Scattering of a PEC Wedge in Anisotropic Media: Wiener-Hopf Model

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***Abstract*— The study of electromagnetic wave in interaction of Perfect Electrically Conducting (PEC) wedge in anisotropic media is of great interest in propagation models, GPR technologies and aerospace applications. In this work we present a general method based on the application of an extended version of the Wiener-Hopf (WH) technique for the analysis of the electromagnetic scattering and diffraction. This method is part of the semi-analytical spectral methods that allow physical interpretation of the solution in terms of field components.**

I. INTRODUCTION

This work presents a novel formulation of the scattering problem constituted by a wedge immersed in anisotropic media with perfect electrically conducting (PEC) boundary conditions and illuminated by plane waves. The structure is presented in Fig. 1 where both cylindrical coordinates (ρ,φ,z) and Cartesian coordinates (x,y,z) are used. The PEC wedge is with aperture angle $2\pi-2\gamma$ and the anisotropic surrounding medium is with tensor dielectric permittivity $\underline{\underline{\varepsilon}}$ and tensor magnetic permeability $\underline{\underline{\mu}}$. The surrounding region is divided into two sub-regions defined respectively by 1) $0<\varphi<\gamma$ and 2) $-\gamma<\varphi<0$.

The proposed technique is a semi-analytical spectral methods capable (as the analytical methods) of physical interpretation in terms of field components. Therefore, the technique preserves the benefits of analytical solution and acquire the some of the geometrical flexibility of numerical methods. The technique is based on the application of boundary conditions to spectral functional equations, yielding a set of Generalized Wiener-Hopf Equations (GWHEs). The spectral functional equations for angular region problem (sub-regions 1 and 2) in complex media are obtained by the application of a revisited version of Bresler-Marcuvitz transversalization method [1] applied to angular regions filled by an arbitrary linear medium [2-3]. The extension of the Wiener-Hopf (WH) technique in angular regions [4-5] demonstrated its efficacy to solve wave scattering problems in presence of geometries containing isotropic angular regions and/or stratified planar regions, see for instance [6-7]. In this work we develop the formulation of the problem for different kind of anisotropic media, uniaxial (1) and biaxial (2) media, and we provide some initial proof of validation in simple cases. In this framework, the literature reports some attempt of analytical/semi-analytical solutions in case of simple anisotropies for half-plane and wedges, see [8]-[15] and references therein; however the semi-analytical/analytical solution of a PEC wedge immersed in biaxial or more complicated anisotropies is not available, while the Wiener-Hopf (WH) formulation allows this possibility. In the paper, we assume $e^{j\omega t}$ time-harmonic dependence of field.

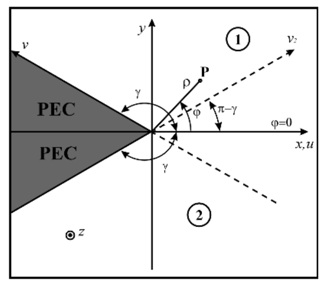

Figure 1. PEC wedge immersed in anisotropic medium with $2\pi-2\gamma$ aperture

$$\underline{\underline{\varepsilon}}_u = \begin{vmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix}, \underline{\underline{\mu}}_u = \begin{vmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{vmatrix}. \tag{1}$$

$$\underline{\underline{\varepsilon}}_b = \begin{vmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix}, \underline{\underline{\mu}}_b = \begin{vmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{vmatrix}. \tag{2}$$

II. FUNCTIONAL EQUATIONS

In [1] Bresler and Marcuvitz proposed a transversalization method for rectangular/layered regions in arbitrary linear media. We extend this procedure for angular regions considering oblique Cartesian coordinates (u,v,z for region 1, u_2,v_2,z for region 2) as on [2]-[3], see Fig.1. In this framework Maxwell's equations are re-written in terms tangent (at φ=0) components

$$-\frac{\partial}{\partial v}\psi_t = \mathcal{M}_e(\frac{\partial}{\partial z},\frac{\partial}{\partial u})\cdot\psi_t = \mathcal{M}_e(-j\alpha_o,\frac{\partial}{\partial u})\cdot\psi_t \cdot \tag{3}$$

with $\psi_t = |\,\mathbf{E}_t\,\mathbf{H}_t\,|' = |\,E_z\,E_x\,H_z\,H_x\,|'$ and where we have considered field dependence specified by the factor $e^{-j\alpha_o z}$ due to invariance of the problem along z. After the application of Laplace transform along u

$$\tilde{\psi}_t(\eta,v) = \int_0^\infty e^{j\eta u}\psi_t(u,v)du \cdot \tag{4}$$

we get an ordinary differential vector problem of first order

$$-\frac{d}{dv}\tilde{\psi}_t(\eta,v) = M_e(\gamma,\eta)\cdot\tilde{\psi}_t(\eta,v) + \psi_s(v) \cdot \tag{5}$$

where the s-term contains initial/boundary conditions and constitutes the forcing function. The solution is built combining a particular integral with a complementary function which is a linear combination of homogeneous equation solutions. The particular integral is obtained using the characteristic Green's dyadic of the problem in term of the eigenvectors and eigenvalues of the matrix operator $M_e(\gamma,\eta)$ of dimension 4. This non-explicit solution is found before imposing the boundary conditions as reported in [3] based on the theory of [16-17]. The solution of these oblique transverse differential equations in Laplace domain is then projected on the reciprocal eigenvectors of the algebraic matrix operator. It yields the functional equations of an angular region with arbitrary boundary conditions and medium. In region 1 we obtain

$$\begin{aligned} \nu_3(\eta)\cdot\tilde{\psi}_t(\eta,0) &= \nu_3(\eta)\cdot\breve{\psi}_s(-m_{a1}(\gamma,\eta)) \\ \nu_4(\eta)\cdot\tilde{\psi}_t(\eta,0) &= \nu_4(\eta)\cdot\breve{\psi}_s(-m_{a2}(\gamma,\eta)) \end{aligned} . \tag{6}$$

where ν_i are reciprocal eigenvectors and $m_{ai} = j\lambda_{ei}$ are quantities related the eigenvalues λ_{ei} of $M_e(\gamma,\eta)$ and

$$\breve{\psi}_s(\chi) = \int_0^\infty e^{j\chi v}\psi_s(v)dv \cdot \tag{7}$$

The eqs. (6) are functional equation in spectral domain that relate spectral quantities defined at φ=0 (left hand side) to quantities defined at φ=γ (right hand side). Similar expressions can be analogously obtained for region 2.

III. GWHEs, Preliminary Validations, Conclusions

Once obtained the functional equations we impose boundary conditions, in this case PEC, by nullifying the tangent electric field components on the faces of the wedge. For region 1 in biaxial medium it yields at normal incidence (α_o=0)

$$\begin{aligned} &\frac{1}{2}\left(\tilde{\mathrm{H}}_{\mathrm{x}}(\eta) - \frac{\tilde{\mathrm{E}}_z(\eta)\xi_1}{\omega\sqrt{\mu_x\mu_y}}\right) = \frac{\breve{\mathrm{H}}_{\mathrm{va}}(-m_{a1})}{2} \\ &\frac{1}{2}\left(\frac{\omega\sqrt{\varepsilon_x\varepsilon_y}}{\xi_2}\tilde{\mathrm{E}}_{\mathrm{x}}(\eta) + \tilde{\mathrm{H}}_z(\eta)\right) = \frac{1}{2}\left(\cos(\gamma) + \frac{1}{\xi_2}\sqrt{\frac{\varepsilon_x}{\varepsilon_y}}\eta\sin(\gamma)\right)\breve{\mathrm{H}}_{\mathrm{za}}(-m_{a2}) \end{aligned} . \tag{8}$$

where on the left (right) we have spectral field components defined at φ=0 (φ=γ) and

$$\begin{aligned} &m_{a1} = -\eta\cos(\gamma) + \sqrt{\frac{\mu_{\mathrm{x}}}{\mu_{\mathrm{y}}}}\xi_1\sin(\gamma),\ m_{a2} = -\eta\cos(\gamma) + \sqrt{\frac{\varepsilon_{\mathrm{x}}}{\varepsilon_{\mathrm{y}}}}\xi_2\sin(\gamma) \\ &\xi_1 = \sqrt{k_1^{\,2} - \eta^2}, k_1 = \omega\sqrt{\mu_{\mathrm{y}}\varepsilon_z}, \xi_2 = \sqrt{k_2^{\,2} - \eta^2}, k_2 = \omega\sqrt{\mu_{\mathrm{z}}\varepsilon_y} \end{aligned} \cdot \tag{5}$$

A system of equations similar to (8) are obtained also for region 2. The coupling of these equations yields a system of GWHEs for the problem of biaxial medium amenable of exact solution. GWHEs are different from classical ones, since the WH plus/minus unknowns are defined into different complex planes but related together. Procedure for their solutions are reported in [5-6]. As a preliminary validation the GWHEs we resort to the PEC wedge in isotropic medium yielding the same equation reported in [5-6], whose solution is well-known in literature in closed form. As a second validation we again consider isotropic medium at normal incidence but with scalar impedance boundary conditions at φ=±γ. This validation example is the classical Malyuzhinets' problem and the equations coincides with the ones reported in [5-6]. Again this problem has closed form solution. Further validation examples will be the PEC wedge in uniaxial medium and finally the analysis of the PEC wedge in biaxial and general anisotropic medium with the help of the semi-analytical Fredholm factorization method [4-5], [18] when exact factorization procedure are not available. Asymptotics is then applied to the spectral solution to get practical field information.

Acknowledgment

This work is supported by the PRIN Grant 2017NT5W7Z GREEN TAGS.

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