POLITECNICO DI TORINO
Repository ISTITUZIONALE

Holographic evidence for nonsupersymmetric conformal manifolds

Original
Holographic evidence for nonsupersymmetric conformal manifolds / Giambrone, A., Guarino, A., Malek, E., Samtleben, H., Sterckx, C., Trigiante, M.. - In: PHYSICAL REVIEW D. - ISSN 2470-0010. - ELETTRONICO. - 105:(2022), p. 066018. [10.1103/physrevd.105.066018]

Availability:
This version is available at: 11583/2972409 since: 2022-10-18T16:07:56Z

Publisher:
APS physics

Published
DOI:10.1103/physrevd.105.066018

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)

27 November 2026

Holographic evidence for nonsupersymmetric conformal manifolds

Alfredo Giambrone,[1,*] Adolfo Guarino,[2,3,†] Emanuel Malek,[4,‡] Henning Samtleben,[5,6,§] Colin Sterckx,[7,2,‖] and Mario Trigiante[1,¶]

[1]*Department of Applied Science and Technology, Politecnico di Torino, Corso Duca degli Abruzzi 24, I-10129 Torino, Italy and INFN–Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy*
[2]*Departamento de Física, Universidad de Oviedo, Avenue Federico García Lorca 18, 33007 Oviedo, Spain*
[3]*Instituto Universitario de Ciencias y Tecnologías Espaciales de Asturias (ICTEA), Calle de la Independencia 13, 33004 Oviedo, Spain*
[4]*Institut für Physik, Humboldt-Universität zu Berlin, IRIS Gebäude, Zum Großen Windkanal 2, 12489 Berlin, Germany*
[5]*University Lyon, Ens de Lyon, CNRS, Laboratoire de Physique, F-69342 Lyon, France*
[6]*Institut Universitaire de France (IUF), France*
[7]*Université Libre de Bruxelles (ULB) and International Solvay Institutes, Service de Physique Théorique et Mathématique, Campus de la Plaine, CP 231, B-1050, Brussels, Belgium*

(Received 23 December 2021; accepted 4 March 2022; published 29 March 2022)

We provide the first holographic evidence for the existence of a nonsupersymmetric conformal manifold arising from exactly marginal but supersymmetry-breaking deformations of a superconformal three-dimensional field theory. In particular, we construct a 2-parameter nonsupersymmetric deformation of a supersymmetric AdS nongeometric background in type IIB string theory. We prove that the nonsupersymmetric backgrounds are perturbatively stable and also do not suffer from various nonperturbative instabilities. Finally, we argue that diffeomorphism symmetry protects our solutions against higher-derivative string corrections.

DOI: 10.1103/PhysRevD.105.066018

Among quantum field theories, conformal field theories (CFTs) play a distinguished role. For example, CFTs are important in statistical mechanics, where they provide a description of many phase transitions. Moreover, CFTs are fixed points of the renormalization-group flow thus introducing a notion of universality. Finally, because of the constraints imposed by conformal invariance, strongly coupled CFTs can provide an insight into nonperturbative QFTs more generally.

An important question to ask when studying CFTs is whether they are isolated fixed points of the renormalization-group flow, or belong to a family of CFTs, known as a conformal manifold. The conformal manifold is spanned by exactly marginal deformations of the CFT, i.e., marginal operators whose β-functions vanish exactly to all orders. Over the last decade, much insight has been gained into local properties of conformal manifolds of supersymmetric conformal field theories [1–5]. In particular, it is not uncommon for four-dimensional $\mathcal{N}=1$ and three-dimensional $\mathcal{N}=2$ CFTs to possess conformal manifolds, whose dimensions can be deduced from the symmetry of the CFTs, without need to compute β-functions or even having a Lagrangian description.

On the other hand, no example is known of a nonsupersymmetric conformal field theory in more than two dimensions featuring a conformal manifold. Indeed, they are widely believed not to exist, since it is unclear how the precise cancellations in the β-functions will be achieved without supersymmetry. However, there are no "no-go theorems" that forbid nonsupersymmetric conformal manifolds. As a result, the existence of nonsupersymmetric conformal manifolds has been largely the subject of speculation, with only few systematic analyses performed recently [6–9].

The AdS/CFT correspondence [10–12] between anti–de Sitter (AdS) solutions of string theory and CFTs provides a

*alfredo.giambrone@polito.it
†adolfo.guarino@uniovi.es
‡emanuel.malek@physik.hu-berlin.de
§henning.samtleben@ens-lyon.fr
‖colin.sterckx@ulb.ac.be
¶mario.trigiante@polito.it

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

powerful tool to address this question, at least in the "large-N limit" where the rank of the gauge group of the CFT is taken to be large. The correspondence maps the conformal manifold of a CFT to a continuous family, known as the "moduli space," of AdS solutions of string theory. As yet, no continuous family of nonsupersymmetric AdS solutions of string theory has been constructed, with the possible exception of [13] (see discussion below). Indeed, nonsupersymmetric AdS solutions of string theory are conjectured to be unstable [14], with only a handful of *isolated* potentially stable nonsupersymmetric AdS vacua known [15].

In this paper, we will provide the first holographic evidence for a three-dimensional nonsupersymmetric conformal manifold. We do this by constructing a 2-parameter nonsupersymmetric deformation of an $\mathcal{N}=4$ supersymmetric AdS_4 vacuum describing a nongeometric solution of type IIB superstring theory. We will prove that the entire 2-parameter family is perturbatively stable in IIB supergravity, and show that it does not suffer from various nonperturbative instabilities. We note that just as for the supersymmetric deformations considered in [16,17], the nonsupersymmetric deformations we study here can also locally be absorbed by coordinate redefinitions, which are, however, not globally well defined. This implies that any local diffeomorphism-invariant quantities, such as those controlling higher-derivative corrections of string theory, are independent of the deformations. This provides hope that our conformal manifold may also exist beyond the large-N limit of the CFT.

We construct our nonsupersymmetric 2-parameter family of AdS_4 vacua of IIB string theory by uplifting the corresponding family of AdS_4 vacua of four-dimensional $[SO(6)\times SO(1,1)]\ltimes \mathbb{R}^{12}$ supergravity [18] using the truncation Ansatz of [19]. Our family of AdS_4 vacua depends on two "axionic" parameters χ_1,χ_2 [18]. For generic values of $\chi_{1,2}$, the AdS_4 vacua are nonsupersymmetric and preserve a $U(1)^2$ symmetry. Three patterns of (super) symmetry enhancement occur at special loci of the (χ_1,χ_2) parameter space. For $\chi_1=\pm\chi_2$, there is an $\mathcal{N}=2$ supersymmetry enhancement whereas a $U(1)^2$ symmetry is still preserved. These AdS_4 vacua belong to the family considered in [16,20,21]. For $\chi_1=0$ or $\chi_2=0$, the vacua are nonsupersymmetric but the residual symmetry gets enhanced to $SU(2)\times U(1)$. Lastly, an $\mathcal{N}=4$ and SO(4) symmetric AdS_4 vacuum appears at the special point $\chi_1=\chi_2=0$. As a result, $\chi_{1,2}$ parameterise nonsupersymmetric deformations of the $\mathcal{N}=4$ AdS_4 S-fold vacuum of IIB string theory [19].

The ten-dimensional geometry we obtain is a nonsupersymmetric "S-fold" of the form $AdS_4\times S^1_\eta\times S^5$, where $S^5=\mathcal{I}\times S^2_1\times S^2_2$ and $\mathcal{I}$ is an interval with angular coordinate $\alpha\in[0,\frac{\pi}{2}]$. The term S-fold refers to the fact that the 10-dimensional solution has an $SL(2,\mathbb{Z})$ S-duality monodromy of IIB string theory as we move around the S^1_η circle. The corresponding dual CFT is known as a J-fold CFT obtained by compactifying $\mathcal{N}=4$ super Yang-Mills theory on a circle with an $SL(2,\mathbb{Z})$ twist [22]. Holography has recently proven powerful in studying supersymmetric AdS_4 vacua of these types and their supersymmetric deformations [16–19,21,23–27].

More concretely, the S-fold solution can be constructed out of the following 10-dimensional solution of classical type IIB supergravity:

$$ds_{10}^2=\Delta^{-1}\Big[\frac{1}{2}ds^2_{AdS_4}+d\eta^2+d\alpha^2+\frac{\cos^2\alpha}{2+\cos(2\alpha)}d\Omega_1+\frac{\sin^2\alpha}{2-\cos(2\alpha)}d\Omega_2\Big],\quad(1)$$

where χ_i-twisted two-spheres Ω_i have metrics

$$d\Omega_i=d\theta_i^2+\sin^2\theta_i d\varphi_i'^2\quad\text{with}\quad d\varphi_i'=d\varphi_i+\chi_i d\eta,\quad(2)$$

and the nonsingular warping factor is

$$\Delta^{-4}=4-\cos^2(2\alpha).\quad(3)$$

The two-form potential B_2 and C_2 take the form

$$B_2=-2\sqrt{2}e^{-\eta}\frac{\cos^3\alpha}{2+\cos(2\alpha)}\mathrm{vol}_{\Omega_1},\qquad C_2=-2\sqrt{2}e^{\eta}\frac{\sin^3\alpha}{2-\cos(2\alpha)}\mathrm{vol}_{\Omega_2},\quad(4)$$

whereas the dilaton $g_s=e^{\Phi}$ and the axion C_0 read

$$e^{\Phi}=\sqrt{2}e^{-2\eta}\frac{2-\cos(2\alpha)}{\sqrt{7-\cos(4\alpha)}},\quad\text{and}\quad C_0=0.\quad(5)$$

The four-form potential C_4, yielding a self-dual field strength $\tilde{F}_5=dC_4+\frac{1}{2}(B_2\wedge dC_2-C_2\wedge dB_2)$, reads

$$C_4=\frac{3}{2}\omega_3\wedge\Big(d\eta+\frac{2}{3}\sin(2\alpha)d\alpha\Big)-\frac{1}{2}f(\alpha)d\alpha\wedge(A_1\wedge\mathrm{vol}_{\Omega_2}+\mathrm{vol}_{\Omega_1}\wedge A_2),\quad(6)$$

where $d\omega_3=\mathrm{vol}_{AdS_4}$ with AdS radius $L_{AdS_4}=1$. The function $f(\alpha)$ in (6) is given by

$$f(\alpha)=\sin^2(2\alpha)\frac{\cos(4\alpha)-55}{(7-\cos(4\alpha))^2},\quad(7)$$

where we have introduced one-forms $A_i=-\cos\theta_i d\varphi_i'$ so that $dA_i=\mathrm{vol}_{\Omega_i}$. Note that the function $f(\alpha)$ in (7) vanishes at $\alpha=0,\frac{\pi}{2}$, where each of the S^2 shrinks to zero size in a smooth way so that the compact space is topologically $S^1_\eta\times S^5$. We have explicitly verified that the above class of

backgrounds satisfies the ten-dimensional type IIB equations of motions and sourceless Bianchi identities.

The S-fold solution, characterized by an $\mathrm{SL}(2,\mathbb{Z})$ monodromy along S^1_η, can then be obtained from the above solution through a suitable $\mathrm{SL}(2,\mathbb{R})$-transformation together with an appropriate choice of the period T, according to the prescription given in [19,22]. In this way the monodromy can be chosen, for instance, to be a hyperbolic element of the form

$$J_k = \begin{pmatrix} k & 1 \\ -1 & 0 \end{pmatrix}, \qquad k > 2. \tag{8}$$

This choice requires the S^1_η radius to be

$$T = \log\left(k + \sqrt{k^2-4}\right) - \log 2. \tag{9}$$

Moreover, k can be chosen such that the supergravity approximation remains valid throughout the S-fold solution, because the dilaton and derivatives of the axio-dilaton remain small throughout [22]. We shall refrain from further discussing these aspects of the solution, since they do not affect our present analysis, which focuses on the 2-parameter deformation of the background and is independent of the duality twist.

The $\chi_{1,2}$ deformations only appear in the background via the combination (2) and thus can locally be absorbed by the coordinate redefinition

$$\varphi_i' = \varphi_i + \chi_i \eta. \tag{10}$$

However, due to the periodicity of $\eta \to \eta + T$, this is only a well-defined coordinate transformation when $\chi_i = \frac{2\pi k_i}{T}$ for $k_i \in \mathbb{Z}$. This suggests that the deformation parameters are periodic with period $\frac{2\pi}{T}$. However, there is a subtlety because of how the spinors are defined on the S^1_η. In fact, by looking at the spinors, as we will demonstrate later in (15) through the Kaluza-Klein spectrum, we see that the correct periodicity is in fact $\chi_i \in [0, \frac{4\pi}{T})$. This means that the nonsupersymmetric conformal manifold is compact and has topology $T^2/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ corresponds to the interchange $\chi_1 \leftrightarrow \chi_2$.

An alternative description of the parameters χ_i comes from their oxidation to the five-dimensional supergravity obtained by reducing IIB string theory on S^5. As noted in [17,18] (see also [25,27]) the χ_i define nontrivial one-form deformations (Wilson loops) for the vector fields along S^1_η. For the $\mathcal{N}=4$ S-fold, this corresponds to turning on Wilson loops for the $\mathrm{SU}(2)\times\mathrm{SU}(2)$-valued gauge fields breaking the symmetry down to its Cartan subgroup.

It is instructive to compare the deformation of the $\mathcal{N}=4$ S-fold solution analysed here, with the deformation, discussed in [13], of the maximally supersymmetric $\mathrm{AdS}_5\times\mathrm{S}^5$ type IIB background, which generalizes the Lunin-Maldacena construction [28]. The holographic dual to this solution is conjectured to be a nonsupersymmetric marginal deformation of $\mathcal{N}=4$ four-dimensional SYM theory. However, [29] suggested that conformal symmetry of this dual theory is absent, while [30,31] hint at the existence of a tachyonic instability in the corresponding superstring background. In [13], the deformation parameters γ_I, $I=1,2,3$, were the effect of shift transformations in the $\mathrm{O}(3,3)$ group acting on the three angular directions associated with translational isometries [32] along internal angular coordinates. These shift transformations were, however, preceded and followed by T-dualities ("factorized dualities") of the kind $R_I \to 1/R_I$ along all the three directions. Just as S^5 in the $\mathrm{AdS}_5\times\mathrm{S}^5$ background, the internal manifold $\mathcal{I}\times\mathrm{S}^2_1\times\mathrm{S}^2_2\times\mathrm{S}^1_\eta$ of the $\mathcal{N}=4$ S-fold solution features three angular coordinates $\xi^I = \varphi_1, \varphi_2, \eta$ each associated with a translational symmetry of the internal metric, although, in the latter case, a constant translation along η is not a symmetry of the whole solution due to the $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$-twist. As opposed to the construction of [13], the χ_i-deformation discussed here only results from a shift transformation in $\mathrm{GL}(3,\mathbb{R}) \subset \mathrm{O}(3,3)$, with no T-dualities. This is effected by the $\mathrm{GL}(3,\mathbb{R})$ matrix

$$A = \begin{pmatrix} 1 & 0 & \chi_1 \\ 0 & 1 & \chi_2 \\ 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

which acts linearly on the I-component of all the fields. The components $g = (g_{IJ})$ of the internal metric along the angular directions ξ^I, for instance, transforms as follows:

$$g \to A^t g A. \tag{12}$$

Our χ_i deformations thus change the metric on the $\mathrm{S}^5\times\mathrm{S}^1_\eta$ compactification, while leaving the fibration structure unchanged. This is analogous to complex structure deformations of $T^2 \sim \mathrm{S}^1\times\mathrm{S}^1$, which can also locally be absorbed by diffeomorphisms which are, however, not globally well defined. Indeed, our χ_i appear like the real part of complex structure deformations of the $\varphi_i \times \mathrm{S}^1_\eta$ tori. A more precise definition is in terms of the mapping torus of S^5 [18]: the χ_i deformations imply that as we move around S^1_η, we end up in a different point on S^5. If $\chi_i \to \chi_i + 2\pi k_i/T$, $k_i \in \mathbb{Z}$, the deformation is in $\mathrm{GL}(3,\mathbb{Z})$ and the internal geometry is not affected. Invariance of the full spectrum, however, due to the presence of states with half-integer j_1, j_2, extends the periodicity of χ_i to $4\pi/T$, as will be discussed below.

Via the AdS/CFT correspondence, our family of nonsupersymmetric AdS_4 vacua of IIB string theory suggests that the dual "J-fold" CFT_3 should belong to a nonsupersymmetric conformal manifold. However, this is not the case if the nonsupersymmetric AdS_4 vacua are

unstable, as conjectured in [14]. These instabilities could arise due to some scalar fluctuation in the Kaluza-Klein spectrum violating the Breitenlohner-Freedman bound, or via a nonperturbative phenomenon. Let us now address these concerns.

First, we will prove that the Kaluza-Klein spectrum has no tachyons, i.e., the AdS_4 vacua are perturbatively stable. To do this, we use the technology developed in [33,34] to compute the full Kaluza-Klein spectrum around the family of nonsupersymmetric AdS_4 vacua we consider here.

It is easiest to express the Kaluza-Klein spectrum as a deformation of the spectrum of the $\mathcal{N}=4$ vacuum. The full $\mathcal{N}=4$ spectrum was computed in [16,35]. Note that our S^1 radius differs from the convention of [16] such that $T_{\text{there}}=\frac{T_{\text{here}}}{2}$. The conformal dimension of the highest weight state of each supermultiplet is given by

$$\Delta=\frac{3}{2}+\frac{1}{2}\sqrt{9+2\ell(\ell+4)+4\sum_{i=1,2}\ell_i(\ell_i+1)+2\left(\frac{2n\pi}{T}\right)^2}, \tag{13}$$

where ℓ denotes the S^5 Kaluza-Klein level, n the S^1 level and ℓ_1, ℓ_2 the $SO(4)$ spin of the highest weight state (in this case, the graviton). These $\mathcal{N}=4$ supermultiplets are counted by the generating function for their highest weight states:

$$\nu=\frac{1}{(1-q^2)(1-qu)(1-qv)}\frac{1+s}{1-s}, \tag{14}$$

where the exponent of q and s determine the Kaluza-Klein levels on the S^5, ℓ, and S^1, n, while the exponents of u and v count the $SU(2)\times SU(2)$ spins, ℓ_1 and ℓ_2. The effect of the $\chi_{1,2}$ deformations is to shift the conformal dimension of each field by replacing

$$\frac{2n\pi}{T}\to\frac{2n\pi}{T}+(j_1+j_2)\chi_+ +(j_1-j_2)\chi_-, \tag{15}$$

in (13), where j_1, j_2 are the charges of the field under the $U(1)\times U(1)$ Cartan of $SO(4)$ and we defined $\chi_\pm=\frac{1}{2}(\chi_1\pm\chi_2)$. Note from (14) that, while j_1, j_2 are half-integers, $j_1\pm j_2$ are always integers. Thus, we manifestly see that the full background has periodicity $\chi_\pm\to\chi_\pm+\frac{2\pi}{T}$, upon which the Kaluza-Klein spectrum gets mapped back to itself with an appropriate reshuffling of the fields among the S^1 level with $n\to n-(j_1\pm j_2)$, just like in [16]. Notice that χ_1,χ_2 separately have period $4\pi/T$, which can only be seen from the spinors on the AdS_4 background which have half-integers charges under the $U(1)\times U(1)$ Cartan.

Even more importantly, we can see that the masses for all the fields are bounded from below by the masses of the fields of the four-dimensional supergravity at the $\mathcal{N}=4$ vacuum, i.e., where $\ell=\ell_1=\ell_2=n=\chi_i=0$. This in particular implies that all scalars have masses above the Breitenlohner-Freedman bound for any value of χ_i. Thus, the nonsupersymmetric vacua are perturbatively stable.

One may also wonder whether the AdS_4 vacua are secretly supersymmetric in 10 dimensions, with some gravitinos among the higher Kaluza-Klein modes becoming light, akin to the "space invaders" scenario [16,36,37]. However, from (13), (15), we can easily see that such gravitinos can only restore supersymmetry when the combination $\frac{2n\pi}{T}+j_1\chi_1+j_2\chi_2=0$. This can only occur when either $n=0$ and $\chi_1=\pm\chi_2$, corresponding to supersymmetry enhancement that already occurs in the four-dimensional supergravity [18], or $\chi_\pm=\frac{2\pi k_\pm}{T}$, for $k_\pm\in\mathbb{Z}$ when some gravitinos at S^1 level $n=-(j_1+j_2)k_+-(j_1-j_2)k_-$ become massless. This latter condition is precisely when the background is mapped back to itself, so that for $0<\chi_\pm<\frac{2\pi}{T}$, $\chi_1\neq\pm\chi_2$, the AdS_4 vacua are not supersymmetric in the full type IIB string theory.

Next we investigate the nonperturbative stability of the nonsupersymmetric AdS_4 vacua. Since the AdS_4 vacua arise as near-horizon limits of certain brane configurations, one may worry that for the nonsupersymmetric vacua the corresponding brane configurations become unstable [38]. We search for signs of such instabilities by considering single probe $Dp-$branes (and single probe NS5-branes) with rigid embeddings in our AdS_4 vacua. In particular, we check whether the branes are unstable due to a greater repulsive force of the fluxes coming from the WZ term than the attractive (i.e., toward the interior of the AdS spacetime) gravitational force due to the DBI term. Indeed, [14] conjectures that there should always be some branes that are unstable in this way, see also [39]. However, we find that single probe $Dp-$branes and NS5-branes without world volume flux remain stable when placed in the nonsupersymmetric backgrounds (1)–(7).

The stability of these probe branes can be understood in the following way. First, note that we can perform the diffeomorphism (10) to remove the χ_i deformation from the metric. However, now the coordinates respect the deformed periodicities

$$\begin{aligned}&\varphi_i'\to\varphi_i'+2\pi,\\ &\eta\to\eta+T,\qquad \varphi_i'\to\varphi_i{}'+\chi_i T.\end{aligned} \tag{16}$$

As a result, the only well-defined embeddings of branes wrapping η must also wrap φ_i'. In particular, let us denote by $\xi\sim\xi+T$ the relevant wrapped world volume coordinate on the brane. Then, the only well-defined embeddings are given by

$$\eta(\xi)=q\xi,\qquad \varphi_i'(\xi)=\left(p_i\frac{2\pi}{qT}+\chi_i\right)\xi, \tag{17}$$

with $p_i \in \mathbb{Z}$. We see that as χ_i is turned on, a brane wrapping S^1_η must also wrap increasing amounts of φ_i', so that the DBI part of the action increases. At the same time, for p-branes, with $p \neq 5$, the WZ coupling is insensitive to wrapping along φ_i', unless the brane is completely internal. Therefore, these branes either become more stable as χ_i are turned on or they are completely internal branes, which cannot trigger nonperturbative instabilities in the usual way. Finally, an explicit computation for NS5- and D5-branes shows that they also remain stable as χ_i are turned on in the backgrounds (1)–(7).

Finally, nonsupersymmetric vacua may also decay due to bubbles of nothing [40], which requires a circle or sphere [41] to collapse. However, our internal space $S^5 \times S^1_\eta$ is topologically protected from such a collapse: the S^5 cannot collapse as it is supported by flux, whereas the S^1_η cannot collapse since the spinors do not have antiperiodic boundary conditions on it [40], but instead general periodicities along S^1_η, provided $(\chi_1, \chi_2) \neq (\frac{2\pi}{T}, 0), (0, \frac{2\pi}{T})$. This means that a straightforward bubble of nothing cannot occur. Still, our vacua could decay semi-classically via more complicated bubbles of nothing containing defects, e.g., a D3-brane in S^5 similar to [42,43] or an O7-plane in S^1 [44]. However, because the volume form of the compactification is independent of the χ_i deformations, our nonsupersymmetric AdS_4 vacua are likely to be stable against the instanton decay constructed in [43], which is *delocalized* on the compactification space. On the other hand, constructing the *localized* instanton solutions is extremely technically challenging. Moreover, the mechanism of [43] treats a shrinking dilaton as equivalent to a shrinking S^1. Aside from the validity of this equivalence, a similar shrinking dilaton would be problematic for our S-fold vacua, where the dilaton is not well defined due to the $SL(2,\mathbb{Z})$ monodromy along S^1_η.

So far, we have proven that our AdS_4 vacua are perturbatively stable and have provided evidence that they may also be stable against semiclassical decay. However, one may worry that while our AdS_4 geometries are solutions of IIB supergravity, the higher-derivative corrections of IIB string theory will spoil our solutions. In the dual CFT, this would imply that some $\frac{1}{N}$ corrections lift the conformal manifold. However, the deformations χ_i can always be locally absorbed by the coordinate redefinition (10), which however is not globally well defined. Therefore, all local diffeomorphism-invariant quantities are independent of the χ_i. In particular, this means that each term of the higher-derivative corrections of string theory, involving powers of the curvature tensor or the fluxes, are also independent of $\chi_{1,2}$. Thus, our nonsupersymmetric AdS_4 vacua are equally valid solutions of IIB string theory as the $\mathcal{N} = 4$ vacuum. Moreover, the χ_i deformations actually correspond to parity-odd (pseudo) scalars in the maximal supergravity [18], so the potential $1/N$ tadpole destabilization of [45] cannot take place for our backgrounds.

There could still be some string corrections, e.g., from branes wrapping the compactification, which are sensitive to χ_i and which could thus spoil our solutions. For example, $Dp-$ instantons could wrap some $(p+1)$-cycle of the compactification, and depend on χ_i. However, our solutions are also protected against such instanton corrections, since the compactification $S^5 \times S^1_\eta$ only has nontrivial 1-, 5- and 6-cycles. Therefore, we can only have D5-instantons wrapped on the full $S^5 \times S^1_\eta$. But since the volume form is independent of χ_i, these instantons give no corrections to our solutions. Nonetheless, one could expect some other extended state to do so, corresponding to some $\frac{1}{N}$ correction in the dual CFT.

According to the proposal put forward in [22], the SCFT dual to the $\mathcal{N} = 4$ background emerges as the effective IR description of a $3d$ $T[U(N)]$ theory [46] in which the diagonal subgroup of the $U(N) \times U(N)$ flavor group has been gauged using an $\mathcal{N} = 4$ vector multiplet. In addition, a Chern-Simons term at level k must be introduced which is defined by the $J_k = -\mathcal{S}\mathcal{T}^k \in SL(2,\mathbb{Z})_{\text{IIB}}$ monodromy along the S^1_η. The effective $\mathcal{N} = 4$ superpotential [47] $W_{\text{eff}} = (2\pi/k)\text{Tr}(\mu_H \mu_C)$ is marginal in the IR and, in [21], a shift $W_{\text{eff}} \to W_{\text{eff}} + \lambda \text{Tr}(\mu_H \mu_C)$ with $\lambda \in \mathbb{C}$ was proposed as an exactly marginal deformation preserving $\mathcal{N} = 2$. The scalar superconformal primary operators μ_H and μ_C are respectively described by the moment maps of the Higgs and Coulomb branch of $T[U(N)]$. Each of the μ_H and μ_C fields is a component of a vector in the adjoint representation of the corresponding $SU(2)$ subgroup of the $SO(4)$ R-symmetry group (to be denoted by $SU(2)_H$ and $SU(2)_C$, respectively). We can therefore associate with μ_H the quantum numbers $j_1 = 1$, $j_2 = 0$ and with μ_C the values $j_1 = 0$, $j_2 = 1$, having identified j_1, j_2 with the eigenvalues of the Cartan generators of $SU(2)_H$ and $SU(2)_C$, respectively. Note that χ_1 (χ_2) only breaks $SU(2)_H$ ($SU(2)_C$) to its $U(1)_H$ ($U(1)_C$) subgroup. The combination $(\chi_1 - \chi_2)/2$ of these two parameters, for $\chi_1 = -\chi_2$, should already be encoded in the λ parameter of the $\mathcal{N} = 2$ exactly marginal deformation proposed in [21]. We suggest that the orthogonal combination $(\chi_1 + \chi_2)/2$, be encoded in the conjectured exactly marginal deformation of the $3d$ Lagrangian:

$$\partial_\alpha \mathcal{O} \partial^\alpha \bar{\mathcal{O}}, \tag{18}$$

where $\mathcal{O} \equiv \text{Tr}(\mu_H \bar{\mu}_C)$ is an operator with $j_1 = 1, j_2 = -1$ and ∂_α denote the partial derivatives with respect to the (real) scalar fields. As opposed to $\text{Tr}(\mu_H \mu_C)$, the above term does not originate from a holomorphic deformation of the superpotential and thus would break all supersymmetries. The exact marginality of the operator (18) is here conjectured in light of the holographic evidence we put forward. Note that the resulting $\mathcal{N} = 0$ theory would be parity symmetric in both the Higgs and the Coulomb sector:

By performing, for instance, a parity transformation in the Coulomb sector which changes sign to the complex structure of the hyper-Kähler manifold (described as a complex Kähler space), $\mu_C \to \bar{\mu}_C$, and $\mathcal{O}$ would be exchanged with the exactly marginal operator $\mathrm{Tr}(\mu_H \mu_C)$ in the superpotential proposed in [21]. The same transformation would correspond in the bulk to a parity $\varphi_2 \to -\varphi_2$ in S_2^2 and, correspondingly, to $\chi_2 \to -\chi_2$. It is therefore the simultaneous presence of the deformations $\mathcal{O}$, $\bar{\mathcal{O}}$ and $\mathrm{Tr}(\mu_H \mu_C)$ in the Lagrangian which breaks supersymmetry. Also, the $\chi_1 \leftrightarrow \chi_2$ symmetry of the supergravity backgrounds amounts to an exchange symmetry between the Higgs and Coulomb branches in the dual nonsupersymmetric CFT's.

Finally, our computation of the Kaluza-Klein spectrum (13), (15) reveals not only the $\frac{4\pi}{T}$ periodicity of the exactly marginal deformations parametrized by χ_i. It also gives the anomalous dimensions of all operators of the CFT dual to supergravity modes along the nonsupersymmetric conformal manifold.

In this paper, we provided the first holographic evidence for the existence of a nonsupersymmetric conformal manifold. We did this by constructing a 2-parameter family of nonsupersymmetric S-fold AdS_4 vacua of IIB string theory, and proving that they are perturbatively stable. Moreover, we excluded several potential nonperturbative instability mechanisms, and showed that our solutions are even protected against some higher-derivative corrections.

Our findings here can be generalized and applied to other settings. For example, an analogous nonsupersymmetric 2-parameter family of S-fold AdS_4 vacua can be obtained by performing similar axionic deformations to the $U(1)$ R-symmetry and $SU(2)$ flavor symmetry of the $\mathcal{N}=2$ $SU(2)\times U(1)$ AdS_4 S-fold vacuum of IIB string theory [20]. This moduli space has a one dimensional locus of $\mathcal{N}=0$ deformations of the $\mathcal{N}=2$ $SU(2)\times U(1)$ vacuum, also contains the supersymmetric deformation studied in [16] and should be connected to our conformal manifold since there is an exactly marginal deformation, connecting the $\mathcal{N}=2$ and $\mathcal{N}=4$ vacua [21]. We explicitly verified that this second 2-parameter family is also perturbatively stable and has the same protection against nonperturbative mechanisms as was shown by our brane-jet computation and topological arguments. Moreover, the axionic deformations can again be reabsorbed by local coordinate redefinitions that fail to be globally well-defined [18], yielding the same space-invaders scenario as here which leads to a T^2 moduli space. This also protects this 2-parameter family of AdS_4 vacua against higher-derivative corrections. Moreover, this same argument can be applied to the recently constructed moduli space of $\mathcal{N}=1$ CFT_3's [25], which would suggest that also this $\mathcal{N}=1$ moduli space is protected against some higher-derivative corrections of string theory. The methods laid out here should also apply to a related class of S-folds where S^5 is replaced by a Sasaki-Einstein manifold.

The fate of our family of nonsupersymmetric AdS_4 vacua deserves further investigation. The brane-web whose near-horizon limit corresponds to the AdS_4 vacua could still suffer from some other instability mechanism. For example, it could feature some tachyon in its fluctuation spectrum, see, e.g., [48,49] for recent discussions. However, because we do not know the brane-web that would give rise to the AdS_4 vacua, it is currently unclear which probe branes to use for this computation. Still, the existence of a continuous limit to the $\chi_i = 0$ supersymmetric case could help in taming such potential instabilities. Also, some nonperturbative string corrections could lift the moduli space. Finally, the CFT_3 interpretation of the χ_i deformations deserves further exploration. We leave these exciting questions for future work.

We are grateful to V. Bashmakov, M. Bertolini, A. Faedo, J. Gauntlett, H. Raj and D. Waldram for helpful discussions. A. G. is supported by the Spanish government Grant No. PGC2018-096894-B-100 and by the Principado de Asturias through the Grant No. FC-GRUPIN-IDI/2018/000174. E. M. is supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) via the Emmy Noether program "Exploring the landscape of string theory flux vacua using exceptional field theory" (Project No. 426510644). C. S. is supported by Interuniversity Institute for Nuclear Sciences (IISN)-Belgium (convention 4.4503.15) and is a Research Fellow of the F. R. S.-Fonds de la Recherche Scientifique (FNRS) (Belgium).

[1] R. G. Leigh and M. J. Strassler, Exactly marginal operators and duality in four-dimensional $N=1$ supersymmetric gauge theory, Nucl. Phys. **B447**, 95 (1995).
[2] B. Kol, On conformal deformations, J. High Energy Phys. 09 (2002) 046.
[3] D. Green, Z. Komargodski, N. Seiberg, Y. Tachikawa, and B. Wecht, Exactly marginal deformations and global symmetries, J. High Energy Phys. 06 (2010) 106.
[4] B. Kol, On conformal deformations II, arXiv:1005.4408.

[5] M. Baggio, N. Bobev, S. M. Chester, E. Lauria, and S. S. Pufu, Decoding a three-dimensional conformal manifold, J. High Energy Phys. 02 (2018) 062.

[6] V. Bashmakov, M. Bertolini, and H. Raj, On non-supersymmetric conformal manifolds: Field theory and holography, J. High Energy Phys. 11 (2017) 167.

[7] C. Behan, Conformal manifolds: ODEs from OPEs, J. High Energy Phys. 03 (2018) 127.

[8] S. Hollands, Action principle for OPE, Nucl. Phys. **B926**, 614 (2018).

[9] K. Sen and Y. Tachikawa, First-order conformal perturbation theory by marginal operators, arXiv:1711.05947.

[10] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999); Adv. Theor. Math. Phys. **2**, 231 (1998).

[11] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).

[12] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).

[13] S. Frolov, Lax pair for strings in Lunin-Maldacena background, J. High Energy Phys. 05 (2005) 069.

[14] H. Ooguri and C. Vafa, Non-supersymmetric AdS and the swampland, Adv. Theor. Math. Phys. **21**, 1787 (2017).

[15] A. Guarino, E. Malek, and H. Samtleben, Stable Non-supersymmetric Anti-de Sitter Vacua of Massive IIA Supergravity, Phys. Rev. Lett. **126**, 061601 (2021).

[16] A. Giambrone, E. Malek, H. Samtleben, and M. Trigiante, Global properties of the conformal manifold for S-fold backgrounds, J. High Energy Phys. 06 (2021) 111.

[17] A. Guarino and C. Sterckx, S-folds and holographic RG flows on the D3-brane, J. High Energy Phys. 06 (2021) 051.

[18] A. Guarino and C. Sterckx, Flat deformations of type IIB S-folds, J. High Energy Phys. 11 (2021) 171.

[19] G. Inverso, H. Samtleben, and M. Trigiante, Type II supergravity origin of dyonic gaugings, Phys. Rev. D **95**, 066020 (2017).

[20] A. Guarino, C. Sterckx, and M. Trigiante, $\mathcal{N}=2$ supersymmetric S-folds, J. High Energy Phys. 04 (2020) 050.

[21] N. Bobev, F. F. Gautason, and J. van Muiden, The holographic conformal manifold of 3d $\mathcal{N}=2$ S-fold SCFTs, J. High Energy Phys. 07 (2021) 221.

[22] B. Assel and A. Tomasiello, Holographic duals of 3d S-fold CFTs, J. High Energy Phys. 06 (2018) 019.

[23] I. Arav, K. C. M. Cheung, J. P. Gauntlett, M. M. Roberts, and C. Rosen, A new family of AdS_4 S-folds in type IIB string theory, J. High Energy Phys. 05 (2021) 222.

[24] I. Arav, J. P. Gauntlett, M. M. Roberts, and C. Rosen, Marginal deformations and RG flows for type IIB S-folds, J. High Energy Phys. 07 (2021) 151.

[25] N. Bobev, F. F. Gautason, and J. van Muiden, Holographic 3d $\mathcal{N}=1$ conformal manifolds, arXiv:2111.11461.

[26] M. Cesàro, G. Larios, and O. Varela, The spectrum of marginally-deformed $\mathcal{N}=2$ CFTs with AdS_4 S-fold duals of type IIB, J. High Energy Phys. 12 (2021) 214.

[27] D. Berman, T. Fischbacher, and G. Inverso, New $N=1$ AdS_4 solutions of type IIB supergravity, arXiv:2111.03002.

[28] O. Lunin and J. M. Maldacena, Deforming field theories with $U(1) \times U(1)$ global symmetry and their gravity duals, J. High Energy Phys. 05 (2005) 033.

[29] J. Fokken, C. Sieg, and M. Wilhelm, Non-conformality of γ_i-deformed $N=4$ SYM theory, J. Phys. A **47**, 455401 (2014).

[30] J. G. Russo, String spectrum of curved string backgrounds obtained by T-duality and shifts of polar angles, J. High Energy Phys. 09 (2005) 031.

[31] M. Spradlin, T. Takayanagi, and A. Volovich, String theory in beta deformed spacetimes, J. High Energy Phys. 11 (2005) 039.

[32] A. Giveon and M. Rocek, Generalized duality in curved string backgrounds, Nucl. Phys. **B380**, 128 (1992).

[33] E. Malek and H. Samtleben, Kaluza-Klein Spectrometry for Supergravity, Phys. Rev. Lett. **124**, 101601 (2020).

[34] E. Malek and H. Samtleben, Kaluza-Klein spectrometry from exceptional field theory, Phys. Rev. D **102**, 106016 (2020).

[35] K. Dimmitt, G. Larios, P. Ntokos, and O. Varela, Universal properties of Kaluza-Klein gravitons, J. High Energy Phys. 03 (2020) 039.

[36] M. Duff, B. Nilsson, and C. Pope, Kaluza-Klein supergravity, Phys. Rep. **130**, 1 (1986).

[37] M. Cesaro, G. Larios, and O. Varela, A cubic deformation of ABJM: The squashed, stretched, warped, and perturbed gets invaded, J. High Energy Phys. 10 (2020) 041.

[38] J. M. Maldacena, J. Michelson, and A. Strominger, Anti-de Sitter fragmentation, J. High Energy Phys. 02 (1999) 011.

[39] I. Bena, K. Pilch, and N. P. Warner, Brane-jet instabilities, J. High Energy Phys. 10 (2020) 091.

[40] E. Witten, Instability of the Kaluza-Klein Vacuum, Nucl. Phys. **B195**, 481 (1982).

[41] H. Ooguri and L. Spodyneiko, New Kaluza-Klein instantons and the decay of AdS vacua, Phys. Rev. D **96**, 026016 (2017).

[42] G. T. Horowitz, J. Orgera, and J. Polchinski, Nonperturbative instability of $AdS_5 \times S^5/Z(k)$, Phys. Rev. D **77**, 024004 (2008).

[43] P. Bomans, D. Cassani, G. Dibitetto, and N. Petri, Bubble instability of mIIA on $AdS_4 \times S^6$, arXiv:2110.08276.

[44] J. McNamara and C. Vafa, Cobordism classes and the swampland, arXiv:1909.10355.

[45] M. Berkooz and S.-J. Rey, Nonsupersymmetric stable vacua of M theory, J. High Energy Phys. 01 (1999) 014.

[46] D. Gaiotto and E. Witten, S-duality of boundary conditions in $\mathcal{N}=4$ super Yang-Mills theory, Adv. Theor. Math. Phys. **13**, 721 (2009).

[47] E. Beratto, N. Mekareeya, and M. Sacchi, Marginal operators and supersymmetry enhancement in 3d S-fold SCFTs, J. High Energy Phys. 12 (2020) 017.

[48] F. Apruzzi, G. Bruno De Luca, A. Gnecchi, G. Lo Monaco, and A. Tomasiello, On AdS_7 stability, J. High Energy Phys. 07 (2020) 033.

[49] F. Apruzzi, G. Bruno De Luca, G. Lo Monaco, and C. F. Uhlemann, Non-supersymmetric AdS_6 and the swampland, J. High Energy Phys. 12 (2021) 187.