# Mixture distribution modelling of the sensitivities of a digital 3-axis MEMS accelerometers large batch

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***Abstract*—Huge quantities of low-cost analogue or digital MEMS sensors, in the order of millions per week, are produced by manufacturers. Their use is broad, from consumer electronic devices to Industry 4.0, Internet of Things and Smart Cities. In many cases, such sensors have to be calibrated by accredited laboratories to provide traceable measurements. However, at present, such a massive number of sensors cannot be calibrated and large-scale calibration systems and procedures are still missing. A first step to implementing these methods can be based on the distribution of the sensitivities of the large batches produced. Such distribution is also useful for sensor network end-users who need a single sensitivity, with the associated uncertainty, to be attributed to the whole network. Recently, a large batch of 100 digital 3-axis MEMS accelerometers was calibrated with a primary calibration system developed at INRiM and suitable for 3-axis accelerometers. Distributions of their sensitivities as a function of axis and frequency were analyzed and their non-normal behaviour was shown. However, in the preliminary phase of the study, the calibration uncertainties were not considered in these distributions. Therefore, in this paper, a mixture distribution modelling, based on Monte Carlo simulations and aimed at including the calibration uncertainties in the sensitivity distributions, is implemented and the resulting distributions are compared to the previous ones in histogram form. These distributions are also fitted with Johnson's unbounded and bimodal functions to get continuous distributions. This paper represents a further step towards the development of large-scale statistical calibration methods.**



***Keywords*— *Digital MEMS accelerometers, large-scale, sensitivity, mixture distribution***


## I. INTRODUCTION

In recent years, the production of sensors, in particular MEMS ones, exponentially increased, reaching a massive quantity, in the order of millions per week. At present, these sensors are mainly used in electronic consumer device applications, such as accelerometers, pressure, gyroscopes, microphones, humidity and temperature sensors. However, as underlying technical performance improves, the reliability and accuracy of these sensors are becoming comparable to those of traditional measuring instruments, at least within specific boundary conditions or for certain measuring ranges, while maintaining significantly reduced costs. This feature makes such sensors attractive for measurement applications, such as those rapidly developing in the field of Industry 4.0, Internet of Things (IoT) and Smart Cities, where a large number of traceable sensors is required [1-3]. However, due to the huge amount of produced MEMS, it is not possible to calibrate every single sensor, as currently done in "traditional" metrology. It is necessary to define large-scale calibration methods, schemes or procedures. These can be based on suitable statistical sampling approaches [4] or through in-line calibration systems [5,6], as also emphasized in the BIPM CCAUV strategy document 2019-2029 [7]. In both cases, it is fundamental to characterize the distribution of the sensitivities of these sensors, produced in large batches, to address a traceability strategy. Such distributions are also useful for sensor network end-users. In fact, sensor networks consist of tens, hundreds, or thousands of transducers, thus attributing a sensitivity to each transducer and for each parameter of influence (e.g. frequency and axis, for 3-axis accelerometers) might be difficult to be managed in numerical, computational and consumption terms by end-users [8] and a single sensitivity value to be attributed to the whole sensor network, together with an associated expended uncertainty based on the distribution of the sensitivities, is more preferable.

Recently, a large batch of 100 digital 3-axis MEMS accelerometers was calibrated at INRiM with a recently-developed traceable system and the individual main and transverse sensitivities were provided for each sensitive axis at frequencies between 5 Hz and 1000 Hz [4,9]. It was found that the distribution of the sensitivities for each frequency and each vibrating axis is significantly non-normal in the considered frequency range. However, such empirical distributions cannot take into account the calibration uncertainties, thus they are not accurate enough for representing the actual variability of the sensitivities, essential to implement large-scale methods or to attribute an uncertainty (or at least a variability measure) to a sensor network composed of these sensors. For this reason, in this paper, a method to include the calibration uncertainties in the sensitivity distributions is implemented as a further step towards the development of a statistical approach for large batches calibration.

Such method is based on the modelling of a mixture distribution resulting from the normal distributions of the sensitivities of the individual sensors, having a standard deviation equal to their calibration uncertainty. The resulting mixture distributions are compared to the previous empirical ones (raw data without calibration uncertainty). They are also fitted with two different families of probability distributions (Johnson's [10] and bimodal distributions [11]). Results are shown and compared.

## II. THE DIGITAL 3-AXIS MEMS ACCELEROMETERS BATCH

The batch under study is composed of 100 digital 3-axis MEMS accelerometers (Fig. 1). These sensors were calibrated with a specific system suitable for the simultaneous amplitude calibration of digital 3-axis MEMS accelerometers in the frequency domain by comparison to a reference transducer (in analogy to ISO Standard 16063-21 [12]), traceable to the SI and developed and validated at INRIM [13-18]. Main and transverse sensitivities were provided for each sensitive axis at frequencies of 5 Hz, 10 Hz, 20 Hz, 40 Hz, 80 Hz, 160 Hz, 315 Hz, 630 Hz and 1000 Hz, at nearly-constant peak amplitude of 10 $m/s^2$. The outputs of the MEMS are given in $Decimal_{16\text{-bit-signed}}$ (hereinafter abbreviated as $D_{16\text{-bit-signed}}$) where the digit unit is a signed 16-bit sequence converted into a decimal number. Sensitivities along *x*- and *y*-axis range between 615 $D_{16\text{-bit-signed}}/(m/s^2)$ and 1025 $D_{16\text{-bit-signed}}/(m/s^2)$, with relative expanded uncertainties around 1.2 % at 5 Hz, and around 0.4 % from 10 Hz to 1 kHz, whereas *z*-axis sensitivities decrease at increasing frequencies and range between 251 $D_{16\text{-bit-signed}}/(m/s^2)$ and 896 $D_{16\text{-bit-signed}}/(m/s^2)$, with relative expanded uncertainties around 0.9 % at 5 Hz and 0.3 % from 10 Hz to 1 kHz. It was found that the simple distribution of the sensitivities for each frequency and vibrating axis is significantly non-normal in the considered frequency range. An example is shown for *z*-axis at 5 Hz in Fig. 2.

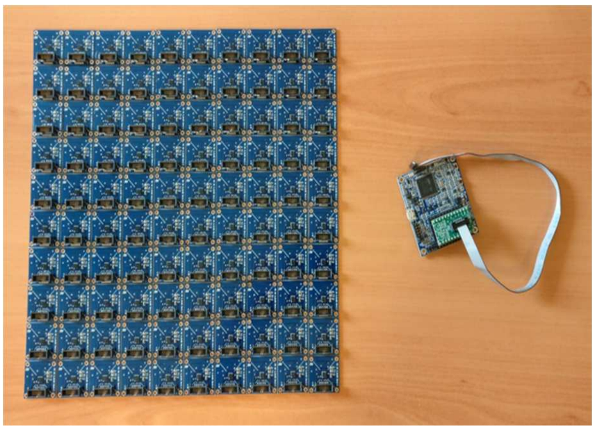

Fig. 1. The 100 digital 3-axis MEMS accelerometers (left) and the external microcontroller (right) [4].

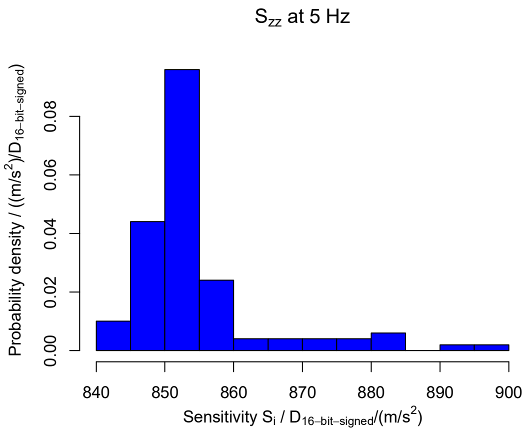


Fig. 2. Simple distribution of the 100 MEMS *z*-axis sensitivities at 5 Hz.

## III. MIXTURE DISTRIBUTION MODELLING

The preliminary investigated histograms refer to the sample distribution of the sensitivity values, $S_i$, each pertaining to the $i^{th}$ MEMS, without the inclusion of the associated expanded uncertainty $U(S_i)$. As a matter of fact, MEMS sensitivities are more accurately represented by a set of normal distributions, whose dispersions depend on the associated calibration uncertainties $u(S_i)$, as schematically shown in Fig. 3, rather than by a set of single values.

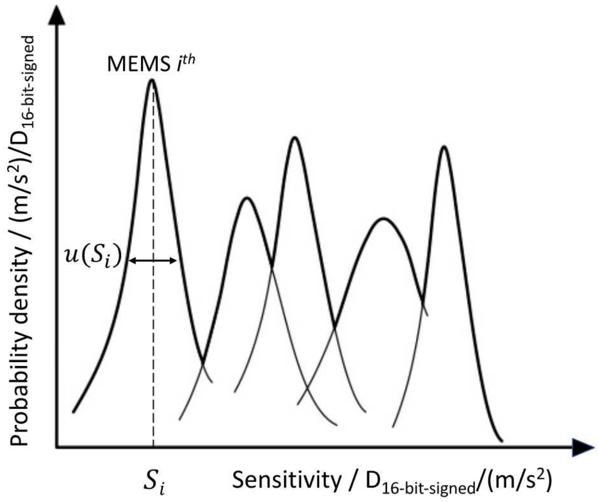


Fig. 3. Schematic representation of the sensitivity of each MEMS, in terms of normal distribution, whose dispersion depends on the associated calibration uncertainty.

To include the calibration uncertainties in the distribution of the sensitivities of the experimental batch, a mixture distribution [19] modelling is implemented. The mixture distribution is obtained from the collection of 100 normally-distributed variables (assigned to the sensitivities of each MEMS in the batch), all having the same weight within the mixture. An important feature of the proposed model is that it can take into account possible covariances between the sensitivity values of different MEMS.

The modelling of the mixture distribution is performed through *R* software by implementing the "rmvnorm" function

[20] that generates data from a multivariate normal distribution, given a vector of mean values, i.e. the 100 sensitivities of the MEMS, and a covariance matrix, i.e. a $100\times100$ matrix where the diagonal terms are the calibration squared uncertainties of each MEMS and the out-of-diagonal terms are the covariance terms of each couple of the MEMS sensitivities. Since all MEMS are calibrated with the same system, whose associated variance weights about 50 % of the overall combined squared uncertainty as shown in [13,21], out-of-diagonal covariance terms are calculated assuming a constant correlation coefficient of 0.5 for all couples of MEMS. This process is numerically performed through a Monte Carlo simulation: for each of the 100 normal distributions, $10^5$ values are extracted and combined into a $10^5\times100$ matrix of randomly generated numbers. The columns of this matrix represent the (marginal) probability density functions of the individual MEMS, which are correlated with each other. Putting all these data together, i.e. mixing the simulated $10^7$ sensitivity values, the final mixture distribution of the possible sensitivity values of the whole MEMS batch is obtained. Such operation can be performed for the sensitivities related to a specific axis and a specific frequency, or for larger groupings, e.g. without any distinction between axis or frequency. As an example, the mixture distributions (with calibration uncertainties) of the main sensitivities along *x*-, *y*- and *z*-axis at 5 Hz are shown and compared to the simple distribution (without calibration uncertainties) in Figs. 4-6. It is worth noting that the dispersion of the mixture distributions is generally higher than that of the simple distributions, due to the inclusion of the individual calibration uncertainties. At higher frequencies, calibration uncertainties along the three sensitive axes are lower, thus the impact on the mixture distribution is less noticeable, as shown in Figs. 7-9 along *x*-, *y*- and *z*- axis at 1000 Hz.

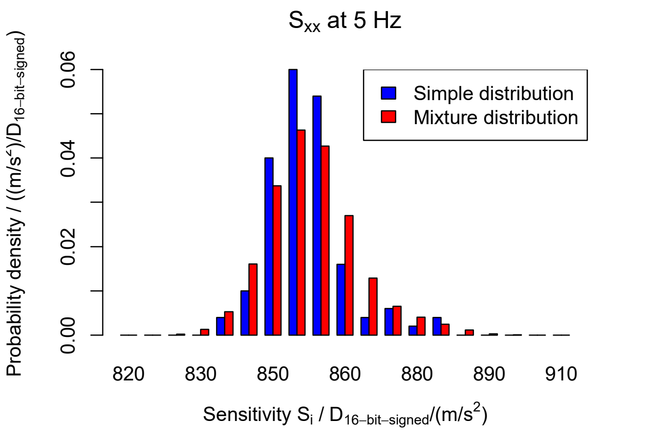


Fig. 4. Mixture and simple distribution of the 100 MEMS *x*-axis main sensitivities at 5 Hz.

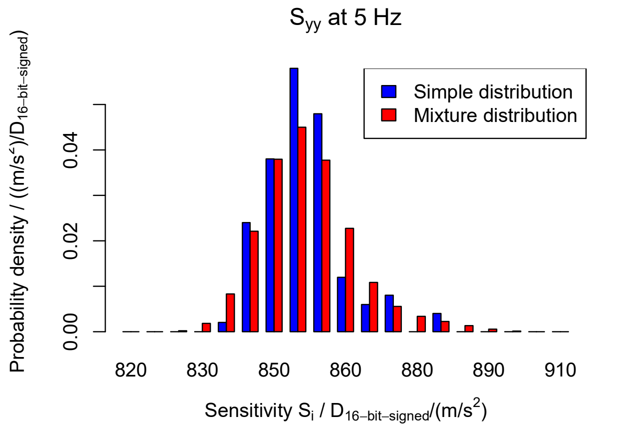


Fig. 5. Mixture and simple distribution of the 100 MEMS *y*-axis main sensitivities at 5 Hz.

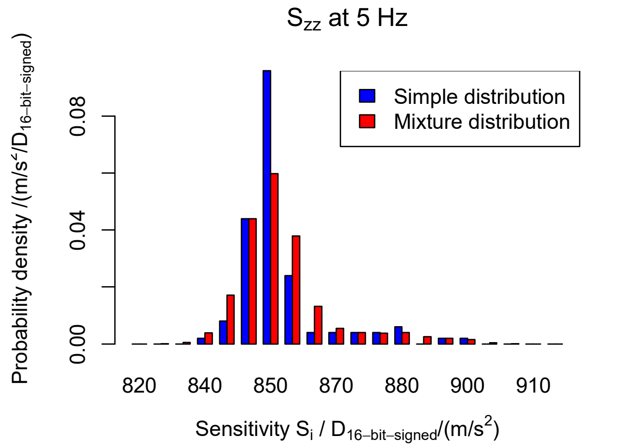


Fig. 6. Mixture and simple distribution of the 100 MEMS *z*-axis main sensitivities at 5 Hz.

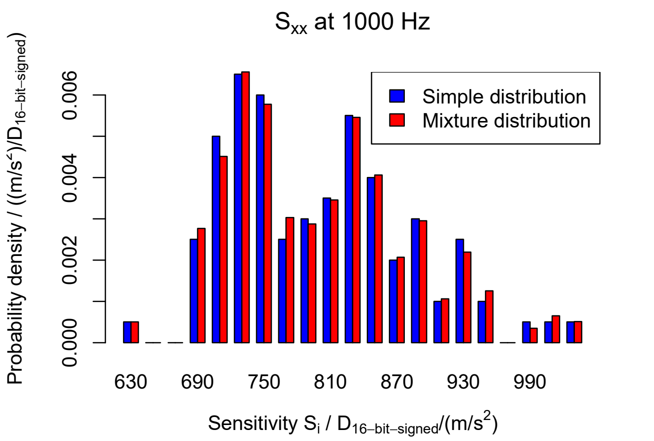


Fig. 7. Mixture and simple distribution of the 100 MEMS *x*-axis main sensitivities at 1000 Hz.

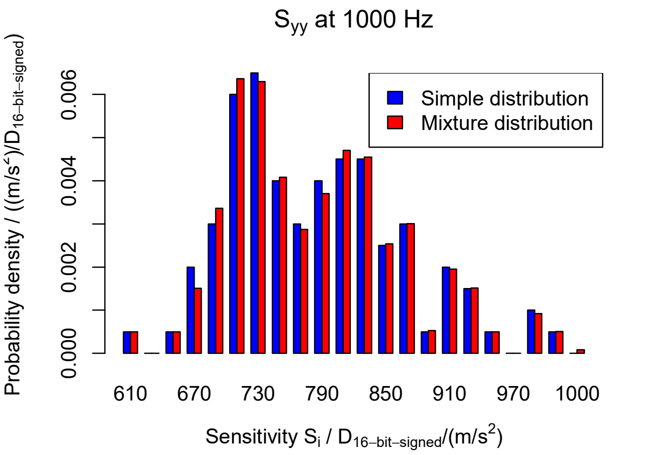


Fig. 8. Mixture and simple distribution of the 100 MEMS *y*-axis main sensitivities at 1000 Hz.

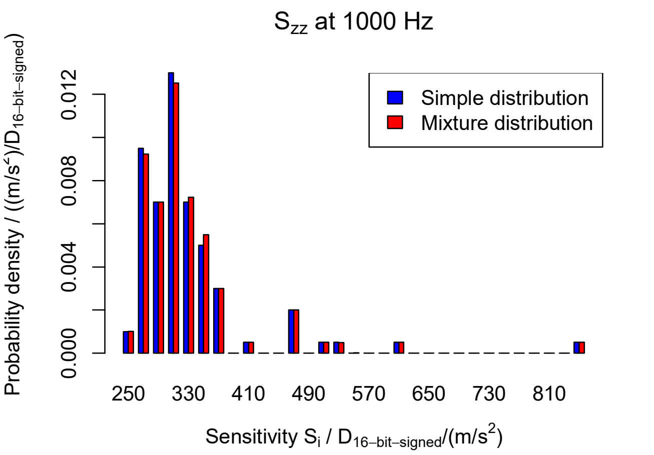


Fig. 9. Mixture and simple distribution of the 100 MEMS *z*-axis main sensitivities at 1000 Hz.

## IV. Influence of the Covariance Terms on Mixture Distributions

As previously described, to rigorously get the mixture distributions, covariance terms should not be neglected since the same calibration system is used for all MEMS. The uncertainty contribution due to the calibration system, in terms of variance, weights around 50 % of the overall combined squared uncertainty. A comparison of the mixture distribution obtained by considering correlated or uncorrelated calibration results is performed. In the second case, covariance terms are set to 0 and the effects on the resulting distributions are shown. By way of example, the mixture distributions of correlated and uncorrelated $S_{zz}$ sensitivities at 5 Hz are shown in Fig. 10. It is found that, in this case, the impact of the correlation is minimal compared with the mixture distribution of uncorrelated values.

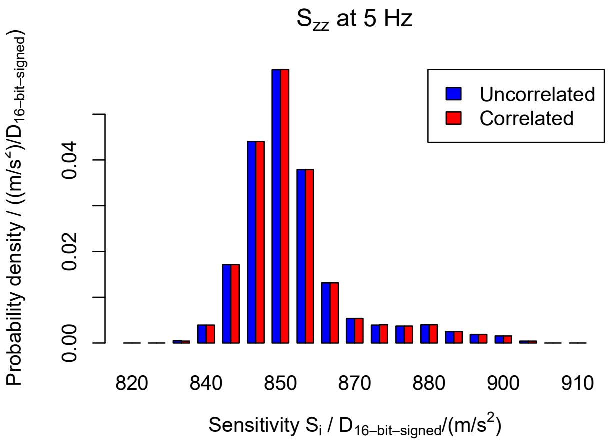


Fig. 10. Mixture distributions of the 100 MEMS *z*-axis main sensitivities at 5 Hz for uncorrelated or correlated values.

## V. Fitting of the Mixture Distributions

The above-reported distributions in histogram form, given their highly non-normal behaviour, are then fitted with Johnson's unbounded SU and bimodal distributions (the latter obtained as a mixture of two normal distributions) in order to get continuous distributions. As an example, Johnson's and bimodal functions fitting is applied to the mixture distributions of *x*-, *y*- and *z*-axis sensitivities at 5 Hz and 1000 Hz, which represent two different cases. Results are shown in Figs. 11-16. In general, it is found that a mixture of two normal distributions fits the histogram more accurately. This is a typical characteristic of a production process affected by a binary factor impacting the process which should be investigated by the manufacturer.

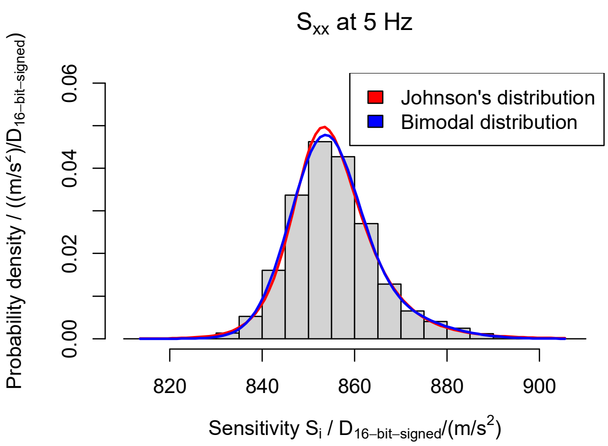


Fig. 11. Mixture distribution of the 100 MEMS *x*-axis main sensitivities at 5 Hz with Johnson's and bimodal distribution fittings

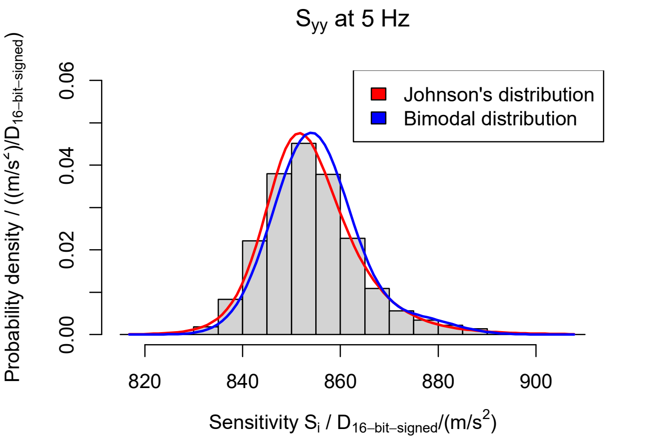


Fig. 12. Mixture distribution of the 100 MEMS *y*-axis main sensitivities at 5 Hz with Johnson's and bimodal distribution fittings

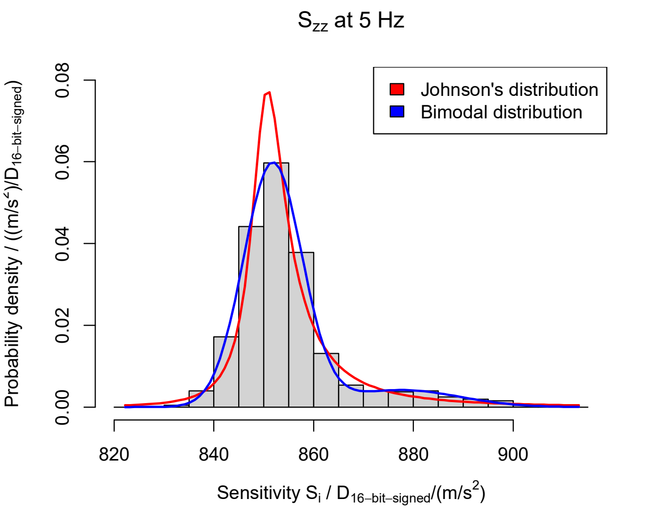


Fig. 13. Mixture distribution of the 100 MEMS *z*-axis main sensitivities at 5 Hz with Johnson's and bimodal distribution fittings

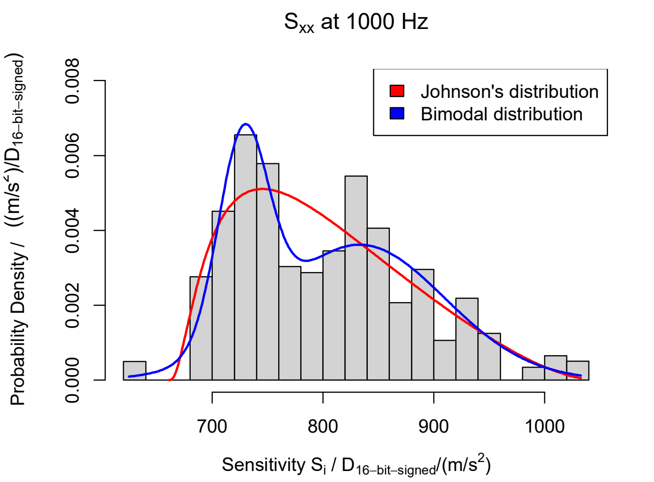


Fig. 14. Mixture distributions of the 100 MEMS *x*-axis main sensitivities at 1000 Hz with Johnson's and bimodal distribution fittings.

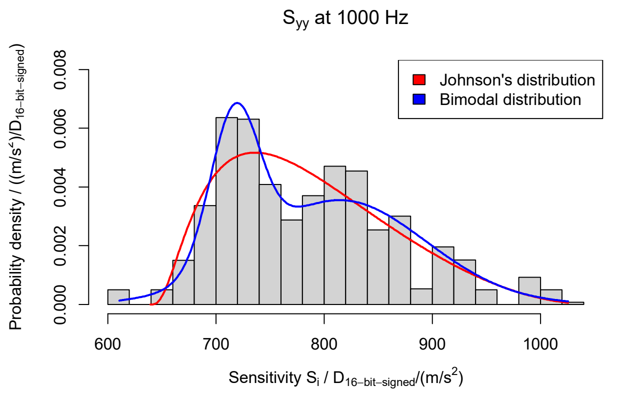


Fig. 15. Mixture distributions of the 100 MEMS *y*-axis main sensitivities at 1000 Hz with Johnson's and bimodal distribution fittings.

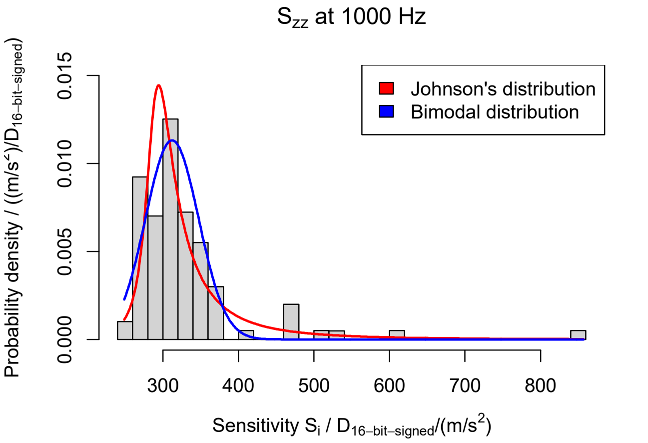


Fig. 16. Mixture distributions of the 100 MEMS *z*-axis main sensitivities at 1000 Hz with Johnson's and bimodal distribution fittings.

## VI. Conclusions

Accurate batch sensitivities distributions are the basis for developing large-scale statistical calibration methods required for low-cost sensors to guarantee traceable measurements. In this work, a mixture distribution modelling of the sensitivities related to a batch of 100 digital 3-axis MEMS accelerometers is implemented. The batch is calibrated along the three axes in a frequency range from 5 Hz to 1000 Hz, hence sensitivities can be expressed as function of axis and frequency. The mixture distribution allows taking into account the calibration uncertainties, associated with each MEMS, as well as the correlations between MEMS sensitivity values. It is found that differences between the mixture and the simple sensitivities distributions (the latter without the inclusion of uncertainties or correlations) are higher at increasing relative uncertainties, as expected. This confirms the necessity to include the uncertainties in the evaluation of the batch sensitivities distribution, although methods to reduce them are under development [22]. Correlation does not seem to play a crucial role, in the present example, but it is important to consider it in the mixture model for the sake of general applicability of the proposed procedure. Mixture distributions are then fitted with Johnson's and bimodal probability density functions. It is found that the bimodal one is more accurate to represent the batch sensitivity distributions. Such behaviour provides also important information to the manufacturer about the MEMS production process which seems to be affected by a binary factor that worth to be investigated in the future.

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