Combined Spectral Methods to Study Complex Scattering Problems Formulated with the Wiener-Hopf Technique: the Semi-infinite Grounded Dielectric Slab problem

Combined Spectral Methods to Study Complex Scattering Problems Formulated with the Wiener-Hopf Technique: the Semi-infinite Grounded Dielectric Slab problem

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In this work we present a new combination of spectral methods that allows to study complex scattering problem in spectral domain containing abrupt discontinuities of materials. The method is applied to a formulation of problem given in terms of incomplete Wiener-Hopf (WH) equations, where, for incompleteness, we intend that some of the physical boundary conditions arising from abrupt discontinuities provide terms in the WH equations not directly related to the plus and minus unknowns of the problem. This is the case of the semi-infinite grounded dielectric slab problem where the semi-infiniteness geometry of the material provide such situation. This problem is of great interest in antennas and propagation community and studies are proposed in different papers with different methods; see references in [1]. The WH incomplete equations are obtained as in [1] starting from the application of unilateral Laplace transform to wave equations defined in three different sub-regions, see Fig. 1.

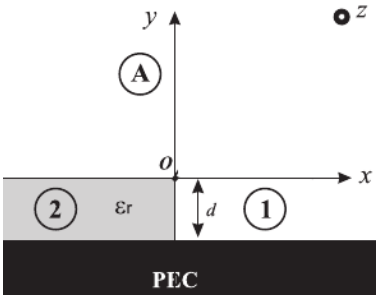

Figure 1. The semi-infinite grounded dielectric slab problem is divided into three sub-regions: region A is a semi-infinite free space region, region 1 is a semi-layer free-space region, and region 2 is a semi-layer dielectric region. Regions 1 and 2 are grounded by a perfectly electrically conducting (PEC) plane.

While considering sub-regions 1 and 2, we need have initial/boundary conditions at the interface between sub-regions at $-d<y<0$ where d is the thickness of the dielectric semi-slab. The solution of the wave equations in Laplace domain is obtained using the characteristic Green's function procedure in the general form reported at [1]. The terms aroused from the integrals of the Green's function and boundary conditions constitute the incomplete parts of the WH problem. To render complete the system of WH equations we study the regularity properties of the incomplete terms. With the application of Mittag-Leffler theorem and Cauchy integral representations, we obtain a complete generalized WH formulation of the problem. The system of equations is then treated with the Fredholm factorization technique that reduces the system of spectral WH equations to integral representations of second kind with compact kernels by eliminating one kind of unknowns (plus or minus). The semi-analytical solution of the system of Fredholm integral equations provide the approximated spectra of the problem in terms of Laplace transforms of field components. The combination of these mathematical tools (characteristic Greens' function, WH and Fredholm factorization) allows extending the capabilities of classical Wiener-Hopf method to a new set of problems, such as the one proposed in this paper. This new development of the WH technique further extends the class of complex scattering problems treated by WH technique, beyond the recent novel applications, see for instance [2-3]. One of the main benefits of the proposed semi-analytical solution is that it allows the computation of field components by using asymptotics similarly to what is done with closed-form spectral solutions when available. Considering plane wave illumination, physical phenomena such as the reflection, diffraction and excitation of surface/leaky waves can be investigated and will be reported during the presentation. Numerical results validate the proposed methodology. The proposed method describes the complete scattering problem with a comprehensive mathematical model in spectral domain avoiding multiple steps of interaction among separated scattering structures. The methodology can be extend to other types grounding or to an ungrounded semi-slab. This work is supported by the PRIN Grant 2017NT5W7Z GREEN TAGS.

1. V. Daniele, G. Lombardi, R.S. Zich, "Physical and Spectral Analysis of a Semi-infinite Grounded Slab Illuminated by Plane Waves", IEEE Trans. Antennas Propagat., submitted April 4, 2022

2. V. Daniele, G. Lombardi, R.S. Zich, "The Electromagnetic Field for a PEC Wedge Over a Grounded Dielectric Slab: 2. Diffraction, Modal Field, Surface Waves, and Leaky Waves," Radio Science, vol 52, pp. 1492-1509, 2017

3. V. Daniele, G. Lombardi, R.S. Zich, "The Double PEC Wedge Problem: Diffraction and Total Far Field," IEEE Trans. Antennas Propagat., vol 66, pp. 6482-6499, 2018.