t<0 \\ \dot{P}t & \text{for } t\geq 0 \end{cases}, \tag{6}$$

where $\dot{P}$ represents the so-called loading rate.

At this point, Eq. (4) can be reformulated into Eq. (7) for the case at hand, where the quasi-static failure load P_f^{FFM} is already determined from the FFM approach (see Eq. (5)) instead of from the TCD-LM criterion.

$$\frac{1}{\tau}\left[\int_{t_f}^{t_f-\tau} P(t)\,\mathrm{d}t\right]=P_f^{FFM}\,. \tag{7}$$

In turn, introducing the definition of the external load $P(t)$ from Eq. (6) into Eq. (7), one obtains the expression in Eq. (8) that results from the coupling of the FFM with the incubation time concept (FFM+τ).

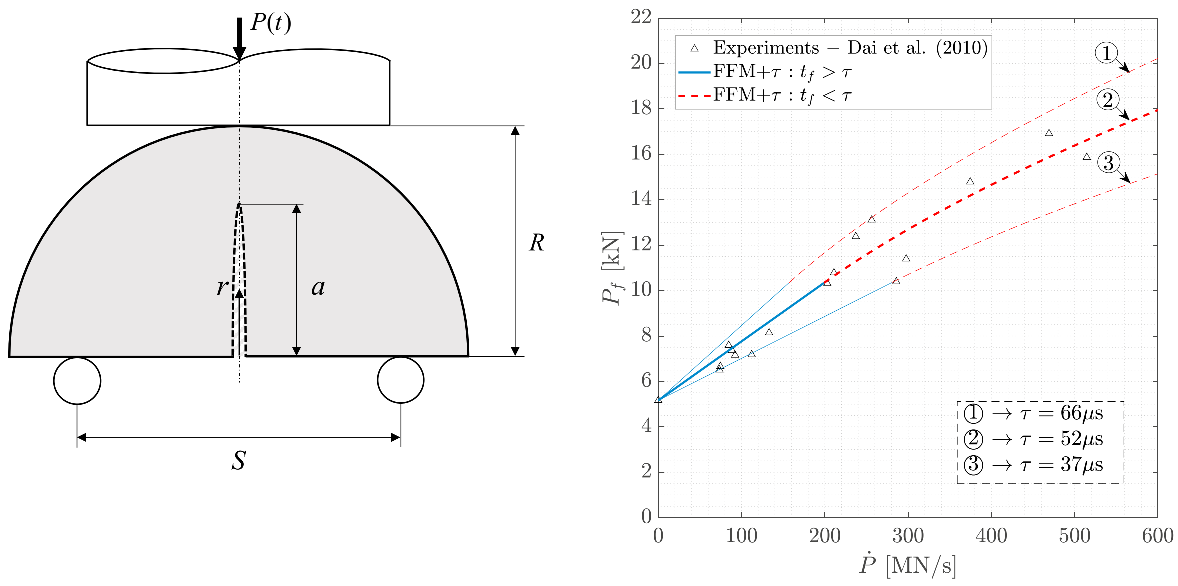
$$P_f^{dyn}(\dot{P})=\begin{cases} P_f^{FFM}+\dfrac{\dot{P}\tau}{2} & \text{for } \dot{P}<\dfrac{2P_f^{FFM}}{\tau} \\ \sqrt{P_f^{FFM}}\sqrt{2\dot{P}\tau} & \text{for } \dot{P}\geq\dfrac{2P_f^{FFM}}{\tau} \end{cases}. \tag{8}$$

Therefore, it is expected that the dynamic failure of the specimen presents two well-differentiated regimes: for low loading rates, the dynamic failure load P_f^{dyn} presents a linear dependence with the loading rate $\dot{P}$; on the other hand, the dynamic failure load P_f^{dyn} becomes proportional to $\sqrt{\dot{P}}$ for high loading rates. Interestingly, the latter regime is triggered once the time to fracture t_f becomes smaller than τ. In any case, the continuity and smoothness at the transition point between both failure regimes is inherently ensured by the formulation.

Henceforth, the values of P_f^{FFM} and τ must be calibrated with the experimental results by Dai et al. (2010). The former magnitude is in turn determined out of the quasi-static material properties σ_c and K_{Ic}. Notice that failure in the present setup occurs in pure Mode I, and so the Stress Intensity Factor and the fracture toughness are used for the FFM's energy balance for being possible and more convenient. Likewise, the expressions required for the stress field and the stress intensity factor are determined through interpolation of several FE punctual solutions.

Regarding the calibration of quasi-static properties, the tensile strength is herein set to the value commonly reported in the literature for Laurentian Granite, i.e. $\sigma_c=12.8$ MPa (see Iqbal and Mohanty (2006)). Concurrently, the value of the fracture toughness is fixed as $K_{Ic}=2.48\ \mathrm{MPa}\sqrt{\mathrm{m}}$ in order to be able to reproduce the reported quasi-static failure load of $P_f=5.16$ kN with the FFM formulation.

On the other hand, the determination of the incubation time on the basis of the results by Dai et al. (2010) is more onerous per the large scatter present in the experimental results. Indeed, as shown in Fig. 2 (b), the overall trend of the experimental results is nicely captured on average by setting $\tau=52.0$ μs, while the lower and upper bounds of τ according to the experiments are $\tau=37.0$ μs and $\tau=66.0$ μs, respectively. From this figure, it results clear that the incubation time modulates the sensibility of the failure load with the loading rate: the higher (lower) the value of τ, the higher (lower) the rate sensibility of the dynamic failure load.



(a) (b)

Fig. 2. (a) Geometrical definition of the SCB specimen; (b) comparison of the dynamic failure loads predicted by FFM+ τ with experimental results.

4. Conclusions

The incorporation of the incubation time concept into the framework of Finite Fracture Mechanics for yielding failure load predictions under dynamic loadings was preliminarily explored. To this end, the relation existent between the Incubation Time failure criterion and the Theory of Critical Distances under certain circumstances was replicated with the Finite Fracture Mechanics approach to obtain a counterpart suitable for dynamic conditions. Indeed, the resultant simplistic procedure showed reasonable accuracy with respect to dynamic experiments on rock specimens, thus resulting useful to conduct first-order analyses of the loading rate effect on the failure load.

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