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A numerical displacement-based approach for the structural analysis of cable nets

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ABSTRACT: A new displacement-based approach is proposed herein to predict the behaviour of pre-tensioned cable nets subjected to vertical loads. The cables are contained in horizontal plane and have a single degree of freedom in each node, where loads are applied. The model is based on the equilibrium equations of an infinitesimal cable-element, which are solved by considering the catenary equation under the hypothesis of (a) zero bending stiffness, (b) linear elastic behaviour of materials, (c) small deformation, and (d) the existence of perfect hinges in each node. More precisely, a finite-difference numerical procedure is introduced in order to evaluate nodal displacements related to a set of applied vertical forces. The effectiveness of the proposed approach is then assessed by comparing the numerical results with those obtained by other models found in the current literature. Finally, the proposed approach is used to design new and more efficient insulating panels for the green house technology.

1 INTRODUCTION

A numerical model able to analyze the non-linear static response of pre-tensioned cable nets, to be used in the feasibility study of an ultra-insulating panel with vacuum technique (Fig. 1), is herein introduced. For this purpose, the model should have the following characteristics:

- easy to use.
- requires few input data.
- reliable.

In other words, a tool for the conceptual design of the panel has to be introduced with the aim of defining the main structural part: the net.

Unfortunately, in the current literature these models cannot be easily found. Indeed, the numerical model proposed by Lewis (1989) concerns the analysis of pre-stressed nets and pit-jointed frame structures. It was based on the principle of minimum energy and was applied to steel trusses using the linear load - displacement constitutive relationship. To simplify the approach, a new numerical procedure, to be used for the analysis of pre-tensioned 2D nets, and based on the displacement method, is herein presented.

The proposed model is then applied to cable nets, loaded in parallel planes, having a single degree of freedom in each node. As results, the nodal displacements, and the induced stress of each cable, are calculated when a set of nodal forces are applied.

To validate the proposed approach, the results of a case study are compared with those obtained with the Lewis' model (Lewis 1989).

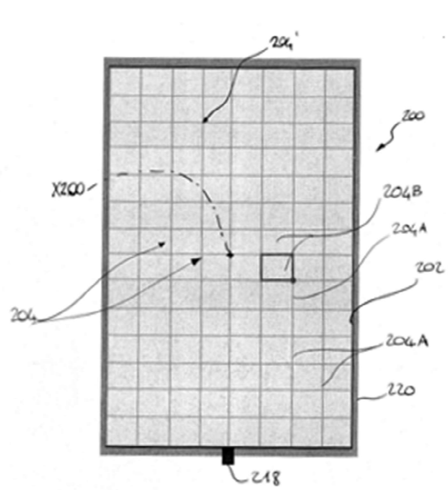

Figure 1: Prototype of ultra-insulating panel.

2 ANALYSIS OF PRE-STRESSED CABLE NETS USING THE DISPLACEMENT-BASED APPROACH

We consider a generic net structure composed by m cable elements connected in n nodes. The structural analysis of this structure is based on following assumptions:

1. the bending stiffness of each cable can be neglected.
2. cable shows a linear-elastic behavior: instability, slackening phenomena, and viscous plastic deformations are not considered.
3. only small deformations of the cable are allowed.
4. nodes are assumed to be perfect hinges.
5. cross-section of each cable is constant throughout its length.
6. self-weight and variable loads are assumed to be concentrated load acting on the nodes.

The cable net is within a plane (x and y) and each node has x, y, z coordinates according to the reference system shown in Fig. 2.1. The cable segment between the two consecutives nodes is indicated as ij (see Fig.2.2), and around the i^{th} node there are four ij segments (Fig. 2.3)

In the initial equilibrium configuration (*status 0 - the undeformed state*), the cable net is fully described by the following parameters (the subscript k indicates the generic node of the net, whereas the suffix 0 is related to *status 0*):

- the coordinates of nodes grouped in vector $\{X\}^0$:

$$\{X\}^0 = \begin{Bmatrix} X_1 \\ \\ X_k \\ \\ X_n \end{Bmatrix}^0 \; ; \; \{X_k\}^0 = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_k^0 ; \quad (k = 1, 2, \dots, n) \quad (1)$$

- the unstrained initial length of each ij segment l_{ij}^0 is defined as follows:

$$l_{ij}^0 = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \qquad (2)$$

- the initial stress in each cable is $S_{ij}{}^0$, to which a possible prestress may be added;
- the loads in the unstrained configuration (*status 0*) $\{\boldsymbol{P}\}^{\boldsymbol{0}}{}_{\boldsymbol{i}}$ are acting on each i^{th} node:

$$\{P\}^0{}_i = \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}^0 ; \qquad (i = 1, 2, \dots, n); \qquad (3)$$

In addition, the *status 0* represents the pure prestress, whereas the weight of the cable and other permanent and variable loads are included within $\{\boldsymbol{P}\}^{\boldsymbol{0}}{}_{\boldsymbol{i}}$.

After applying external loads, the new equilibrium configuration is defined by:

1. a variation of the node coordinates:

$$\begin{cases} \Delta x_{ij} = u_i - u_j \\ \Delta y_{ij} = v_i - v_j \\ \Delta z_{ij} = w_i - w_j \end{cases} \qquad (4)$$

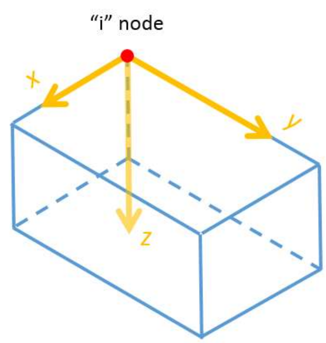

Figure 2.1: Reference system for the cable net.

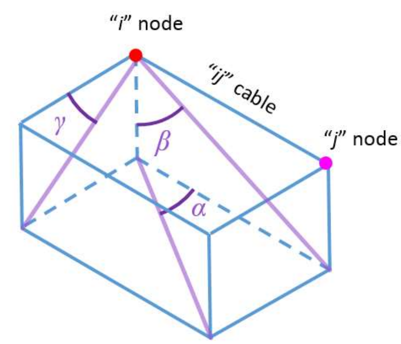

Figure 2.2: Definition of ij cable in the undeformed state (*status 0*).

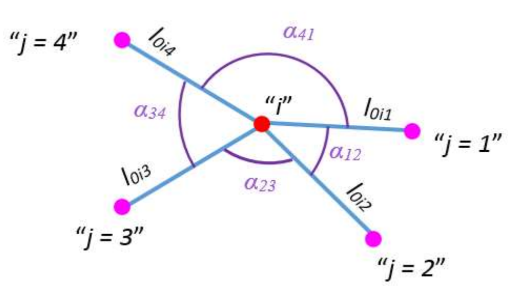

Figure 2.3: Reference geometry in initial unstrained state (where u, v and w indicate the displacements of each single node, according to the reference system x, y, z, respectively, as shown in Figure 2.4).

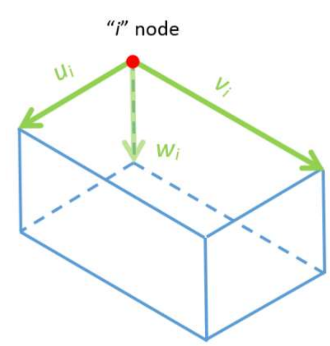

Figure 2.4: Displacements of each node in x, y, z directions.

2. increment of the length of each *ij* segment:

$$\Delta l_{ij} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta z_{ij}^2} \tag{5}$$

3. increment of the stress in each *ij* segment:

$$\Delta S_{ij} = (EA)_{ij} \frac{\Delta l_{ij}}{l_{ij}^0} \tag{6}$$

where *E* and *A* represent the elastic modulus and the cross-sectional area of the *ij* segment, respectively. Considering Eq. (5), Eq. (6) can be re-written as:

$$\begin{cases} \Delta S_{ij}^x = \Delta S_{ij} \frac{\Delta x_{ij}}{l_{ij}^0 + \Delta l_{ij}} \\ \Delta S_{ij}^y = \Delta S_{ij} \frac{\Delta y_{ij}}{l_{ij}^0 + \Delta l_{ij}} \\ \Delta S_{ij}^z = \Delta S_{ij} \frac{\Delta z_{ij}}{l_{ij}^0 + \Delta l_{ij}} \end{cases} \tag{7}$$

and the final stress of single cable can be written as $\{S\}_{ij} = \{S_{ij}^0 + \Delta S_{ij}\}_{ij}$.

4. Increment of the loads applied in each i^{th} node:

$$\{P\}_i = \{P^0 + \Delta P\}_i \tag{8}$$

where *ΔP* represents the load increment with respect to the previous state.

To define the final state (and therefore *ΔP*), the equilibrium and compatibility equations are included in the matrix relationship:

$$\{P\} = [K] \cdot \{X\} \tag{9}$$

where:

$\{P\}$ = vector ($2n \times 1$) of the load on the net node.
$[K]$ = global stiffness matrix ($2n \times 2n$).
$\{X\}$ = vector ($2n \times 1$) of the displacements in a net node.

The stress variation in each single cable is defined by the Equilibrium equations considering the applied loads, according to the reference system:

$$\begin{cases} \sum_{kj=1}^{i} \frac{S_{kj}^0}{l_{kj}^0} \Delta x_{kj}^0 = 0; \quad (= -P_{xk}^0) \quad \textit{in x direction} \\ \sum_{kj=1}^{i} \frac{S_{kj}^0}{l_{kj}^0} \Delta y_{kj}^0 = 0; \quad (= -P_{yk}^0) \quad \textit{in y direction} \\ \sum_{kj=1}^{i} \frac{S_{kj}^0}{l_{kj}^0} \Delta z_{kj}^0 = 0; \quad (= -P_{zk}^0) \quad \textit{in z direction} \end{cases} \tag{10}$$

By analyzing the structure, it is necessary to write three equilibrium equations for each node, leading to *3n* system of equations. Thus Eq. (9) becomes:

$$3n \begin{Bmatrix} i=1 \\ \vdots \\ \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}_i \\ \vdots \\ i=n \end{Bmatrix} = [K] \cdot (3n) \begin{Bmatrix} i=1 \\ \vdots \\ \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_i \\ \vdots \\ i=n \end{Bmatrix} \tag{11}$$

Considering the terms of Eqs. (4) written in the Taylor-McLaurin Series (we neglect the terms beyond the 1^{st} order because of the hypothesis of small deformations), Eq.(5) and Eq.(9) can be written as in Eq.(12) (see the box).

If α_{ij} is the relative extension of the cable:

$$\alpha_{ij} = \frac{l_{ij}^0}{\sqrt{(u_i - u_j)^2 + (v_i - v_j)^2 + (w_i - w_j)^2}} \tag{13}$$

the loading state variation system of the generic i^{th} node can be written as:

$$\begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}_i = \begin{Bmatrix} \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{u_i - u_j}{1 + \alpha_{ij}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{v_i - v_j}{1 + \alpha_{ij}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{w_i - w_j}{1 + \alpha_{ij}} \end{Bmatrix}_i \tag{14}$$

As the net is made with a single material, having a constant elastic modulus (*E*), for each i^{th} node, it is possible to write:

$$\begin{cases} P_x = u_i \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} - \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} u_j \\ P_y = v_i \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} - \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} v_j \\ P_z = w_i \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} - \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} w_j \end{cases} \tag{15}$$

By considering:

$$\begin{aligned} K_{ix} &= \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} \\ K_{iy} &= \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} \\ K_{iz} &= \sum_{j=1}^{4} \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} \end{aligned} \tag{16}$$

and

$$\begin{aligned} P_x^0 &= \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} u_j \\ P_y^0 &= \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} v_j \\ P_z^0 &= \frac{EA_{ij}}{l_{ij}^0 \cdot (1 + \alpha_{i1})} w_j \end{aligned} \tag{17}$$

$$\begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}_i = \begin{Bmatrix} \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{\Delta x_{ij}}{1 + \frac{l_{ij}^0}{\sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta z_{ij}^2}}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{\Delta y_{ij}}{1 + \frac{l_{ij}^0}{\sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta z_{ij}^2}}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{\Delta z_{ij}}{1 + \frac{l_{ij}^0}{\sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta z_{ij}^2}}} \end{Bmatrix}_i = \begin{Bmatrix} \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{u_i - u_j}{1 + \frac{l_{ij}^0}{\sqrt{(u_i - u_j)^2 + (v_i - v_j)^2 + (w_i - w_j)^2}}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{v_i - v_j}{1 + \frac{l_{ij}^0}{\sqrt{(u_i - u_j)^2 + (v_i - v_j)^2 + (w_i - w_j)^2}}} \\ \sum_{j=1}^{N} \frac{(EA)_{ij}}{l_{ij}^0} \cdot \frac{w_i - w_j}{1 + \frac{l_{ij}^0}{\sqrt{(u_i - u_j)^2 + (v_i - v_j)^2 + (w_i - w_j)^2}}} \end{Bmatrix}_i \tag{12}$$

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the Equilibrium of the net (composed by 9 equations) can be re-written as:

$$\begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}_i = \begin{bmatrix} K_{ix} & 0 & 0 \\ 0 & K_{iy} & 0 \\ 0 & 0 & K_{iz} \end{bmatrix} \cdot \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_i + \begin{Bmatrix} P_x^0 \\ P_y^0 \\ P_z^0 \end{Bmatrix}_i \tag{18}$$

Accordingly, the displacements of each i^{th} node can be calculated as:

$$\begin{Bmatrix} u_i = (P_{ix} - P_{ix}^0) \cdot \frac{1}{K_{ix}} \\ v_i = (P_{iy} - P_{iy}^0) \cdot \frac{1}{K_{iy}} \\ w_i = (P_{iz} - P_{iz}^0) \cdot \frac{1}{K_{iz}} \end{Bmatrix} \tag{19}$$

From the analytical model previously described , a simple numerical algorithm has been developed to calculate displacements and stresses induced by the deformation of the cable net.

3 VALIDATION OF THE NUMERICAL MODEL

To validate the proposed algorithm, it is applied to ideal nets of simple geometry (i.e., those reported in Lewis 1989). Each net is in the plane (*x*, *y*) and the displacements are in the *z* direction. The cables are developed in *x* or *y* direction, at equidistant spacing, whereas only the central internal nodes are loaded with the forces P_z. The displacement will be calculated in these nodes. According to Table 1 (Lewis 1989), the following examples are analyzed:

- Example 1: simple net. The net consists of only two cables (one in *x* direction and one in *y* direction) with three degrees of freedom. Thus, there are 4 square meshes, whereas a single load is applied in the central node (Fig. 3a).
- Example 2: cable net 2 × 1. A net with a rectangular mesh loaded in the two internal central nodes (Fig.4a). The net is built with three cables (two in *x* direction and one in *y* direction), with six degrees of freedom (Buchholdt 1985).
- Example 3: cable net 2 × 2. A net with square mesh and load applied in the central nodes (symmetrical load). The net is built with four cables (two in *x* direction and two in *y* direction), with twelve degrees of freedom (see Fig.5a).

Table 2 shows the results obtained with proposed model and by Lewis (1989) in terms of z-displacement of the central nodes. Moreover, the total displacements of the net computed by the numerical algorithm is shown in Figg.3b, 4b and 5b.

To give a quantitative rate of the model, the following parameters are computed:

- percentage of the maximum deflection (at the central nodes of the net) of the cable:
  $$\Delta l_{\%} = 100 - \left[\left(l^0 - \Delta l \;/_{l^0}\right) * 100\right] \tag{20}$$
- the deviation of the calculated deflection from that of Lewis (1989):
  $$\Delta l_{Lewis} - \Delta l_{proposed\ model} \tag{21}$$
- percentage of deviation from Lewis' (1989) displacement:
  $$Dev_{\%} = \left[\left(\frac{\Delta l_{Lewis} - \Delta l_{proposed\ model}}{\Delta l_{Lewis}}\right) * 100\right] \tag{22}$$

As shown in Table 2, we note that the deflection given by the proposed numerical model and those computed by Lewis (1989) are coincided in nets of Example 1 and 3. Only for the net of the Example 2, the deflection of the central nodes given by the proposed numerical model differ from the values computed by Lewis (1989) of about 7%. The latter error can be acceptable, because the model is used to the feasibility study of the panel illustrated in Fig.1.

Table 1. Geometric and material characteristics of three ideal nets (Lewis 1989).

	Example 1 simple net	*Example 2* 2 × 1 cable net	*Example 3* 2 × 2 cable net
Size of the net (cm × cm)	80 × 80	600 × 800	120 × 120
Cross-section area of cables *A* (mm^2)	0.785	2.00	0.785
Young's modulus *E* (GPa)	124.80	110.0	124.80
Pretension load in the cables S^0 (kN)	0.2	0.5	0.2
External load applied on the internal node P^0 (kN)	0.015	0.200	0.015
Diameter of the cable *d* (mm)	1	1.596	1
Cable initial length l^0 (mm)	400	2.000	400
Total number of nodes	5	8	12
Number of nodes on the perimeter	4	6	8
Number of internal nodes	1	2	4

Table 2. Comparison between the results obtained with the proposed model and those computed by Lewis (1989).

	Deflection (mm)	$\Delta l_{\%}$ (%)	$Dev_{\%}$ (%)
Example 1			
Proposed model	6.98	+1.745	+0.01
Lewis (1989)	6.97	+1.743	+0.14
Example 2			
Proposed model	184.78	+9.24	+14.92
Lewis (1989)	199.70	+9.99	+7.47 %
Example 3			
Proposed model	12.78	+3.20	-0.56
Lewis (1989)	12.22	+3.06	+5.75 %

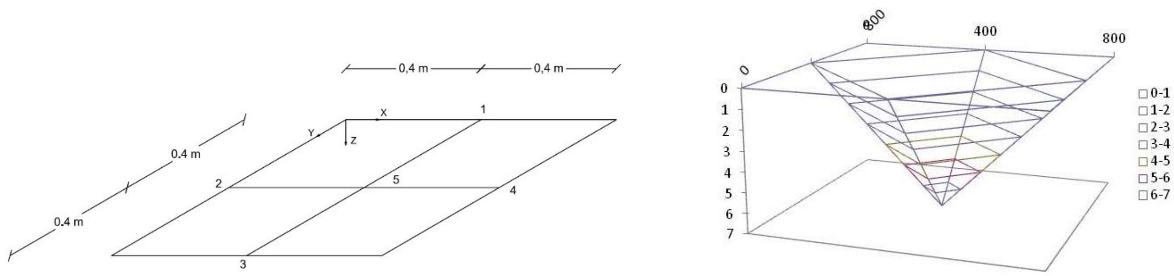
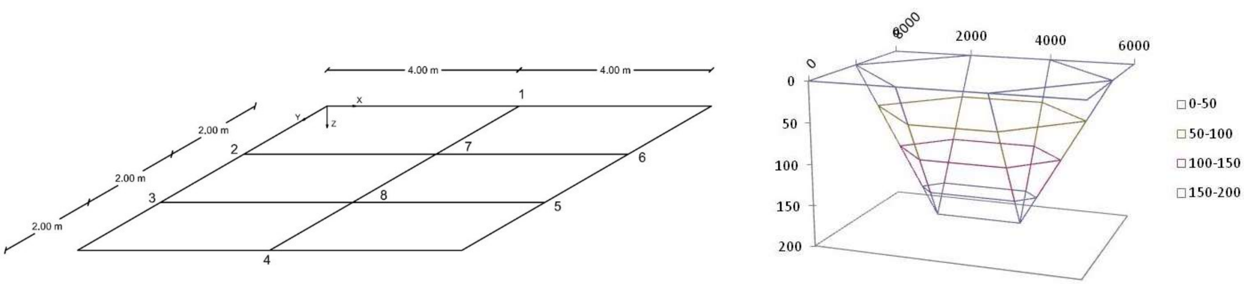


Figure 3. (a) Geometric scheme of the 2 × 2 cable net with total dimension equal to 80 cm × 80 cm and cable length of 400 mm (Example 1 - Lewis 1989) and (b) the vertical displacements (along the z axis) of the nodes.



(a) (b)

Figure 4. (a) Geometric scheme of the 2 × 1 cable net with total dimension equal to 6 m × 8 m and cable length of 2.000 mm (Example 2 - Lewis 1989) and (b) the vertical displacements (along the z axis) of the nodes.

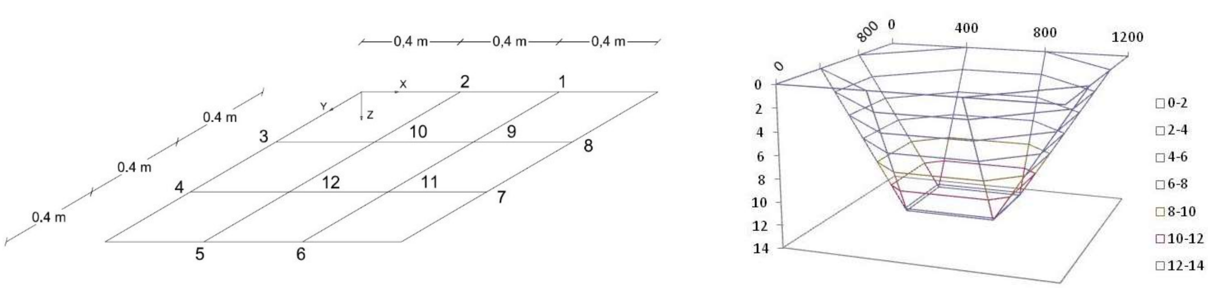


Figure 5. (a) Geometric scheme of the 2 × 2 cable net with total dimension equal to 120 cm × 120 cm and cable length of 400 mm (Example 3 - Lewis 1989) and (b) the vertical displacements (along the z axis) of the nodes.

4 CONCLUSIONS

Based on the results of a simplified nonlinear procedure for the analysis of cable nets, the following conclusion can be drawn:

- the equations of the dynamic equilibrium of a body in motion (the D'Alembert principle), leads to the formulation of the dynamic relaxation algorithm, as obtained by Lewis (1989).
- in the same way, in the proposed model the linear load- displacement relationship can be obtained by means of a stiffness matrix algorithm.
- both the models calculate approximately the same maximum vertical displacement of the nets made with cables.
- Thus, the proposed simplified can be effectively used to design the nets made with steel cables.

Future paper will be devoted to use the proposed model for the feasibility analysis of prototype of a vacuum ultra-high insulating panel.

ACKNOWLEDGMENTS

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