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Landscapes" by Litwin et al

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09 April 2024

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Comment on "Groundwater Affects the Geomorphic and Hydrologic Properties of Coevolved Landscapes" by Litwin et al.

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Abstract The objective of this comment is to correct two sets of statements in Litwin et al. (2022,
https://doi.org/10.1029/2021JF006239), which consider our research work (Bonetti et al., 2018, https://doi.
org/10.1098/rspa.2017.0693; Bonetti et al., 2020, https://doi.org/10.1073/pnas.1911817117). We clarify here
that (a) the specific contributing area is defined in the limit of an infinitesimal contour length instead of the
product of a reference contour width (Bonetti et al., 2018, https://doi.org/10.1098/rspa.2017.0693), and (b)
not all solutions obtained from the minimalist landscape evolution model of Bonetti et al. (2020, https://doi.
org/10.1073/pnas.1911817117) are rescaled copies of each other. We take this opportunity to demonstrate
that the boundary conditions impact the obtained solutions, which has not been considered in the dimensional
analysis of Litwin et al. (2022, https://doi.org/10.1029/2021JF006239). We clarify this point by using
dimensional analysis and numerical simulations for a square domain, where only one horizontal length scale
(the side length l) enters the physical law.

Plain Language Summary This comment is focused on rectifying the statements regarding the
previous works of Bonetti et al. (2018, https://doi.org/10.1098/rspa.2017.0693) and Bonetti et al. (2020,
https://doi.org/10.1073/pnas.1911817117) in Litwin et al. (2022, https://doi.org/10.1029/2021JF006239). We
clarify here the definition of the specific contributing area and the role of boundary conditions in the obtained
solutions of the landscape evolution model, which has been left unnoticed by Litwin et al. (2022, https://doi.
org/10.1029/2021JF006239). Using dimensional analysis and simulations for a square domain, we show that
only those solutions remain rescaled copies of each other where the non-dimensional group containing the
influence of boundary conditions remains the same. This result was already incorporated in the dimensional
analysis of Bonetti et al. (2020, https://doi.org/10.1073/pnas.1911817117), where a non-dimensional index C_I
was derived explicitly.

Litwin et al. (2022) wrote a very interesting paper on the problem of the co-evolution of groundwater hydrology
and land-surface dynamics. The authors are to be commended for this very nice research contribution, which
quantifies the feedback between the spatial patterns in the emerging topography and sub-surface properties.

This comment is intended only to rectify some statements in this paper that regard our work (Bonetti
et al., 2018, 2020). The first one considers the definition of the specific contributing area a and its relation to
the total contributing area A. The second one is related to the dimensional analysis and the scaling properties of
the landscape evolution model (LEM) solutions. We feel that these corrections are necessary to avoid misinter-
pretations of our results as well as to clarify important aspects regarding the scaling properties of the obtained
solutions.

The statements in question concern the LEM with governing equations

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K\sqrt{a}|\nabla z| + U, \tag{1}$$

$$-\nabla \cdot \left(a\frac{\nabla z}{|\nabla z|}\right) = 1, \tag{2}$$

where z is the topographic elevation, t is time, D is a diffusion coefficient, K is an erosion coefficient, a is the specific contributing area, and U is the uplift rate. For constant K, the model—referred to as the *NoHyd* model—is a specific version of the minimalist LEM of Bonetti et al. (2020).

Regarding the definition of the specific contributing area, Litwin et al. (2022) write on page 5: "To make the conversion between A and a, we represent A as the product of a and a characteristic contour width v_0, which is a chosen constant value." In reality, a is defined in the limit of an infinitesimal contour length (Bonetti et al., 2018; Gallant & Hutchinson, 2011), not as the product of a reference contour width. As a result, Equation 2 is only valid for the specific contributing area defined in this limit, as discussed in Bonetti et al. (2018).

With reference to similarity solutions and dimensional analysis, they write on page 4, "Additionally, the nondimensionalization generalizes our results and reconciles conflicting dimensional analyses provided by Theodoratos et al. (2018) and Bonetti et al. (2020)." Further, they mention on page 27, "We show that contrary to Bonetti et al. (2020) there is a single typology of the *NoHyd* model which can be rescaled to obtain all results the model may produce." These conclusions need to be revised because the role of the boundary conditions is not included in the dimensional analysis. While the dimensional analysis of the LEM is facilitated by the fact that some of the main variables and parameters are clearly listed in the governing equations, the obtained solutions depend on the initial and boundary conditions as much as they do on the equations (Bursten, 2021).

Considering the domain geometries for which only one horizontal length scale enters the physical law (more complicated geometries will add further governing quantities), the LEM solution can be formally written as

$$z = f(K, D, U, x, y, t, l). \tag{3}$$

For concreteness, we consider here a square domain of side length l with boundaries at fixed zero elevation values that is,

$$z(0, y) = z(l, y) = 0, \qquad 0 \le y \le l, \tag{4}$$

$$z(x, 0) = z(x, l) = 0, \qquad 0 \le x \le l. \tag{5}$$

Taking L, H, and T as primary dimensions for horizontal directions, vertical direction, and time, respectively (Porporato, 2022), the dimensions of the quantities involved in Equation 3 are $[z] = H$, $[K] = L^{1/2}T^{-1}$, $[D] = L^2T^{-1}$, $[U] = HT^{-1}$, $[x] = L$, $[y] = L$, and $[l] = L$. Choosing K, D, U as the repeating variables and applying the Π theorem for the formulated physical law of Equation 3 yields

$$\Pi_z = \Phi\left(\Pi_x, \Pi_y, \Pi_t, \Pi_l\right), \tag{6}$$

with

$$\Pi_z = \frac{z}{\left(\frac{DU^3}{K^4}\right)^{1/3}} = \frac{z}{\mathcal{H}} = \hat{z}, \tag{7}$$

$$\Pi_x = \frac{x}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{x}{\mathcal{L}} = \hat{x}, \tag{8}$$

$$\Pi_y = \frac{y}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{y}{\mathcal{L}} = \hat{y}, \tag{9}$$

$$\Pi_t = \frac{t}{\left(\frac{D}{K^4}\right)^{1/3}} = \frac{t}{\mathcal{T}} = \hat{t}, \tag{10}$$

$$\Pi_l = \frac{l}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{l}{\mathcal{L}} = C_I{}^{2/3}, \tag{11}$$

where C_I is the "channelization index" derived in Bonetti et al. (2020).

Using the above-defined scales $\mathcal{H}$, $\mathcal{L}$, and $\mathcal{T}$, the non-dimensionalized form of the LEM (governing equations: Equation 1 and Equation 2; boundary conditions: Equation 4 and Equation 5) can be written as

$$\frac{\partial \hat{z}}{\partial \hat{t}} = \hat{\nabla}^2 \hat{z} - \sqrt{\hat{a}}|\hat{\nabla}\hat{z}| + 1, \tag{12}$$

$$-\hat{\nabla} \cdot \left(\hat{a} \frac{\hat{\nabla}\hat{z}}{|\hat{\nabla}\hat{z}|} \right) = 1, \tag{13}$$

$$\hat{z}(0, \hat{y}) = \hat{z}\left(C_I^{2/3}, \hat{y}\right) = 0, \qquad 0 \le \hat{y} \le C_I^{2/3}, \tag{14}$$

$$\hat{z}(\hat{x}, 0) = \hat{z}\left(\hat{x}, C_I^{2/3}\right) = 0, \qquad 0 \le \hat{x} \le C_I^{2/3}. \tag{15}$$

The absence of $\Pi_l \left(= C_I^{2/3}\right)$ in the non-dimensionalized governing Equations 12 and 13 does not mean that the solutions for this model do not depend on it. Π_l appears in the non-dimensionalized boundary conditions (Equation 14 and Equation 15) used to compute the steady-state solutions. While there is freedom in selecting the repeating variables, choosing one set over another for defining the relevant scales does not change the physics of the model at hand (Porporato, 2022). Selecting l, D, and U keeps Π_l in the dimensionless governing equations (Bonetti et al., 2020), while Π_l appears in the dimensionless boundary conditions when using K, D, and U for the non-dimensionalization.
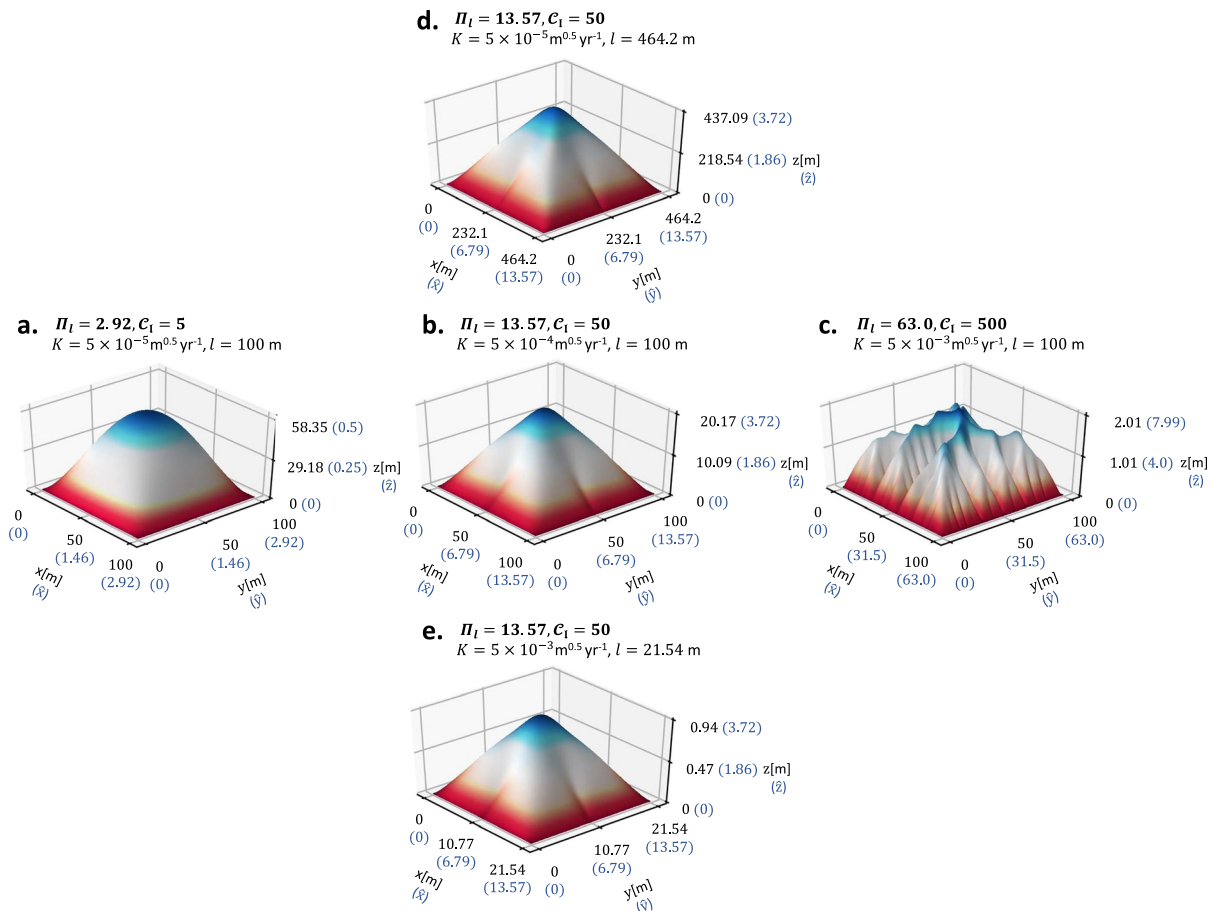

Figure 1. Steady-state solutions for a square domain with fixed elevation boundary conditions, $D = 10^{-2}$ m² yr⁻¹, and $U = 10^{-3}$ m yr⁻¹. The dimensional form $z(x, y)$ is labeled in black, while the non-dimensional form $\hat{z}(\hat{x}, \hat{y})$ computed using scales $\mathcal{H}$ and $\mathcal{L}$ is written in blue and enclosed in parentheses. In panels a, b, and c (left to right), the domain length is fixed as $l = 100$ m with increasing values of erosion coefficient K that lead to distinct topographies; also explained by the varied values of $\Pi_l = 2.92$, 13.57, and 63.0. The non-dimensional solution $\hat{z}(\hat{x}, \hat{y})$ is also different in these cases. In panels d, b, and e (top to bottom), increasing values of K are offset by the decreasing values of domain length l so that the dimensionless number $\Pi_l = 13.57$ ($C_I = 50$) remains invariant and the topographies are rescaled version of each other (same non-dimensional form $\hat{z}(\hat{x}, \hat{y})$). The simulations were performed using the numerical algorithm developed in Anand et al. (2020).

The dependency of the non-dimensional solutions on the boundary conditions through Π_l signifies that not all the solutions that the model can produce are rescaled copies of each other. This is true only for those solutions in which the non-dimensional group Π_l remains invariant. A set of steady-state simulations for a square domain, represented in both dimensional and non-dimensional forms, demonstrates this point (Figure 1). In panels a, b, and c, the value of Π_l is changed by increasing the erosion coefficient K for a domain of fixed side length $l = 100$ m. This is the case discussed in Bonetti et al. (2020), which shows different solutions ranging from the unchannelized case (Figure 1a) to a case with multiple channels (Figure 1c). The non-dimensional solutions $\hat{z}(\hat{x}, \hat{y})$ computed by employing suitable scales $\mathcal{H}$ and $\mathcal{L}$ are also dissimilar in these cases. On the contrary, panels d, b, and e have different values of K, but they preserve the same Π_l because of the different length scale l, and therefore, are scaled versions of each other (similar non-dimensional forms labeled in the blue text). Theodoratos et al. (2018) consider this case only.

In reality, the reason why many solutions of LEMs look alike is that they have an interesting property of complete self-similarity that emerges at very large channelization index values $\left(C_I = \frac{K l^{3/2}}{D} \right)$ (Hooshyar, Anand, & Porporato, 2020; Hooshyar, Bonetti, et al., 2020; Hooshyar et al., 2021; Porporato, 2022). For these conditions, several spatially averaged quantities become invariant with respect to C_I, having reached a self-similar regime. This is similar to what happens to turbulent flows in the fully rough regime as they become independent of the Reynolds number or to regular polygons, which tends to circles as the number of sides tends to infinity (Barenblatt, 1996).

Data Availability Statement

The details of the numerical algorithm used for the simulations is described in Anand et al. (2020) and the well-commented Python code is available on GitHub https://github.com/ShashankAnand1996/LEM (Anand, 2022).

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