Preprints, IFAC CAMS 2022
Kgs. Lyngby, Denmark. September 14-16, 2022

Energy-maximising model predictive control for a multi degree-of-freedom pendulum-based wave energy system

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Abstract: Renewable energy sources can be a solution for the recent pollution increasing scenario and the need for diversification of the energy market. Among such alternative sources, wave energy represents a viable solution, due to the its high power density and accessibility. Nonetheless, wave energy is still in phase of development, and a key stepping stone towards commercialisation is strongly linked to the availability of optimal control strategies for maximum energy harvesting. With its ability to handle system constraints and optimise power absorption directly, model predictive control (MPC) has gained popularity within the WEC community as a potential solution for the corresponding energy-maximising problem. In this study, an MPC strategy is developed for real-time control of the so-called PeWEC energy harvesting system, providing also a solution for the wave excitation estimation and forecasting problem, inherently required by the MPC controller to achieve optimal performance. Improved computational requirements are obtained via definition of a reduced control-oriented model, describing the dynamics of the system in a compact form. The performance of the proposed strategy is illustrated via a comprehensive numerical appraisal.

Keywords: Wave Energy Converter, Optimal Control, Model Predictive Control, Wave Estimation, Wave Prediction, Model Reduction

1. INTRODUCTION

The recent rise of fossil fuel costs and the increase of carbon dioxide emissions are pushing research towards efficient use of alternative energy sources. Among possible alternatives, ocean wave energy has shown great potential thanks to the excellent power density in ocean sea states, and the massive spread of the primary source (Mattiazzo, 2019). Although wave energy represents a viable solution for lowering greenhouse emissions, it is still in development, requiring a number of steps in order to allow effective commercialisation of wave energy converters (WECs) (Ruehl and Bull, 2012). As of today, production, installation and maintenance costs, are hindering the possibility of large-scale production. A fundamental pathway towards reducing wave energy production costs resides in so-called energy-maximising optimal control strategies (Ringwood et al., 2014), capable of maximising resource conversion, while respecting the intrinsic physical constraints characterising both the device, and the power take-off (PTO) actuator components.

Led by those considerations, one of the most appealing strategies is model predictive control (MPC), being able to provide optimal control actions and explicitly handling imposed constraints on the system to avoid potential damage on any sensible device components, while maximising energy harvesting (Faedo et al., 2017; Cretel et al., 2011; Mayne et al., 2000).

Successful implementation of MPC controllers within the WEC application requires knowledge of the device dynamics to optimise the corresponding control action over a given (user-defined) prediction horizon. Nonetheless, accurate prediction of the WEC motion depends upon knowledge of the so-called wave excitation force, *i.e.* the force exerted on the device by virtue of the incoming wave field. Given its unmeasurable nature (Peña-Sanchez et al., 2020b), suitable unkown-input estimation and prediction algorithms must be employed to compute instantaneous and future values of such external excitation, respectively (see *e.g.* Fusco and Ringwood (2010); Peña-Sanchez et al. (2020a)).

The resulting control loop, composed of the interconnection between controller, and wave excitation estimator and forecaster, is intended to be applied in real-time, and standard hydrodynamic modelling can lead to high computational complexity for the resulting system (Li and Belmont, 2014). As such, it is often desired to define a suitable control-oriented model, capable of representing the main device dynamics in a simplified form, facilitating real-time implementation of the numerical optimisation routines required for the effective implementation of MPC algorithms.

This study presents a computationally efficient MPC formulation, with specific application to the so-called Pendulum Wave Energy Converter (PeWEC) device (Pozzi et al., 2017), described in detail within Section 2. In particular, inspired by recent advances in impedance-matching for WEC systems (Faedo et al., 2022b), a control-oriented model is defined, describing the dynamics involved within the MPC process in a simplified form, allowing for real-time implementation of the composite control loop.

The remainder of this article is structured as follows. Section 2 provides an account of the considered control-oriented WEC dynamics. Section 3 introduces the adopted control strategy, including MPC controller, prediction, and estimation of external forces. Section 4 offers a numerical appraisal of the proposed strategy, while Section 4 discusses the main conclusions of this study.

2. CONTROL-ORIENTED SYSTEM MODEL

The pitching device adopted in this study is the so-called PeWEC device (Pozzi et al., 2017), and is schematically depicted in Fig. 1. This offshore WEC exploits the relative motion between the hull and the internal pendulum to extract power about the PTO hinge axis, namely ε.

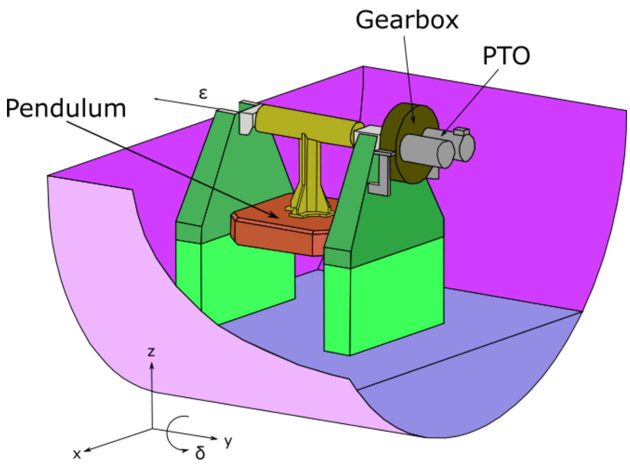

Fig. 1. PeWEC device schematic representation

This system can be reasonably described in terms of 4 degrees-of-freedom (DoFs), namely x (surge), z (heave), δ (roll), and ε (PTO axis). Following linear potential flow theory, the WEC system can be described in terms of a multiple-input multiple-output (MIMO) linear time-invariant (LTI) operator $G : \mathbb{C} \to \mathbb{C}^{4\times4}$, $s \mapsto G(s)$, with inputs defined by the forces applied on each corresponding DoF, and outputs defined in terms of the velocities associated with each mode of motion, *i.e.* $\{v_x, v_z, v_\delta, v_\varepsilon\}$. In particular, the first three DoFs are affected by a corresponding wave excitation force $\{f_x, f_z, f_\delta\}$, arising as a consequence of the action of the incoming wave field on the hull. In contrast, the PTO axis ε is only (directly) affected by the control torque u, which is, effectively, the variable to be optimally designed via appropriate optimal control technology. To be precise, the control problem for wave energy power extraction aims at computing the optimal control torque u to maximise energy extraction at the PTO axis over a given time-horizon $T \in \mathbb{R}^+$. Noting that the (instantaneous) absorbed power is given by $P_{abs} = -uv_\varepsilon$, the objective is to design u such that the map

$$J(u) = -\frac{1}{T}\int_{\Xi} u(\tau)v_\varepsilon(\tau)\mathrm{d}\tau, \tag{1}$$

with $\Xi = [0, T] \subset \mathbb{R}^+$ is maximised.

Given the nature of the control objective (1), and based upon recent results on impedance-matching for multi-DoF WEC systems (Faedo et al., 2022b), it is not necessary to consider each 'mode' described by $G(s)$ to solve for the maximiser of (1), since the optimal control conditions for underactuated systems (*i.e.* devices which can directly absorb energy from a limited number of the available modes of motion), can be described in terms of the dynamics associated to the *controlled* DoFs, subject to a particular external perturbation. For the PeWEC case, we are hence interested in the map $u \mapsto \varepsilon$, which can be described (in the Laplace domain) as [1]

$$V_\varepsilon = [G_x \ G_z \ G_\delta \ G_\varepsilon] \begin{bmatrix} F_x \\ F_z \\ F_\delta \\ U \end{bmatrix}, \tag{2}$$

with G_ε,G_x,G_y and G_δ the mappings describing the 'contribution' of each corresponding DoF to the PTO axis output v_ε. Note that equation (2) can be equivalently expressed as

$$V_\varepsilon = G_\varepsilon[\tilde{F}_e + U] \tag{3}$$

with the map

$$\tilde{F}_e = \frac{G_x}{G_\varepsilon}F_x + \frac{G_z}{G_\varepsilon}F_z + \frac{G_\delta}{G_\varepsilon}F_\delta, \tag{4}$$

the so-called *total* wave excitation force (see Faedo et al. (2022b)). Via realisation, one can derive a time-domain equivalent of equation (3) in terms of a stable, strictly proper, minimum-phase, single-input single-output (SISO), finite-dimensional state-space model. The discrete time representation of system G_ε can be written in state space form as:

$$G_\varepsilon \equiv \begin{cases} x(k+1) = Ax(k) + B[\tilde{f}_e(k) + u(k)], \\ \quad v_\varepsilon(k) = Cx(k), \end{cases} \tag{5}$$

where the mapping between continuous and discrete domain is expressed through $t = kT_s$, with T_s sampling time, and $x(k) \in \mathbb{R}^{32}$. Note that the relatively high dimension, required to represent (5), arises from parameterisation of particular hydrodynamic effects, *i.e.* so-called radiation forces ((Cummins, 1962)).

It is stressed that the modelling approach chosen in this paper differ from standard MPC formulations provided in the literature of WEC control (see, *e.g.* Pozzi et al. (2017)). In particular, the control-oriented model, considered herein, is of a SISO nature (instead of MIMO), and 'condenses' the effect of the (multiple) wave excitation forces $\{f_x, f_y, f_\delta\}$ into a single 'disturbance' $\tilde{f}_e$. This has a significant impact on the design and computational requirements associated to both estimation and forecaster algorithms required by the MPC controller, as detailed throughout Section 3.

3. CONTROL ARCHITECTURE

3.1 Model predictive Control

MPC resolves the optimisation problem, with objective function (1), directly over the selected horizon, finding

[1] From now on, we omit the dependence on s when clear from the context.

an optimal control sequence. Then, in a receding-horizon fashion, only the first element of the optimised solution is applied to the system, and the entire procedure is repeated each sampling instant (Li and Belmont, 2014). In order to manage the energy associated with the optimal control action, it is convenient to introduce an additional term in the objective function (1), weighted in terms of a design parameter $r \in \mathbb{R}^+$ (see Eq. 6). Furthermore, aiming at adding a soft constraint on the maximum PTO angular velocity v_ε, a quadratic term, weighted adequately via $q \in \mathbb{R}^+$ is also considered, *i.e.* the map (1) is modified as

$$J(u) = \frac{1}{T}\int_{\Xi} u(\tau)v_\varepsilon(\tau) + ru(\tau)^2 + qv_\varepsilon(\tau)^2 \mathrm{d}\tau, \quad (6)$$

where note that the sign of $J(u)$ (as originally written in (1)) has been changed without any loss of generality, and simply to consider the (more standard) minimisation convention, rather than its maximisation counterpart.

The design stage for r and q is crucial: the lower are their absolute values, the higher is the implicit weight on the power production term (i.e. the higher will be energy harvest). Notwithstanding, r must be chosen considering the minimum value for achieving convexity of the function, avoiding possible numerical errors and allowing the solver to achieve in time the solution (for real time purposes it depends on the available hardware). The weight q is selected to be small, since for the application it is only necessary to limit the maximum value of the PTO axis via explicit hard constraints.

Regarding hard constraints, a saturation limit on v_ε is imposed to avoid high loads on PTO bearings, and a saturation limit on the maximal control torque u reflects the actuation limit of the system. Given the objective function and defined set of constraints, the energy-maximising optimisation problem can be written as

$$\begin{aligned} &u_{opt} = \arg\min_u J(u), \\ &\text{subject to:} \\ &\text{PeWEC dynamics in (5)} \\ &|v_\varepsilon| < v_{max},\ |u| < u_{max},\ \forall t \in \Xi. \end{aligned} \quad (7)$$

Following (Li and Belmont (2014)), at a given instant k, defining $U_i^j := [u(k+i), u(k+i+1), ..., u(k+j)]^T$, $W_i^j := [\tilde{f}_e(k+i), \tilde{f}_e(k+i+1), ..., \tilde{f}_e(k+j)]^T$ and considering the control input array U_0^N and the wave excitation force array W_0^{N-1}, the optimal control problem in (7) can be rewritten in terms of a quadratic programming (QP) problem in standard form, *i.e.*

$$\begin{aligned} &U_{0\,opt}^N = \arg\min_{U_0^k} \frac{1}{2}(U_0^N)^\mathsf{T} H (U_0^N) + FU_0^N, \\ &\text{subject to:} \\ &A_{con}U_0^N \leq b, \end{aligned} \quad (8)$$

where

$$\begin{aligned} H &= \Phi_U + \Phi_U^T + 2R + 2\Phi_U^T Q \Phi_U \\ F &= (\Lambda + 2\Phi_U^T Q \Lambda)x(k) + (\Phi_W + 2\Phi_U^T Q \Phi_W)W_0^{N-1}, \end{aligned} \quad (9)$$

with

$$\Lambda = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix},$$

$$\Phi_U = \begin{bmatrix} 0 & & & & \\ -CB & 0 & & & \\ -CAB & -CB & 0 & & \\ \vdots & \vdots & \ddots & \ddots & \\ -CA^{N-1}B & -CA^{N-2}B & \dots & -CB & 0 \end{bmatrix}, \quad (10)$$

$$\Phi_W = \begin{bmatrix} 0 & & & \\ CB & 0 & & \\ CAB & CB & 0 & \\ \vdots & \vdots & \ddots & \ddots \\ CA^{N-1}B & CA^{N-2}B & \dots & CB \end{bmatrix},$$

and

$$\begin{aligned} &A_{con} = \begin{bmatrix} \mathbb{I} \\ -\mathbb{I} \\ \Phi_U \\ -\Phi_U \end{bmatrix},\ b = \begin{bmatrix} U_{max} \\ U_{max} \\ V_{max} - \Lambda x(k) - \Phi_W W_0^{N-1} \\ V_{max} + \Lambda x(k) + \Phi_W W_0^{N-1} \end{bmatrix}, \\ &R = r\mathbb{I}_{N+1}, \qquad Q = q\mathbb{I}_{N+1}, \\ &U_{max} = \mathbf{1}_{N+1,1}u_{max}, \quad V_{max} = \mathbf{1}_{N+1,1}v_{max} \end{aligned} \quad (11)$$

and the matrix $\mathbf{1}_{i,j} \in \mathbb{R}^{i\times j}$ is a matrix with all its entries equal to one, while $\mathbb{I}_{N+1} \in \mathbb{R}^{N+1\times N+1}$ is the identity matrix of size $N+1$, with N the discrete prediction horizon ($T = NT_s$).

A critical aspect resides in the class (*i.e.* properties) of the cost function (8): Typically, the power maximisation application leads to an intrinsically non-convex formulation under zero-order hold discretisation of the WEC dynamics (Faedo et al. (2017)). Nonetheless, convexity can be restored via appropriate selection of r (see *e.g.* Li and Belmont (2014)). A schematical representation of the considered control strategy is presented in Fig. 2. The

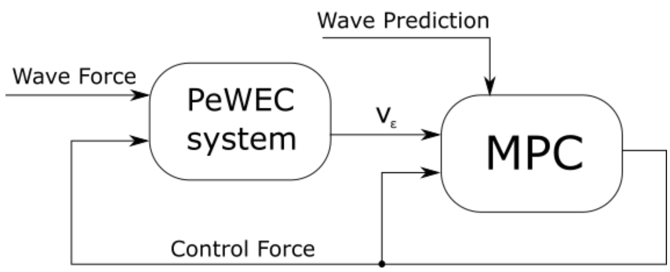

Fig. 2. Control loop architecture

reason behind the choice to feedback only the velocity v_ε at the PTO shaft, is discussed in the following sections.

3.2 Model Reduction

One of the main reasons to include weight r in the cost function is related to the non-convexity issue. Non-convex optimization supposes underlying global solvers that do not support real-time implementation features (due to the time taking computational burden), while convex solvers (Wright, 1996) have shown their effectiveness for real-time purposes in many application fields.

Model reduction (MR) techniques (Moore, 1981) further assist in moving the corresponding QP problem in this fast optimization direction. In particular, MR for linear systems exploits internal properties of the corresponding model to find a representative approximation of lower order (dimension). This computational lightness also opens the possibility of enlarging the MPC prediction horizon, which is a necessary condition to approach optimal performance with given stability guarantees (Mayne et al. (2000)). Within this study, MR via balanced truncation (see Safonov and Chiang (1989)) has been considered to further reduce the model presented in (5), aiming at improving the real-time capabilities of the proposed strategy.

In Fig. 3 it is shown the frequency response comparison of the original PeWEC model (red), as in (5), and the reduced model by balanced truncation (yellow). It is worth noticing

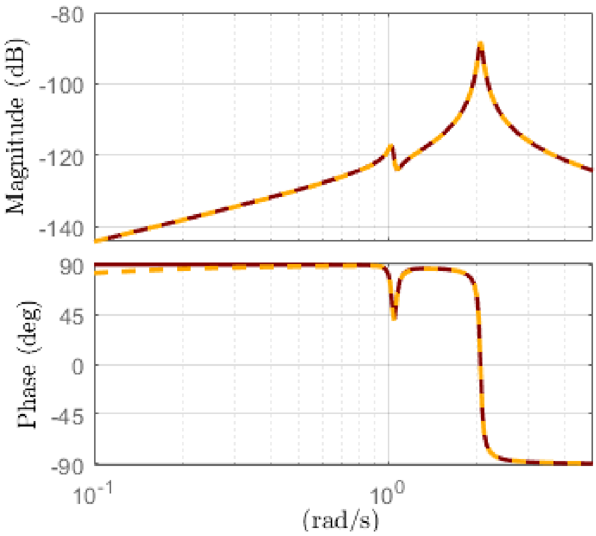

Fig. 3. Frequency response of the original model (red) and the reduced order model (yellow)

how both systems present virtually identical input-output responses, having the original PeWEC system a dimension of 32, against the significantly reduced number of 5 states after applying MR.

3.3 Prediction

One of the most critical aspects of MPC applied to wave energy conversion is the necessity to predict the wave excitation force over the prediction horizon accurately.

At each sample time, the optimization performed in Eq. 7 presumes full knowledge of the system dynamics, which are strongly influenced by the instantaneous value of the total excitation force in (5), which is not available in practice. As such, forecasting model must be employed in order to predict its future behaviour.

An Auto-Regressive model (AR), as considerd in (Peña-Sanchez et al., 2020a) is employed within this study. The model assumes that the total wave force $\tilde{f}_e$ at a given time instant k is a linear combination of its past values. So, the p-step ahead prediction is:

$$\hat{f}_e(k+p) = \sum_{i=1}^{n} a_i \hat{f}_e(k+p-i) \tag{12}$$

where n is the model order, and the parameters a_i are computed each 1 [s] through minimisation of the one-step prediction error, on refreshed sets of wave data of length M:

$$J_{LS} = \sum_{k=n+1}^{M} \left(f_e(k) - \hat{f}_e(k) \right). \tag{13}$$

New data are produced recursively from predicted data.

3.4 Estimation

Every prediction algorithm requires a set (at least unitary) of data to train its parameters and produce a forecast. Training sets are filled with instantaneous values of wave excitation force, either collected directly from pressure probes installed on the WEC hull, or indirectly with estimators. Since pressure probes are practically impossible to use in real applications (due to the fact that separating the wave excitation from the device motion within the corresponding pressure field is technically unfeasible), a number of unknown-input estimation techniques have been developed in literature (Peña-Sanchez et al., 2020b). This study exploits a recent innovative approach (Faedo et al. (2022a)), which treats the input estimation problem in terms of a classical tracking control loop.

Suppose v_m is the measured velocity of the controlled PeWEC DoF, and consider the model G_ε, associated with such a mode of motion. Via solution of a velocity tracking objective,

$$\lim_{t\to\infty} \|v_m - v_\varepsilon\| = 0, \tag{14}$$

facilitated by a control signal u_{est}, is straightforward to show that u_{est} can be used to compute an approximation of the total wave force component $\tilde{f}_e$, *i.e.*

$$\tilde{f}_e \simeq -u + u_{est}. \tag{15}$$

To achieve (14), we note that, as discussed in Section 2, the transfer function G_ε of the controlled DoF is stable, strictly proper, and minimum phase. Such properties allows for direct consideration of the so-called Youla parameterisation for the design of the control law u_{est} in (15).

In particular, we design a (SISO) controller $K : \mathbb{C} \to \mathbb{C}$, in terms of a corresponding Youla parameter $Q(s)$ (Youla et al., 1976), *i.e.*

$$K(s) = \frac{Q(s)}{1 - G_\varepsilon(s)Q(s)}. \tag{16}$$

If $Q(s)$ is proper and stable, the closed-loop resulting from the controller $K(s)$ is stable. Based on the principle of plant inversion, and given the aformentioned properties associated with G_ε, $Q(s)$ is chosen as the filtered inverse response of the plant, *i.e.*

$$Q(s) = \frac{F_q(s)}{G_\varepsilon(s)} \quad F_q(s) = \frac{\omega_c^{n_q}}{(s+\omega_c)^{n_q}}, \tag{17}$$

where F_q is a low-pass shaping filter, with $n_q = 2$ and $\omega_c = 30$ [rad/s]. An overview of the estimator loop can be appreciated in Fig. 4. Note that, via the definition of the 'condensed' map $\tilde{f}_e$ in (5), the estimator problem solved herein is of a SISO nature, and a single estimated signal is required to solve the MPC optimization procedure.

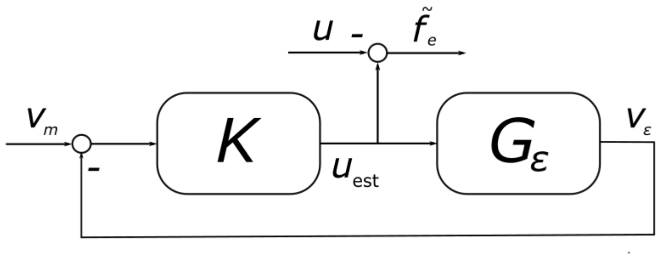

Fig. 4. Employed estimator architecture.

3.5 A note on estimator-forecaster interconnection

Since, within real-time applications, the MPC optimizer can potentially fail to find a definitive solution if the number of iterations is restricted to alleviate computational demand, we suggest adding a suitable random white noise characterisation to the prediction training data set. This technique is used to inform the AR on the potential existence of high frequency components (induced by potential 'jumps' between non-converged control solutions, hence preventing an undesired output from the forecaster model.

4. RESULTS

In this section, the results of the proposed control loop are presented, together with a detailed account on the choice of controller, estimator, and forecaster parameters. The simulations have been performed within irregular wave scenarios, selected from the Mediterranean spot of Pantelleria. From now on, figures refer to simulations considering waves with a significant wave height $H_s = 3$ [m], and energetic period $T_e = 6$ [s]. Every wave is tested for $t = 600$ [s], a significant time length to produce statistically consistent performance results.

For this PeWEC application case, the MPC prediction horizon T_p is set to 100 steps (5 [s] sampled at 20 [Hz]), the control action is computed every 50 [ms], and the cost function parameters in Eq. 6 $\{q, r\}$ are set to $q = 10^{-9}$, $r = 10^{-6}$, aiming to maximise power extraction, while keeping a convex objective J. The AR model order is selected as $n = 40$, and trained on 20 [s] of past wave estimated values.

In the following, we report and discuss estimation, prediction and power extraction results, obtained with the proposed controller architecture. To begin with, note that, as can be appreciated from Fig. 5, the proposed estimation strategy can provide an accurate account of the total excitation force $\tilde{f}_e$, with a maximum measured delay of 0.1 [s], and a maximum amplitude normalised root mean square error of 3.6%.

Fig. 6 shows instead the prediction performance of the AR model at simulation time $t = 314$ [s], proving the effectiveness of the algorithm in forecasting the true shape of the excitation force (which influences directly the quality of the algorithm optimality performance).

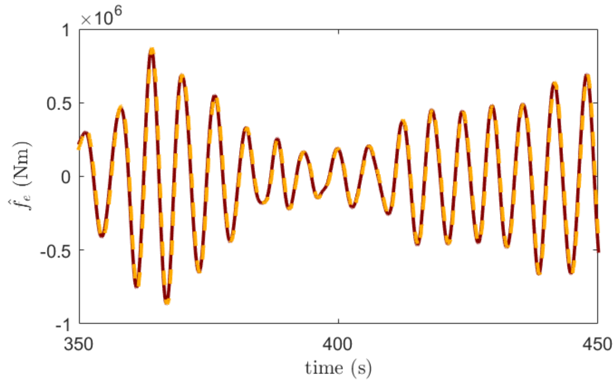

Fig. 5. Sample of the force produced by the total wave excitation force $\tilde{f}_e$ on the PTO (red), and the estimated signal (yellow).

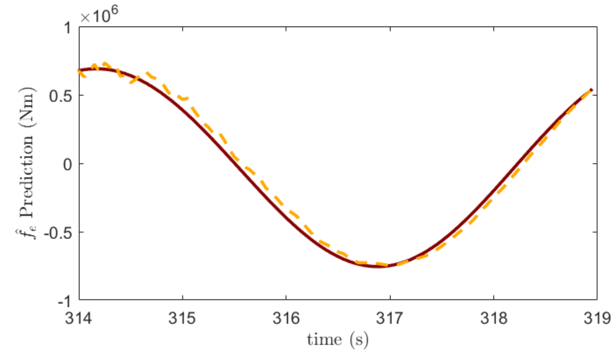

Fig. 6. Wave prediction at $t = 314$ [s] simulation time. In red the exact incoming wave, in yellow the instantaneous AR prediction fed to the MPC

To provide a fair comparison in terms of energy-maximising performance, let us introduce the well-known and standard reactive proportional-integral control law:

$$T_{cont} = -k_c\varepsilon - c\dot{\varepsilon} \tag{18}$$

in which the set of parameters $\{k_c, c\} \subset \mathbb{R}^+$ are chosen via exhaustive parametric simulations. To highlight the ability of the MPC control strategy to keep the variables of interest within the constraint limits ($\dot{\varepsilon}_{max} = 1.2217\ rad/s$ ans $u_{max} = 1140\ kNm$, Fig 7 shows the PTO velocity $v_\varepsilon = \dot{\varepsilon}$ obtained with c and k_c parameters which maximize the power production neglecting explicit limitations on ε and u magnitude (for this particular wave realisation), and the velocity obtained with the optimal model-based control. It can be easily recognised how the MPC manages to keep the imposed constraints on the velocity $\dot{\varepsilon}$, while reactive control oversteps imposed limits. This constraint overcoming in the real application is directly implies PTO system malfunction.

Differently, Fig. 8 shows the power extraction comparison between c and k_c parameters capable of maximising energy production while keeping between given bounds the constrained variables (only for this particular wave realization), and the MPC power extraction capacity. The MPC controller outperforms the reactive controller in (18) in terms of power extraction, while keeping the corresponding constraints on the $\dot{\varepsilon}$ variable.

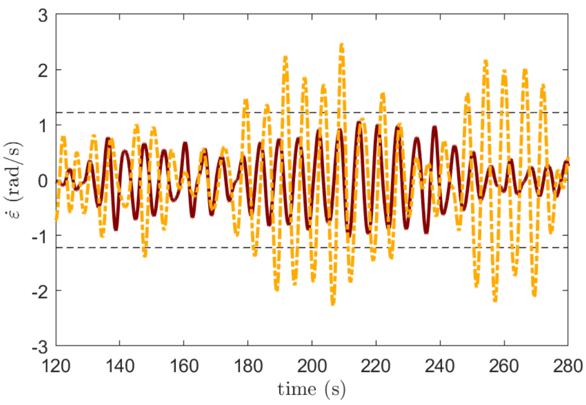

Fig. 7. PTO velocity $\dot{\varepsilon}$ in case of MPC control (red) and optimal reactive control (yellow). In black, the constraint limits set on $\dot{\varepsilon}$

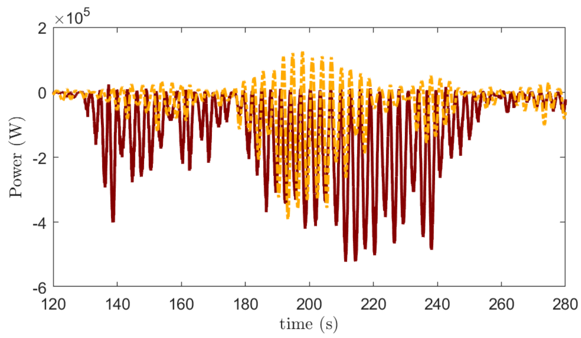

Fig. 8. Produced Power. In red, the MPC control strategy, in yellow the c-k control capable to respect given constraints

5. CONCLUSIONS

This study implements a MPC control strategy for the multi-DoF PeWEC offshore wave energy harvester. Together with the control algorithm itself, we develop a suitable (total) wave force estimator and corresponding predictor, assessing the interconnected control loop, with the MPC designed based upon a specific SISO control-oriented reduced model. This paper also discusses central issues of MPC implementation for WECs, including *e.g.* convexity of the cost function, and suitable tuning of the associated estimator and AR parameters.

Finally, we show the potential of the proposed strategy in maximising power extraction for the specific device (the result can be easily extended for underactuated multi DoF systems), while keeping the variables of interest within prescribed constraint bounds, being able to outperform well-established WEC control architectures.

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