Abstract

This research is focused on the development of new methods and algorithms for solving polynomial optimization problems arising in the framework of robust control. More specifically, the main objective of the research is to exploit powerful tools and recent mathematical results in polynomial optimization to effectively address some problems widely recognized to be complex ones and/or partially addressed in the past by means of other less effective tools, with particular reference to the problem of fixed-order/fixed-structure (FOFS) robust controller design. The thesis is divided into three parts: the first two parts focus on model-based and direct data-driven design of FOFS robust controller, respectively. Part III deals with application of convex relaxation technique for the computation of frequency response envelopes for linear-time-invariant (LTI) single-input single-output (SISO) plants affected by parametric uncertainty. Although the last part of the thesis is not directly focused to the design of FOFS controller, but it represents another interesting application of convex-relaxation techniques in the context of robust frequency domain control. Furthermore, the computed frequency response envelope bounds can be used for the design of FOFS controllers by frequency different domain methods.

1. **Model-based FOFS control design:** We propose a unified approach for designing model-based robust FOFS H_∞ mixed-sensitivity controllers, both for CT and DT systems. First, we define the feasible controller parameter set, which is the set of the controller parameters that guarantee robust stability of the closed-loop system and the achievement of the nominal performance requirements. We then translate the constraints in the feasible controller parameter set to polynomials in unknown controller parameters and frequency. Then, we compute a convex relaxation of the nonconvex feasible controller parameter set and formulate the original FOFS H_∞ mixed sensitivity control design problem as the non-emptiness test of the feasible controller parameter set. We present three algorithms for the convex relaxation of the feasible controller parameter set. The first algorithm uses Putinar's positivstellensatz for relaxing the feasible controller parameter set. It is a single-shot algorithm and provides the inner approximation of the feasible controller parameter set. The second algorithm uses a generalization of the S-procedure for SOS relaxation of the feasible controller parameter. It also a single-shot algorithm and provides the inner approximation of the feasible controller parameter set. The third algorithm is based on the exchange method

and solves the optimization problems iteratively using moment relaxation. This method provides the outer approximation of the feasible controller parameter set.

The main advantages of the proposed algorithms compared to existing methods are: (i) existing local optimization-based algorithm provide only local solutions and thus can typically trap in local minima, whereas the proposed algorithms look for controller parameters in a relaxed convex set, and (ii) the existing global optimization-based algorithms are applicable only for special plant/ controller structure.

Finally, we provide two simulation examples and one experimental application to show the efficiency and comparison of the proposed algorithms on both CT and DT systems.

2. **Direct data-driven FOFS control design:** We propose a unified approach for designing robust FOFS H_∞ mixed-sensitivity direct data-driven controllers, both for CT and DT systems. The proposed method does not require any mathematical model of the plant and only requires the frequency response samples collected from the plant (for stable systems) or its co-prime factors (for the unstable system) at discrete frequencies. For a given controller structure and/or order and a set of frequency-domain input-output data, we define the feasible controller parameter set as a semi-algebraic set of all the controller parameters that guarantee the robust stability and fulfillment of nominal performance. The problem of designing the H_∞ mixed sensitivity direct data-driven controller is then reformulated as a polynomial optimization problem. The polynomial optimization problem is then relaxed to convex SDP using moment relaxation. The resultant SDP can be solved efficiently by the readily available softwares. We also developed the necessary and sufficient conditions for achieving nominal stability in the direct data-driven framework without linear parameterization of the controller. We extend this methodology to systems subjected to frequency-domain uncertainties. Two types are uncertainties are considered in this work (i) unstructured uncertainty described by the frequency domain transfer function and (ii) unknown but bounded uncertainty affecting the experimentally collected input-output data. For the unstructured description of uncertainty, a single shot moment relaxation-based algorithm is provided for computing the parameters of the FOFS controller. For unknown but bounded uncertainty, an iterative algorithm based on the exchange method is provided to design robust FOFS H_∞ mixed-sensitivity direct data-driven controllers.

   The main advantages of the proposed algorithms compared to existing methods are: (i) It does not require linear parameterization of the controller or its co-prime factors,

(ii) existing local optimization-based algorithm provide only local solutions whereas the proposed algorithm provides the globally optimal solution, and (iii) for low order fixed-structure controllers, the existing global optimization-based algorithms either does not converge to global optimal solution or convergence is achieved by fixing the poles of the controller a priori.

Finally, we provide three simulation examples to show the efficiency of the proposed algorithms on both CT and DT systems.

3. **Computation of Bode envelope bounds:** We propose a novel approach, motivated by recent results in the framework of set membership identification, for computing the Bode plots bounds for uncertain LTI SISO systems whose parameters belong to a semialgebraic set. Based on the description of the parametric uncertainty implicitly provided by the feasible parameter set, suitable polynomial optimization problems are formulated whose global optima can be approximated arbitrarily well using the convex SDP relaxation. In the proposed approach, the uncertainty can enter the numerator and denominator polynomials nonlinearly and the coefficient space is a semialgebraic set. Thus, the proposed approach generalizes previously available results. The reported simulation examples show the effectiveness of the proposed methodology in the computation of tight bounds on the Bode plots envelope.