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# A study on the statistical convergence of turbulence simulations around a cylinder

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**Abstract.** The turbulent flow around a circular cylinder at a Reynolds number equal to 3900 is studied by an implicit Large Eddy Simulation performed by means of a discontinuous Galerkin finite element solver. The average velocity field in the wake is evaluated and compared with experimental data from the literature. The focus of the present work is on the estimation of the statistical uncertainty which is related to the use of a finite time window for the averaging operation. This topic represents an open problem for both Direct Numerical Simulations and Large Eddy Simulations in which it is difficult to define a priori the size of the time window which gives statistically converged averaged quantities. Different techniques to estimate this uncertainty are compared in order to get a quantitative criterion for checking the convergence of statistics. In particular, the Non-Overlapping Batch Means and the Batch Means Batch Correlations techniques are applied to the present test case.

## INTRODUCTION

The simulation of turbulent phenomena in many flows of industrial interest has been based on Reynolds Averaged Navier-Stokes (RANS) equations for several decades. The growing computational power which has become available in the last years makes it possible to describe turbulent flows with scale resolving approaches (like Direct Numerical Simulations (DNS) or Large Eddy Simulations (LES)) which improve significantly the prediction capability in several flows characterised by separation and laminar to turbulent transition. However, new challenges are related to the use of DNS or LES approaches. For example, the computation of the average field is not trivial when DNS or LES are performed. While the RANS approach gives directly the averaged fields, DNS and LES give unsteady fields on which time averaging must be performed. However, it is usually difficult to know a priori the proper time window size.

In order to perform a quantitative check on the statistical convergence level of the results it is possible to apply several algorithms which allow to estimate the variance of the computed average field: among them, the Non-Overlapping Batch Means (NOBM) method [1], the Overlapping Batch Means (OBM) method [2] and the Batch Means Batch Correlations (BMBC) method [3] represent similar approaches which share a common idea. In particular, all these methods require to define a set of subsamples (batches) from the computed time history.

In the present work the turbulent flow around a circular cylinder at a Reynolds number equal to 3900 is studied by means of an implicit Large Eddy Simulation with a discontinuous Galerkin solver. This flow was experimentally investigated by Lourenco and Shi [4] and by Parnaudeau et al. [5] who provided the average velocity field in three control stations in the wake. The numerical predictions are compared with the experimental data and the variance of the average velocity profile is estimated by the NOBM and BMBC methods for different sizes of the time window. The OBM method is not considered here because of its larger computational cost [2].

## NUMERICAL FRAMEWORK

The flow field is described by the compressible Navier-Stokes equations with an ideal fluid with constant specific heat ratio  $\gamma = 1.4$ . The far field Mach number is set to  $M_\infty = 0.2$ . No subgrid model is applied so the following computations can be classified as Implicit Large Eddy Simulation (ILES). The computational domain is reported in Figure 1a. The external boundary is set at 16 diameters from the cylinder while the spanwise length is equal to 3 diameters. Periodic boundary conditions are applied in the spanwise direction. An unstructured mesh with 300000 hexahedra elements is generated by means of Gmsh [6], following the indications on wall resolution and domain size from [5]. Simulations are carried out by means of a parallel unstructured Fortran 90 code which was tested on both compressible and incompressible problems [7, 8, 9, 10]. The parallelisation is based on the Message Passing Interface (MPI) framework and it is managed by the DMplex class provided by the PETSc library [11]. Computations are carried out on the CINECA Marconi cluster with 1088 physical cores. The spatial discretisation of the equations is performed by means of a discontinuous Galerkin method in which an orthonormal modal basis is defined in each element, following [12]. A third order accurate reconstruction is adopted for the present work: this means that 10 degrees of freedom are used for each equation in each element and 3 millions of degrees of freedom for the entire solution. Convective fluxes are evaluated by means of an approximate Riemann problem solver [13] while diffusive fluxes are computed by means of a recovery based approach [14]. An explicit second order Runge-Kutta scheme is adopted for time integration.

## STATISTICAL UNCERTAINTY

Consider a time dependent variable  $x(t)$  defined on the interval  $[t_0, t_f]$ . The algorithms described in the following require to split the time interval in a set of batches with the following size:  $\Delta t = (t_f - t_0)/K$  where  $K$  is the number of batches. The global average  $\hat{\mu}$  and the local average  $\bar{X}_k$  performed on the  $k$ -th batch are defined as

$$\hat{\mu} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} x(t) dt, \quad \bar{X}_k = \frac{1}{\Delta t} \int_{t_{k-1}}^{t_k} x(t) dt \quad 1 \leq k \leq K \quad (1)$$

where  $t_k = k\Delta t$ . The average distance from  $\hat{\mu}$  is defined for each batch as

$$\bar{x}_k = \frac{1}{\Delta t} \int_{t_{k-1}}^{t_k} (x(t) - \hat{\mu}) dt \quad 1 \leq k \leq K \quad (2)$$

If a finite number of samples is considered, it is possible to define a correlation matrix following the approach of Russo and Luchini [3]. In this way the contributions from the diagonal and near-diagonal values of the matrix are summed in the terms  $S_0$  and  $S_1$ , respectively:

$$S_0 = \sum_{k=1}^K (\bar{X}_k^2 - \hat{\mu}^2) = \sum_{i=1}^K \bar{x}_i^2, \quad S_1 = \sum_{k=1}^{K-1} \bar{x}_k \bar{x}_{k+1} \quad (3)$$

### Non-Overlapping Batch Means (NOBM) Method

In the NOBM method only the terms which belong to the blocks on the diagonal of the correlation matrix are considered. In this way the statistical variance related to the average  $\hat{\mu}$  can be computed as

$$\hat{\sigma}_{NOBM}^2 = \frac{S_0}{K(K-1)} \quad (4)$$

This approach can be straightforwardly implemented. However, the results are influenced by the choice of the batch size ( $\Delta t$ ) and the optimal choice is problem dependent [15]. Furthermore, the NOBM method introduces a bias in the variance estimation [3].

### Batch Means Batch Correlation (BMBC) Method

The BMBC method keeps more terms of the correlation matrix with respect to the NOBM method. In particular the variance is estimated as

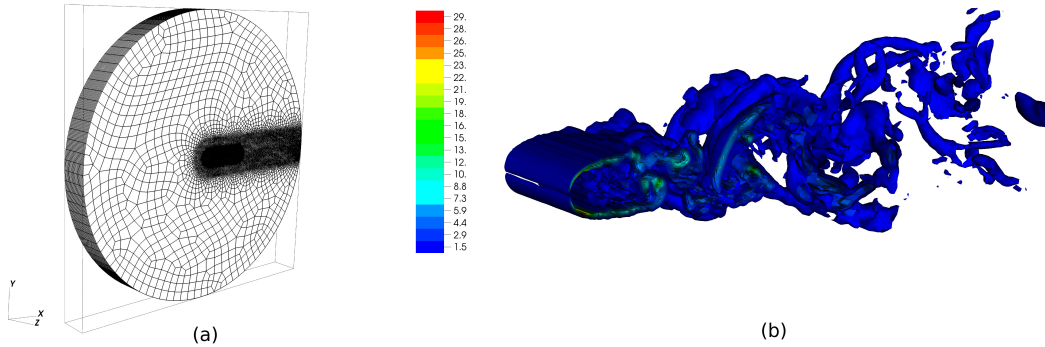
$$\hat{\sigma}_{BMBC}^2 = \frac{S_0 + 2S_1}{(K-1)(K-2)} \quad (5)$$

The additional terms included in  $S_1$  give the possibility to avoid the bias in the variance estimation. However, the problem of choosing the optimal batch size  $\Delta t$  remains. Russo and Luchini [3] suggest to perform a study for different values of the batch size and to choose a value which gives approximately  $S_1/S_0 = 0.5$ . In particular, they observe that if the batch size is chosen too small then many near-diagonal terms are neglected and this introduces a significant bias error in the variance estimation. On the other hand, if the batch size is too large then a loss of information could occur. In the following this criterion will be tested on the problem under investigation.

## RESULTS AND CONCLUSIONS

A preliminary simulation is performed with a second order accurate scheme starting from a uniform field. After 200 convective times the solution is used to initialise a third order accurate simulation which is carried out for 100 convective times. Starting from this point, statistics are computed for different sizes of the time window (12.5, 25 and 50 convective times). The obtained results are spatially averaged in the spanwise direction ( $z$ ) and then the time average is performed. The results are reported in Figure 2 where the velocity profile in the transverse direction ( $y$ ) is shown together with the uncertainty ranges obtained by the NOBM and BMBC methods with a batch size  $\Delta t = 0.25$  convective times. The results refer to a control station at  $x/D = 1.06$  where  $x$  and  $D$  are the streamwise position and cylinder diameter, respectively. The Figure shows clearly that as the time window size is increased the predicted average velocity profile gets closer to the experimental values of [5] and the statistical uncertainty  $\sigma$  is reduced. It is possible to see that the predicted velocity shows a significant error with respect to experimental values at the edges of the wake (approximately at  $y/D = \pm 0.7$ ): this is probably due to insufficient spatial resolution in that region.

In order to investigate the influence of the batch size different values of  $\Delta t$  are considered for the database with 50 convective times. In Figure 3, the effect of the batch size choice on the variance and on the  $S_1/S_0$  term is evaluated. It is clear that when the goal of the average operation is the computation of a spatial distribution it is difficult to choose the batch size: if the same batch size is used for all the spatial points then different values of  $S_1/S_0$  are obtained and so it is difficult to impose the condition  $S_1/S_0 \approx 0.5$  suggested by Russo and Luchini [3]. The present work is a preliminary study which was carried out on a limited time window size: future work will include the study of larger time histories in order to capture possible low frequency oscillations which are not visible in the chosen time window.



**FIGURE 1.** Computational mesh (a) and vorticity magnitude field (normalised with respect to  $u_\infty/D$ ) (b)

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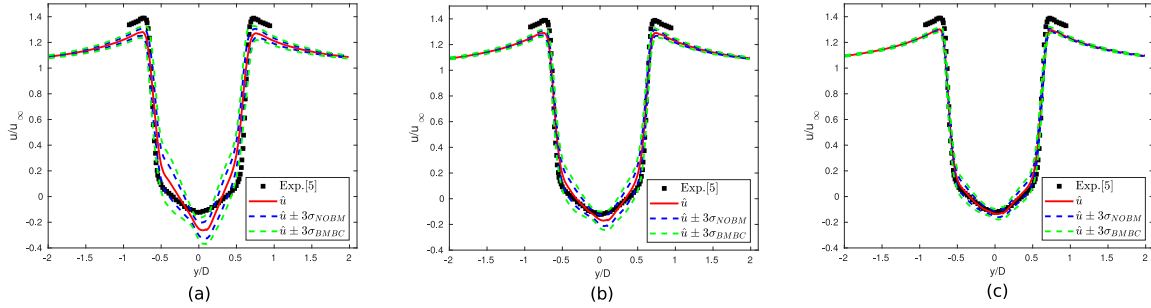


FIGURE 2. Streamwise velocity at  $x/D = 1.06$ : time window with 12.5 (a), 25 (b) and 50 (c) convective times.

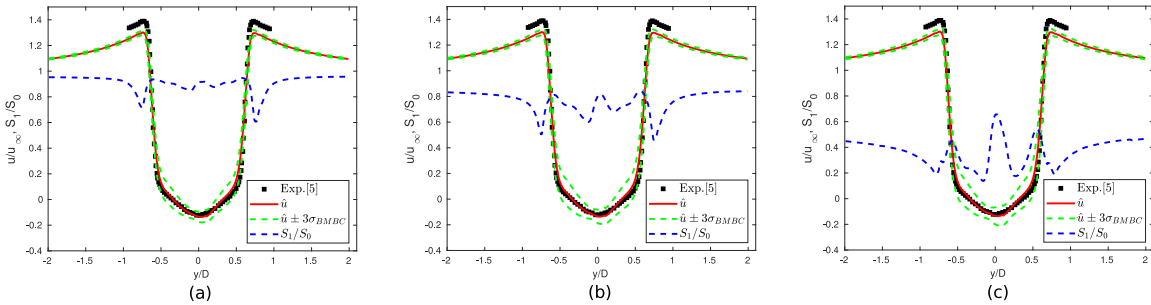


FIGURE 3. Study on the batch size effect with  $\Delta t = 0.25$ (a),  $0.5$  (b) and  $1$ (c) convective times.

## REFERENCES

- [1] R. W. Conway, B. M. Johnson, and W. L. Maxwell, *Management Science* **6**, 92–110 (1959).
- [2] M. S. Meketon and B. Schmeiser, “Overlapping batch means: Something for nothing?” in *Proceedings of the 16th conference on Winter simulation* (IEEE Press, 1984), pp. 226–230.
- [3] S. Russo and P. Luchini, *Journal of Computational Physics* **347**, 328–340 (2017).
- [4] P. Beaudan and P. Moin, “Numerical experiments on the flow past a circular cylinder at sub-critical reynolds number,” Tech. Rep. (Stanford Univ CA Thermosciences Div, 1994).
- [5] P. Parnaudeau, J. Carlier, D. Heitz, and E. Lamballais, *Physics of Fluids* **20**, p. 085101 (2008).
- [6] C. Geuzaine and J.-F. Remacle, *International journal for numerical methods in engineering* **79**, 1309–1331 (2009).
- [7] A. Ferrero and F. Larocca, *Progress in Computational Fluid Dynamics, an International Journal* **16**, 14–25 (2016).
- [8] E. Ampellio, F. Bertini, A. Ferrero, F. Larocca, and L. Vassio, *Advances in aircraft and spacecraft science* **3**, 149–170 (2016).
- [9] A. Ferrero, F. Larocca, and V. Bernaschek, *Advances in aircraft and spacecraft science* **4**, 555–571 (2017).
- [10] A. Ferrero, A. Iollo, and F. Larocca, *International Journal for Numerical Methods in Engineering* **116**, 332–357 (2018).
- [11] M. Lange, M. G. Knepley, and G. J. Gorman, “Flexible, scalable mesh and data management using petsc dmplex,” in *Proceedings of the 3rd International Conference on Exascale Applications and Software* (University of Edinburgh, 2015), pp. 71–76.
- [12] F. Bassi, L. Botti, A. Colombo, D. A. Di Pietro, and P. Tesini, *Journal of Computational Physics* **231**, 45–65 (2012).
- [13] M. Pandolfi, *AIAA journal* **22**, 602–610 (1984).
- [14] A. Ferrero, F. Larocca, and G. Puppo, *International Journal for Numerical Methods in Fluids* **77**, 63–91 (2015).
- [15] A. M. Law, W. D. Kelton, and W. D. Kelton, *Simulation modeling and analysis*, Vol. 3 (McGraw-Hill New York, 2000).