

Shape deformations and instabilities of single bubble rising in liquid metals

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(Article begins on next page)

1 Title: “Shape deformations and instabilities of single bubble rising in liquid metals”

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5 **Abstract**

6 In this study, a Computational Fluid Dynamics (CFD) simulation was used to study single bubble flow in
7 liquid metal. Till now, bubble trajectory and shape (Mougin & Magnaudet, 2002) stability problems in liquid
8 metal have only been insufficiently analyzed in the literature. Because of the difficulty of such an experimental
9 validation, no universal correlations on terminal velocity, shape aspect ratio, and drag force coefficient have
10 been produced to date. The existing bubble shape parameter and terminal velocity correlations with
11 dimensionless numbers are still debatable, mostly because experimental validation is very challenging.

12 The objective of this study was to develop new correlations for bubble stability and bubble deformation in
13 liquid metals. An in-house code, PSI-BOIL, has been used for the simulations. A single bubble rising in a
14 quiescent liquid has been simulated for three different sets of materials (Nitrogen+Mercury, Argon+GalSn,
15 Argon+Steel).

16 The obtained results suggest shape instability phenomena take place in the bubble dynamics in liquid metals
17 for Eötvös numbers > 1.7 . Small bubbles ($Eo < 1.7$) maintain a stable ellipsoidal shape, while the shape and
18 velocity of larger bubbles ($Eo > 1.7$) tend to oscillate with bubbles rising via non-rectilinear trajectories.

19 The inviscid approximation works well for bubbles in liquid metals. It has been confirmed that the dynamics
20 and the shape of small bubbles ($Eo < 1.7$) in liquid metals are only controlled by the Weber number.

21

22 Nomenclature

23 Physical quantities

24	D	bubble diameter (m)
25	g	gravitational acceleration (m/s^2)
26	p	pressure (Pa)
27	F	force (N)
28	t	time (s)
29	u	velocity (m/s)
30	u_T	terminal rising velocity (m/s)
31	X	bubble aspect ratio: bubble height divided by bubble width (-)
32	γ	surface tension coefficient (N/m)
33	μ	dynamic viscosity (Pa·s)
34	ρ	density (kg/m^3)
35	χ	curvature (m^{-1})

36 Dimensionless numbers

37	EO	Eötvös number, $EO = \frac{(\rho_l - \rho_g)gd^2}{\gamma}$
38	MO	Morton number, $MO = \frac{g\mu_l^4(\rho_l - \rho_g)}{\rho_l\gamma^3}$
39	Re	Reynolds number, $Re = \frac{\rho_l u_T D}{\mu_l}$
40	We	Weber number, $We = \frac{\rho_l u_T D}{\gamma}$

41 Subscriptions

42	a	Archimede
43	g	gas
44	l	liquid
45	i	inertia
46	st	surface tension
47	v	viscous

48

49 Introduction

50 Bubble motion in liquid metals plays a determining role in many engineering processes. Gas bubbles are
51 commonly inserted into a liquid metal to enhance the mixing process that takes place in the liquid phase (Liu
52 & Li, 2017) and to improve the heat transfer of the liquid (Lorenzin & Abánades, 2016).. In fact, liquid metal
53 is an excellent type of energy carrier, and it is largely used in many types of nuclear reactors, such as fast
54 reactors, spallation source devices and fusion reactors.

55 Moreover, a bubbly flow in liquid metal is also used in industrial processes, such as in metal stirring and
56 purification, continuous casting and liquid metal chemical reactions, with the purpose of increasing efficiency.
57 A great deal of scientific research has been carried out in this field. In continuum casting (Liu & Li, 2017) ,
58 for example, bubbles are injected into a liquid metal to avoid clogging of the flow and to make the process as
59 continuous as possible. Timmel et al. (2010) presented a new experimental facility (LIMMCAST) focused on
60 the study of gas bubble rising in metals. Yang et al. (2020) produced an advanced break-up/coalescence model
61 to precisely predict bubbles flow in liquid metals. Baake et al. (2017) demonstrates the effectiveness of neutron
62 radiography for the experimental investigation of these phenomena.

63 Because of its innumerable industrial applications, bubble motion in liquid metal has been a central topic for
64 the scientific community over the past 70 years (Haas et al. 2021). Nevertheless, important questions,
65 concerning the stability and velocity of bubbles in this range, still remain unclear and the bubble rising problem
66 remains an open issue in multi-phase fluid mechanics. In particular, no correlations between bubble shape and
67 velocity have been revealed to be accurate in a liquid metal.

68

69 Bubble rising in a quiescent liquid is a well-known problem in multiphase-fluid mechanics.

70 Moreover, correlations between fluid-dynamics dimensionless numbers have been produced in literature for a
71 wide range of Eötvös and Reynolds numbers, but a universal correlation has not yet been found. Nevertheless,
72 many correlations, which are valid over their own ranges, were derived or produced in the past.

73 Levich (Kang & Leal, 1988) analytically derived a drag coefficient that is valid for a inviscid flow (viscosity
74 tends to zero ($\mu \rightarrow 0$)) for spherical bubbles (aspect ratio tends to unity ($X \rightarrow 1$)) in which the viscous
75 component of the drag force is negligible (high Re).

76 Hadamard (J.S., 1911) derived a drag coefficient for a creeping flow ($Re < 1, X \rightarrow 1$), assuming that the
77 pressure component by the wake is negligible. Mei and Klausner (TOMIYAMA et al., 1998) extended
78 Hadamard's correlation for spherical bubbles to an arbitrary Reynolds number.

79 Mendelson (1967) studied the bubble rising problem in inviscid liquid and approximated the bubble velocity
80 to that of the propagation wave velocity of the gas-liquid interface. The phase velocity, u_{phase} , is assumed to be
81 the sum of the velocity resulting from the surface tension and the gravitational term.

82 The aforementioned author computed the bubble rising velocity by determining the principal wavelength, $\lambda =$
83 πd .

84 TOMIYAMA et al. (1998) generalized the drag correlation for a wider range ($10^{-2} < Eo < 10^3, 10^{-14} <$
85 $Mo < 10^7, 10^{-3} < Re < 10^5$) and provided the most general correlation possible. First, he unified Levich

86 and Hadamard's correlation for a wide range of Reynolds numbers. He then calculated the drag coefficient,
87 using Mendelson's theory, to provide a drag coefficient that is only dependent on the initial conditions. Since
88 a bubble no longer tends to be spherical for higher Eo , but is instead ellipsoidal, the terminal velocity is lower
89 than in a spherical case. The maximum of the two drag coefficients is chosen by considering this effect. Yan
90 et al. (2018) produced a more precise empirical solution for the bubble drag coefficient for the same range of
91 correlations as Tomiyama. Y. Zhou et al. (2020) produced a novel correlation for the drag coefficient of a
92 bubble rising in liquid which predicts 93.5% of the available existing data.

93 The main issue in finding a universal correlation is the large scale of change of the dimensionless numbers
94 (Re , Eo , Mo), that is, of several orders of magnitude. Viscous, inertia and tension forces have different degrees
95 of importance in each region in the Grace diagram (Acrivos, 1979), and the bubble behavior changes to a great
96 extent. Thanks to experiments and CFD simulations, a map has been created which predicts the terminal
97 Reynolds number, once the initial conditions are known. Generally, the bubble terminal Reynolds number
98 increases as the Eötvös number increases and decreases as the Morton number decreases.

99

100 The deformation of the bubble shape has also been a widely studied topic over the last few decades, because
101 of its numerous industrial (Liu & Li, 2017) and research applications. Experiments and CFD simulations have
102 led to the formation of maps that describe different types of bubble shape in different dimensionless number
103 regimes (Krull et al., 2016). The shape of a bubble is controlled by two dimensionless numbers, as is its
104 velocity. The most common numbers used in literature are the Reynolds number and the Eötvös number. A
105 bubble with $Re < 1$ tends to keep a spherical shape for all Eo . Such a bubble gradually changes from spherical
106 to ellipsoidal and finally to a spherical cap shape for higher Reynolds numbers ($Re > 1$), thereby increasing the
107 Eötvös number. Furthermore, the bubble shape is highly sensitive to the initial flow condition (Tomiyama et
108 al., 2002). A non-zero velocity field or a residual initial compression of a bubble can easily lead to a different
109 kind of bubble rising behavior (different trajectory and different shape). Many correlations, based on
110 experimental and CFD data, are present in the literature, each with its own range of validity.

111 Besagni & Deen (2020) compared the most famous shape correlations present in the literature and provided
112 his own new correlation, which is the most accurate generalization to date, based on the many experimental
113 data he had available.

114 (Moore, 1959) analytically predicted the bubble shape in liquid metal through a linear theory which, however,
115 loses precision for large bubbles. Sugihara (Besagni & Deen, 2020) extended Moore's correlation to a wider
116 range of Weber numbers. The effectiveness of the latter two correlations is based on assuming that the flow in
117 liquid metals behaves like an inviscid flow. Therefore, the dynamics is controlled entirely by the Weber
118 number. Legendre et al. (2012) extended Moore's correlation to more viscous fluids ($1 < \log Mo < 11$), focusing
119 the study on the bubble-liquid interfacial area.

120 Aoyama et al. (2016) experimentally studied bubble deformation for specific Morton numbers ($\log Mo = -6.6$,
121 -5.5 , -4.9 , -3.9) and produced a correlation for shape deformation for a wide range of Morton numbers. Y.
122 Zhou et al. (2020) also studied the bubble deformation problem and produced the most general correlation for
123 the bubble aspect ratio, which agrees with 90% of the existing experimental and numerical data.

124 Gaudlitz & Adams (2009) investigated bubble path and find a connection between the trajectory and wake
125 structure.

126 J. Zhang & Ni (2014) discovered vertical uniform magnetic field can straighten the bubble trajectory and found
127 a close relationship between fluctuations in rising velocity and shape variations. He (J. Zhang et al., 2016) also
128 investigated bubble velocity and shape under horizontal magnetic field. In this case magnetic field tends to
129 decompress bubble shape, straighten the trajectory and exponentially reduce the terminal velocity.

130 Both W. Zhou & Dušek (2017) and Cano-Lozano et al. (2013) respectively proposed a marginal stability curve
131 of a deformable bubble ascending freely in a viscous Newtonian liquid.

132 Schwarz & Fröhlich (2014) investigated bubble rising under magnetic field and, in accordance with Zhang
133 studies, produced a correlation between horizontal magnetic field and terminal velocity for a wide bubble
134 range.

135 J. Zhang et al. (2021) recently demonstrates the mechanism to control the trajectory of an air bubble under the
136 application of a magnetic field

137 Will et al. (2021) experimentally investigated the effect of geometrical anisotropy for buoyant spheroidal
138 particles rising in a still fluid and discovered bubble shape and trajectory are highly dependent on initial bubble
139 compression.

140 As the Eötvös number increases, a bubble starts to experience instability phenomena that influence the
141 trajectory, the velocity, and the shape of the bubble itself. The stability of a bubble is influenced by the ratio
142 of the surface tension forces ($\sim D^2$) to the inertia forces ($\sim D^3$). The bubble starts to experience 3D dependent
143 forces and it loses its axial symmetry, and this renders the bubble rise a more complex phenomenon. Certain
144 large bubbles in low Morton number regions ($Mo < 10^{-4}$) experience instabilities and the bubble changes shape
145 from ellipsoidal to a wobbling disk. The reason why these instabilities occur has not yet been ascertained
146 completely. Many authors, such as Mougin & Magnaudet (2002), asserted that the main cause of the onset of
147 these instability phenomena are wake vortexes that appear under the bubble. Those vortexes increase the
148 energy dissipation and the flow field on the bottom of the bubble, which loses its axi-symmetry, and this in
149 turn leads to the onset of instabilities.

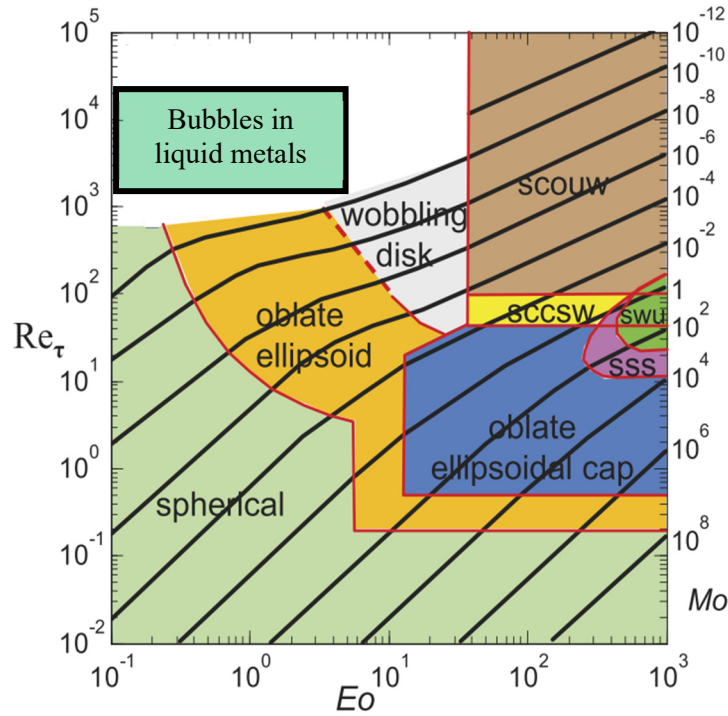
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151 As can be seen in Figure 1, a liquid metal is a region for which no universal correlation has yet been found.
152 The main obstacle is the opacity of the metal that makes an optical inspection of the bubble almost impossible.
153 In the past, such techniques as the electrical triple probe (Mori et al., 1977) were adopted to determine the
154 bubble rising velocity. Nevertheless, due to inaccuracies of the instrumentation, it was not possible to precisely
155 predict the bubble velocity, and only overall estimates could be obtained. Modern techniques, such as UDV
156 (Ultrasound Doppler Velocimetry), Neutron radiography (Wang et al., 2017) and X-ray radiography, seem to
157 be the best tools to experimentally investigate this environment.

158 Keplinger et al. (2017) validated X-ray radiography as an inspection technique by comparing his data with
159 results in water and used it to experimentally study bubble break-up (Keplinger et al., 2019) and coalescence
160 (Keplinger et al., 2018) in liquid metals.

161 Strumpf (2017) & C. Zhang et al. (2005) adopted UDV to investigate bubble velocity and trajectory under
 162 magnetic field, while Richter et al. (2018) experimentally studied path and shape instabilities by mean of X-
 163 ray radiography.

164 Birjukovs et al. (2020) & Birjukovs et al. (2021) demonstrated a new image processing methodology for
 165 resolving gas bubbles travelling through liquid metal from dynamic neutron radiography and adopted it for
 166 studying high Reynold bubble rising. Till now, all the conducted experiments have been insufficient to produce
 167 a robust correlation for the bubble rising velocity, u_{∞} , and aspect ratio, X , in liquid metals. These modern
 168 techniques show a much higher accuracy in detecting the bubble velocity and shape than the older ones, but
 169 they result to be expensive, and their use is somewhat limited.



170
 171 Figure 1. Region of bubbles in typical liquid metals in Grace diagram (Jin et al., 2016). Bubble shape: scouw—
 172 spherical cap with open unsteady wake sccsw—spherical cap with closed steady wake; swu—skirt with wavy
 173 unsteady skirt; sss—skirt with smooth steady skirt.

174
 175 ***Objectives of present study***

176 Bubble stability problems in liquid metal have been insufficiently analyzed in the literature. Because of the
 177 difficulty involved in conducting experimental validations, no universal correlations have been produced as
 178 yet.

179 Similarly, deformations of the bubble shape and their terminal velocities in liquid metal are not fully developed
 180 topics in fluid dynamics, mostly because of the difficulty in their validation.

181 As far as the bubble shape in liquid metals is concerned, Moore (1959) proposed the most famous correlation
 182 but, because of the important assumptions in this correlation, it loses applicability for large bubbles and a more
 183 general theory is therefore required.

184 In this study, a Computational Fluid Dynamics (CFD) simulation was used to study single bubble flow in
 185 liquid metal. More precisely, a new analytic theory on the velocity and deformation of bubble rising has been
 186 produced and compared with the CFD results of the PSI-BOIL code. PSI-BOIL is an in-house code developed
 187 by the Paul Scherrer Institute (PSI). It is a CFD code specialized in high precision simulation of biphasic heat
 188 transfer problems, based on finite discretization technique.

189 The CFD velocities of bubbles in liquid metals are compared with those obtained by means of the existing
 190 correlations in the literatures. A semi-analytical model for bubble shape deformation is also derived for liquid
 191 metal and is compared with the CFD results. Finally, a new stability criterion is proposed for bubbles in liquid
 192 metal.

193 **Governing equations**

194 The studied problem refers to a single bubble rising in a quiescent liquid metal. The governing equations for
 195 the incompressible flow are defined as:

$$196 \quad \nabla \cdot u = 0 \quad (1)$$

$$197 \quad \frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \otimes u) = -\nabla p + \nabla \cdot (\mu(\nabla u + (\nabla u)^T)) + \rho g + \gamma \chi n \quad (2)$$

198 where u [m/s] is the fluid velocity vector, p [Pa] is the pressure, t [s] is the time, ρ [kg/m³] is the density,
 199 μ [Pa·s] is the dynamics viscosity, g [m/s²] is the gravitational acceleration, γ [N/m] is the surface tension
 200 coefficient between the gas and the liquid, χ [N/m] is the local curvature of the interface, and n is the normal
 201 vector of the interface.

202 The energy conservation equation was not solved since the thermal energy exchange was considered
 203 negligible. Thus, the temperature field was assumed to be constant, and the material proprieties of the gas and
 204 the liquid were kept constant.

205 A linear interpolation between liquid and gas was considered, using the volume fraction H , to compute the
 206 material proprieties.

$$207 \quad \mu = H\mu_1 + (1 - H)\mu_2 \quad (3)$$

$$208 \quad \rho = H\rho_1 + (1 - H)\rho_2 \quad (4)$$

$$209 \quad \sigma = H\sigma_1 + (1 - H)\sigma_2 \quad (5)$$

210 The governing equation for the transport of the volume fraction was written as in Eq.(6), and it was solved
 211 with the Volume Of Fluid (VOF) method:

$$212 \quad \frac{\partial H}{\partial t} + \nabla \cdot (uH) = 0 \quad (6)$$

213 The continuity and momentum conservation equations were solved using the fractional step method. An
 214 adaptive time-step was used, where the time step was defined as: $\Delta t = \min\{\Delta t_{CFL}, \Delta t_\gamma\}$, where

$$215 \quad \Delta t_{CFL} = C_{CFL} \frac{\Delta}{|u_{max}|} \quad (7)$$

$$216 \quad \Delta t_\gamma = 5 \cdot \sqrt{\frac{0.5 \cdot \rho_{GAS} \cdot \Delta^3}{2\pi\gamma}} \quad (8)$$

217 C_{CFL} is the Courant-Friedrichs-Lewy (CFL) number, and it was set to 0.25 in this study. u , v and w are the
 218 velocity components in the x-, y- and z-directions, respectively, and Δ is the grid spacing.

219

220

221 **Conditions of simulations**

222 A 3D rectangular computational domain, with $6D \times 6D \times 24D$ dimensions (both the lateral directions and the
223 height), was used, and was discretized with a uniform cube mesh ($dx=D/16$). The liquid was quiescent at the
224 beginning. The height of $24D$ was sufficient for all the bubbles to reach terminal velocity. The width of $6D$
225 made the flow independent of the influence of the surrounding walls.

226 Simulations were conducted with different bubble sizes ($0.1 < Eo < 2$). Three cases of systems with different
227 material combinations were considered: Argon-GalInSn, Nitrogen-Mercury, Argon-iron. These three sets of
228 materials have been used because they are common in metallurgical processes and above all because they have
229 been used in previous experiments and simulations (Moore, 1959), so that the results of PSI-Boil can be
230 compared and validated. The material properties are listed in Table I.

231 Table I "Material properties for three systems with different combination of gas and liquid phases."

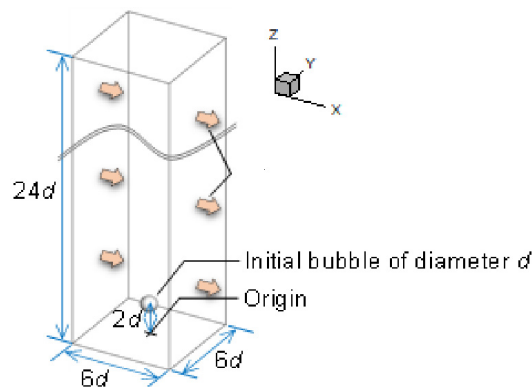
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	μ [Pa·s]	ρ [kg/m ³]	γ [N/m]
Argon	1.176e-5	1.654	0.5330
GalInSn	2.20e-3	6.36e3	
Nitrogen	1.77e-5	1.17	0.4535
Mercury	1.50e-3	1.35e4	
Argon	1.176e-5	1.654	1.200
Iron	6.30e-3	7.00e3	

240

241 At the beginning of the simulation, the velocity field was zero. The pressure field followed Stevino's law and
242 increased linearly with the depth. A spherical bubble was placed in the center in the x- and y-directions and at
243 a $2D$ distance from the bottom, as shown in Figure 2.

244



245

246

Figure 2 "Computational domain and the initial bubble."

247

248 **Analysis of bubble dynamics in liquid metal**

249 **Analytical Description**

250 **Governing parameters**

251 A single bubble rising in liquid metal is characterized by a high Reynolds number, that is, Eq. (9) and
252 considerably low Morton numbers, Eq. (10). The Reynolds number, which was of the order of one thousand,
253 due to the high liquid metal density compared to water, suggests that inertia forces (F_i) were predominant over
254 the viscous forces (F_v) ($Re \gg 1$). The Morton number was particularly low ($\approx 10^{-13}$), which means that the
255 Archimedes (F_a), Eq. (12), and surface tension (F_{st}) forces were dominant over the viscous forces. The two
256 driving factors that characterize bubble rising in liquid metal are the inertia and surface tension forces, while
257 the viscous term, although present, does not play any substantial role.

$$258 \quad Re = \frac{F_i}{F_v} = 10^4 \quad (9)$$

$$259 \quad Mo = \frac{F_v}{F_{st} * F_a} = 10^{-13} \quad (10)$$

260 The system was characterized by only two dimensionless parameters, Eq. (11), in which viscosity was absent,
261 since it no longer had any driving power. The Eötvös number and the Weber number ($We = \frac{\rho \cdot v^2 \cdot D}{\gamma}$) were used
262 in this analysis.

$$263 \quad F(\Pi_1, \Pi_2, \Pi_3) = 0 \rightarrow F(We, Eo) = 0 \rightarrow We = F(Eo) \quad (11)$$

264 Under this approximation, a 2-variable function fully characterized the problem. Furthermore, since Eo was
265 not velocity dependent, the bubble rising velocity could be explicitly expressed as a function of the material
266 and geometry proprieties.

267 **Negligible shear stress field**

268 In a Newtonian fluid, the stress tension is proportional to the velocity gradient $\tau = \mu \cdot \nabla u$. In a solid sphere,
269 since internal flow does not occur, the liquid velocity on the sphere surface is zero and

$$270 \quad \tau \cong \mu \cdot \frac{u_\infty}{L}, \text{ with } L \text{ equal to the characteristic decreasing length.}$$

271 The viscous effect becomes less dominant in bubbles rising in liquid metal. The surface shear stress field on
272 the bubble surface is significantly reduced by two phenomena:

- 273 • First, inner gas recirculation takes place, and the bubble surface velocity is no longer zero. Since there
274 is a huge difference in the gas-liquid density, the outermost layer of the gas is carried by the external
275 liquid, which has higher inertia. Therefore, the velocity gradient and the shear stress are significantly
276 reduced to $\tau \cong \mu \cdot \frac{u_\infty - u_{surf}}{L}$. This effect is relevant for large bubbles for which there is enough space
277 inside to create convective gas movement.
- 278 • Secondly, the purity of the surrounding liquid increases the slipping effect along the surface of the
279 bubble, thereby further decreasing the local velocity gradient. Both contributions significantly reduce
280 the tension stress field across the bubble interface.

281 **Bubble force balance**

282 Under equilibrium, the buoyancy force, which is velocity independent, Eq. (12), is balanced by the drag force,
 283 Eq. (13), which generally increases as the velocity increases. The buoyancy force is the sum of the gravitational
 284 force and the surface integral of the static pressure, h , acting on the bubble. The drag force is computed by
 285 integrating the pressure ($p_{\text{tot}} - p_{\text{static}}$) with the shear stress field.

286 In this analysis, the considered system was the whole bubble, which is the sum of the interface and the inner
 287 gas. Since the surface tension is a mutual force that acts between the surface and the inner gas, it was not
 288 necessary to consider it since it is an internal force of the system.

$$289 \quad F_{\text{buoyancy}} = -mg + \oint p_{\text{static}} \cdot \vec{dA} = -\rho_g Vg + \rho_l Vg = \Delta\rho \cdot V \cdot g = \Delta\rho \cdot \frac{\pi D^3}{6} \cdot g \quad (12)$$

$$290 \quad F_{\text{drag}} = \oint (p_{\text{tot}} - p_{\text{static}}) \cdot \vec{dA} + \oint \tau_{//} \cdot \vec{dA} \quad (13)$$

291 The velocity and pressure distribution over the entire bubble surface needs to be known to compute the drag.
 292 The pressure and the tension terms are generally equally important, and both need to be computed. The pressure
 293 field across the bubble is computed by resolving the Navier-Stokes equations Eq.(14) along a streamline from
 294 a distant point to the surface Eq(15). The pressure on the bubble surface is the sum of a gravitational, a kinetic,
 295 and a viscous term. The gravitation term is not treated since it is already considered in the buoyancy force. The
 296 $p_{\text{tot}} - p_{\text{static}}$ field at the bubble interface is the sum of a kinetic and a viscous pressure, Eq. (15).

$$297 \quad \rho \left[\frac{\partial u}{\partial t} + u \cdot \nabla u \right] = -\nabla p + \mu \nabla^2 u + \rho g \cdot dz \quad (14)$$

$$298 \quad p_{\text{tot}} - p_{\text{static}} = \frac{\rho}{2} \cdot (u^2(s) - u_{\infty}^2) + \mu \int_{\infty}^s \nabla^2 u \cdot dr \quad (15)$$

299 The kinetic pressure in the upper part of the bubble is much larger than the viscous pressure,
 300 $\left(\frac{1}{2} \rho u_{\infty}^2 \gg \mu \int_{\infty}^s \nabla^2 u \cdot dr \right)$, because of the high density of liquid metals and because of the small stress tensor
 301 field, $\tau = \nabla u$.

302 In Figure 3, \vec{r} represents the streamline of the flow and \vec{s} the position on the bubble surface.

$$303 \quad \Delta p_v = \mu \int_{\infty}^s \nabla \cdot \vec{\tau} \cdot dr = \mu \int_{\infty}^s \frac{\partial}{\partial s} \tau_s \cdot dr = \mu \int_{\infty}^s \tau_s(r) \cdot dr \quad (16)$$

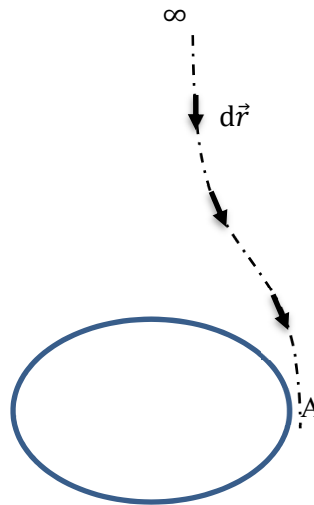


Figure 3 "Flow streamline"

- 304
- $\tau(\infty)$ is zero and increases as it approaches point A, in which τ reaches its maximum value.

- 305 • The tension stress field $\tau(A)$ is negligible, since the velocity changes almost linearly on the side of the
- 306 bubble due to an efficient internal flow [Section 3]
- 307 • Therefore, $\int_s^\infty \tau_s(r) \cdot dr$ is also negligible and the viscous overpressure is zero.
- 308 • Viscous pressure Eq.(16) is negligible for each point in the upper part of the bubble.

309

310 The fluid before the bubble detachment point can be considered inviscid, as there are no energy dissipations,
 311 and all the liquid momentum is transferred into pressure Eq. (17). When the liquid is approaching the bubble
 312 surface, the velocity is decreased, and the kinetic energy is changed into pressure energy. The stagnation point
 313 of the bubble (top point) has the highest pressure, and when approaching the sides of the bubble, the pressure
 314 decreases monotonously. Therefore, the inviscid model was used to model the pressure on the bubble surface.
 315 No boundary layer existed, and only the tangential component of velocity $u_{//}$ was present on the bubble
 316 surface.

$$317 \quad \Delta(p_{\text{tot}} - p_{\text{static}}) + \frac{1}{2}\rho\Delta u^2 = 0 \quad (17)$$

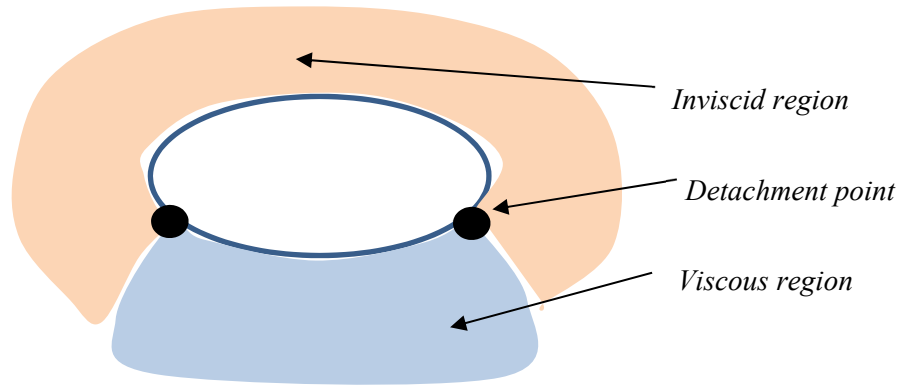
318 How the pressure changes from the stagnation point to the bubble sides depends on the bubble aspect ratio (X),
 319 which in turn depends on the pressure acting on the surface. Nevertheless, it can be established that the surface
 320 pressure field is proportional to the pressure of the stagnation point, which is, according to Bernoulli, equal to
 321 $\frac{1}{2}\rho_l u_\infty^2$. Therefore, kinetic overpressure Eq. (18) depends on bubble velocity u_∞ and on the local velocity
 322 tangential to the surface $u(s)$.

$$323 \quad p_{\text{kinetic}}(s) = \frac{1}{2}\rho(u_\infty^2 - u^2(s)) \quad (18)$$

324 The inviscid approximation is no longer feasible after the detachment point (Figure 4). The fluid on the bottom
 325 of the bubble is no longer irrotational and dissipation phenomena take place. The pressure on the bottom of
 326 the bubble is significantly reduced, because all the liquid momentum is dissipated by the viscosity friction and
 327 does not contribute to hydrodynamic pressure. From the Lagrangian point of view, the fluid particles that are
 328 approaching the bubble surface decelerate and their kinetic energy decreases. In an ideal case, the pressure
 329 increases to conserve kinetic energy but, due to viscous forces, most of the energy is dissipated and the liquid
 330 pressure is drastically reduced, compared the non-viscous case. How much energy is transferred into pressure
 331 and how much is lost depends on the velocity field. In this analysis, a total dissipation model is used for
 332 simplicity. Therefore, all the kinetic energy of the liquid is dissipated, while the pressure field after the
 333 detachment point can be approximated to zero Eq. (19) and has no impact on the drag force (Figure 5).

$$334 \quad p_{\text{tot}}(s) - p_{\text{static}}(s) = \frac{1}{2}\rho(u_\infty^2 - u^2(s)) + \mu \int_\infty^s \nabla^2 u \cdot ds \simeq 0 \quad (19)$$

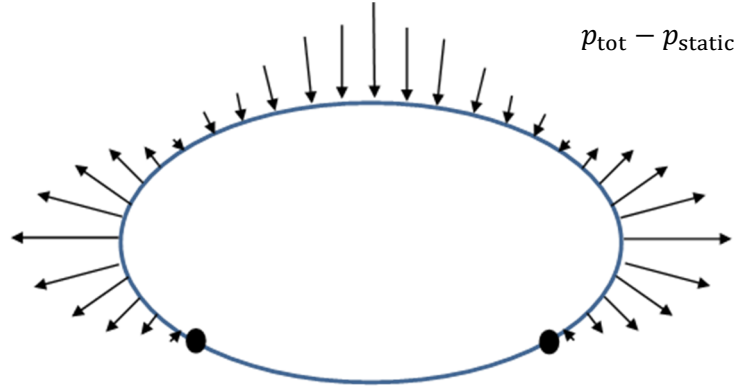
335



336

337

Figure 4 "Top and Bottom of the bubble modeled respectively with inviscid and viscous model."



338

339

Figure 5 Pressure field ($p_{\text{tot}} - p_{\text{static}}$) on bubble surface."

340

The shear stress component was neglected to compute the drag force, since it was much lower than the pressure component. Moreover, the pressure field was only integrated in the "inviscid region", since any pressure on the bottom was assumed to be zero due to dissipation, Eq. (20). Moreover, $p_{\text{tot}} - p_{\text{static}}$ in the inviscid region is the kinetic pressure.

344

Clearly, the viscous pressure was physically the main cause of the drag force but, due to the full dissipation model used at the bottom of the bubble, the liquid viscosity coefficient did not influence the bubble dynamics and the drag force only depended on the kinetic pressure upstream of the detachment point.

347

The point of detachment of the liquid from the liquid surface depends on the bubble shape and on the kinetic energy density of the liquid (ρu^2). Since the bubble was assumed to be ellipsoidal, it can be assumed, without any loss of generality, that the detachment point depended only on the bubble aspect ratio and on the Weber number of the bubble.

351

$$F_{DRAG} = \iint (p_{\text{tot}} - p_{\text{static}}) \cdot \vec{dA} + \iint \tau_{//} \cdot \vec{dA} \approx \iint p_{\text{kinetic}} \cdot \vec{dA} = \iint \frac{1}{2} \rho (u_{\infty}^2 - u^2(s)) \cdot \vec{dA} \quad (20)$$

352

The final balance equation is Eq. (21):

353

$$\Delta \rho \cdot \frac{\pi D^3}{6} \cdot g = \iint \frac{\rho}{2} \cdot (u_{\infty}^2 - u^2(s)) \cdot \vec{dA} \quad (21)$$

354

The liquid velocity on the bubble surface only has the $u_{//}$ component tangential to the surface, while the normal component, u_n , is zero to conserve the flowrate ($\nabla \cdot \vec{u} = 0$). We assume the local velocity on the surface of the bubble is proportional to the rising bubble velocity. $u^*(s)$ is a unknown function that represents the dependency of the velocity on the local position Eq. (22).

358

$$u(s) = u^*(s) \cdot u_{\infty} \quad (22)$$

359
$$\Delta\rho \cdot \frac{\pi D^3}{6} \cdot g = \oint \frac{\rho \cdot u_\infty^2}{2} \cdot (1 - u^{*2}(s)) \cdot d\vec{A} \quad (23)$$

360
$$\Delta\rho \cdot \frac{\pi D^3}{6} \cdot g = \frac{\rho \cdot u_\infty^2}{2} \cdot \overline{1 - u^{*2}(s)} \cdot \pi D^2 \quad (24)$$

361
$$Eo = 3 \cdot We \cdot \overline{1 - u^{*2}(s)} \quad (25)$$

362
$$\overline{1 - u^{*2}(s)} = g(X(We)) = g(We) \quad (26)$$

363
$$Eo = 3 \cdot We \cdot g(We) \quad (27)$$

364
$$F(We, Eo) = 0 \quad (28)$$

365 As can be seen in Eq.(27), the whole system is characterized by only two dimensionless numbers, which are
366 connected.

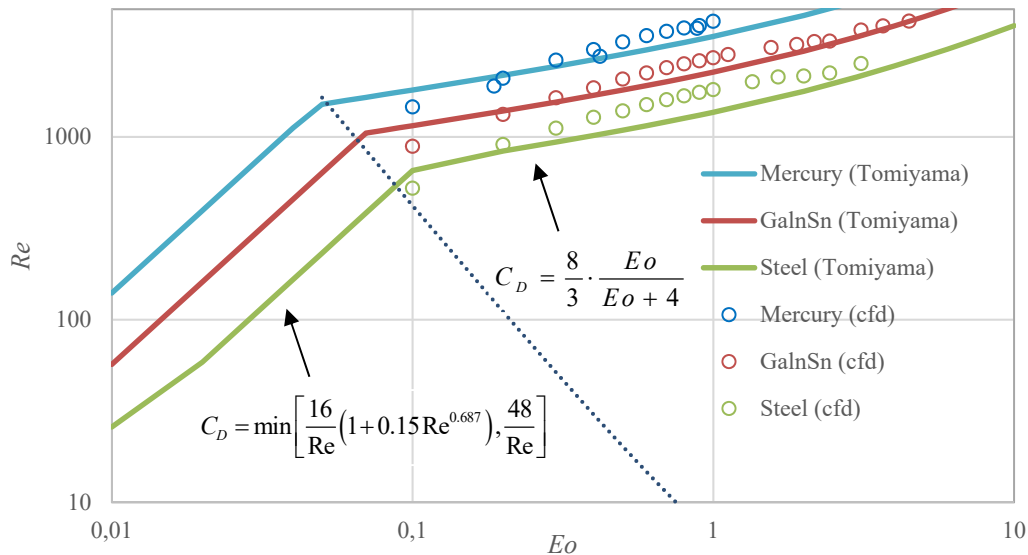
367 Moreover, $\overline{1 - u^{*2}(s)}$ is the surface average of the $1 - u^{*2}(s)$ function in the inviscid region before the
368 detachment point. Such a function can only be known exactly by resolving the flow and the pressure field
369 around the bubble. Nevertheless, by assuming the bubble is axi-symmetric ellipsoidal shaped, $\overline{1 - u^{*2}(s)}$ is a
370 function of only the aspect ratio of the ellipsoid ($X = \frac{a}{c}$). In the next chapter, it is demonstrated that the aspect
371 ratio is only a function of the Weber parameter. This means that the bubble rising problem can be expressed
372 with a single equation that contains only 2 dimensionless numbers $F(We, Eo)$. Therefore, the whole problem
373 is influenced by only 1 dimensionless number.

374 **Terminal rising velocity**

375 **Comparison against Tomiyama's correlation**

376 The computed Reynolds number, based on the terminal rising velocity, is compared with Tomiyama's
377 correlation Eq. (29) in Figure 6 as a function of the Eötvös number. The range simulated here corresponds to
378 Mendelson's formula in Tomiyama's correlation, in which the velocity is only controlled by the Eötvös
379 number. It can be observed that, on first approximation, Tomiyama's correlation and, therefore, Mendelson's
380 equation, are able to clearly predict the bubble behavior and give the right order of magnitude of the terminal
381 Reynolds number.

382
$$C_D = \max \left\{ \min \left[\frac{16}{Re} (1 + 0.15 Re^{0.687}), \frac{48}{Re} \right], \frac{8}{3} \cdot \frac{Eo}{Eo + 4} \right\} \quad (29)$$

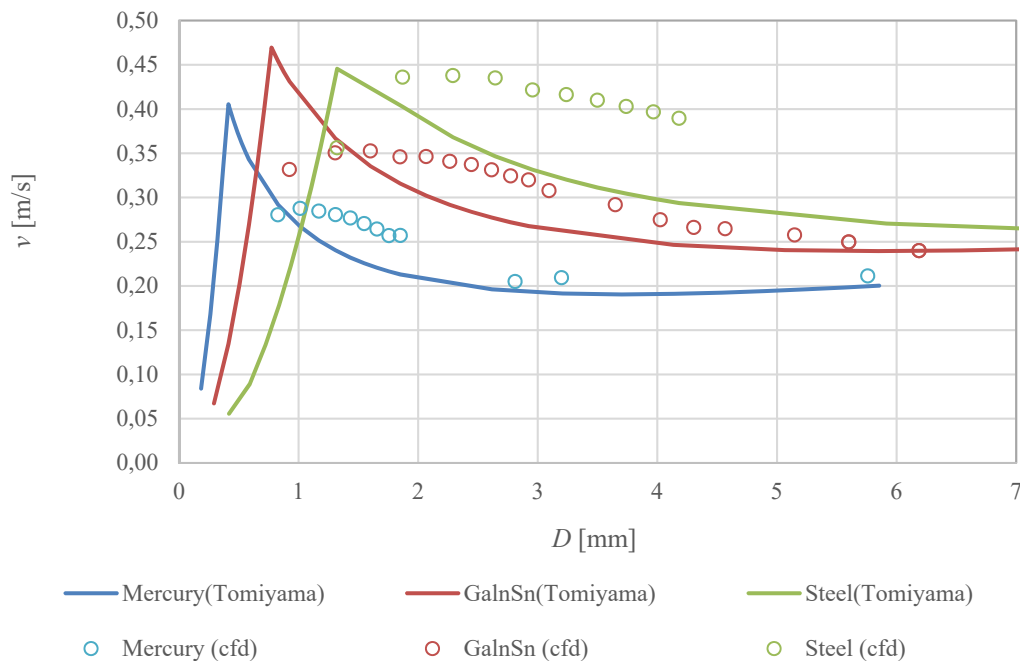


383

384 Figure 6 "Comparison between Tomiyama's correlation and PSI-BOIL simulation results."
 385 There is a rapid change in the velocity profile in Tomiyama's correlation (Figure 7) when passing from the
 386 Eötvös dependent region ($C_D = \frac{8}{3} \cdot \frac{Eo}{Eo+4}$) to the Reynolds dependent region ($C_D = \frac{48}{Re}$). As we consider smaller
 387 bubbles, the Reynolds number acquires more importance, while the Eo loses importance in determining the
 388 drag coefficient. The change from the Eo to Re regions is gradual in the simulations, and the velocity slope
 389 changes smoothly from negative to positive. The Tomiyama curve has been validated in low-intermediate
 390 Reynolds regimes and never in the liquid metal. This could be the reason of significant deviations of CFD
 391 results from the Tomiyama correlation.

392 Unfortunately, due to a CFD parasitic current issue, it was impossible to simulate bubbles with $Eo < 0.05$. The
 393 parasitic current is a computational phenomenon that occurs when there is a low resolution of the surface of
 394 the bubble which compromises the accuracy of the shape of the bubble and the resolution of the surface tension
 395 force. Once a grid size is set, the simulations are no longer reliable over a certain bubble size.

396 Therefore, the $C_D = 48/Re$ dependency region cannot be demonstrated by means of simulations.

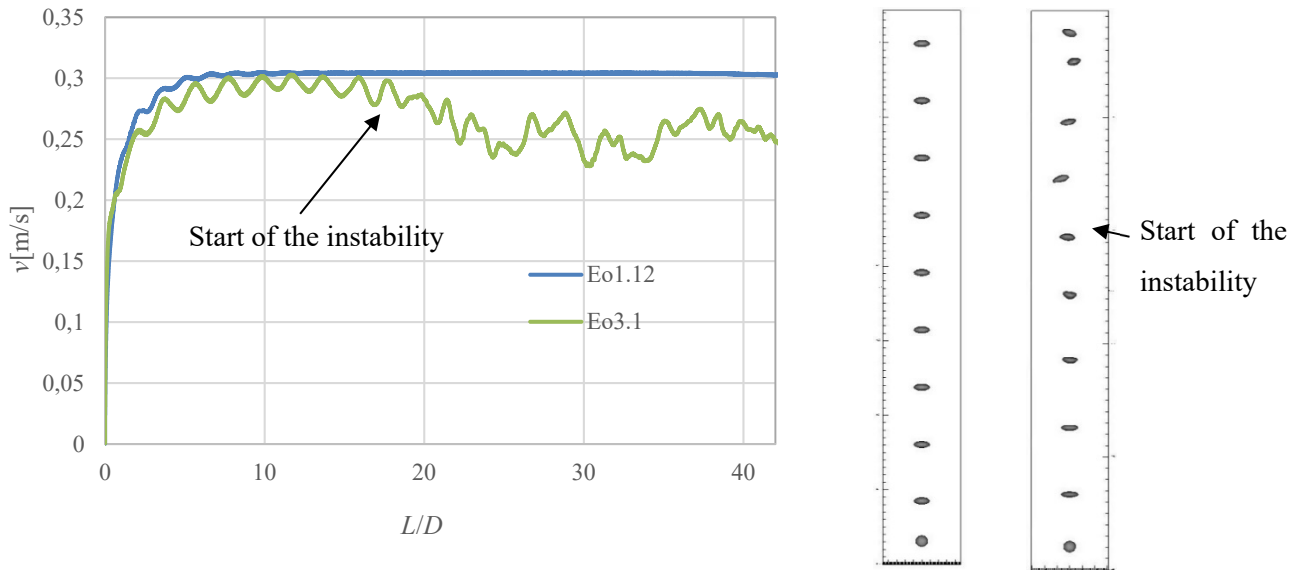


397
 398 Figure 7 "Comparison of terminal velocity as the function of bubble diameter between Tomiyama's correlation
 399 and PSI-BOIL."

400 ***Instabilities of bubble***

401 A certain number of larges bubbles tend to show instable behavior at some points during their rising (Figure
 402 8). The trajectory in no longer rectilinear, and the instant velocity is kept the same; therefore, the average rising
 403 vertical velocity decreases.

404 . Wake flow no longer tends to be axisymmetric and non-uniform velocity oscillations are a direct consequence
 405 of bubble shape oscillations initiated by wake flow asymmetrization.



406
 407 Figure 8 “Right: Velocity for $Eo = 1.12$ and 3.1 bubbles. Left: $x-z$ section of bubble trajectory (left: $Eo = 1.12$
 408 right: $Eo = 3.1$).”

409
 410 The line in the Grace diagram that separates the stable and unstable bubble regions has a negative slope. This
 411 means that, for high Morton numbers, instabilities will occur for larger Eötvös number, while instabilities
 412 occur earlier for low Morton material instabilities. The Morton number can be considered a good parameter to
 413 quantify the stability of a set of materials. According to the definition, the Morton number can be assumed as
 414 a ratio between the viscous force over the inertia to the surface tension forces. It is well known that viscous
 415 forces tend to keep the system stable.

416

$$Mo = \frac{g \cdot \mu_c^4 \cdot \Delta \rho}{\rho_c^2 \cdot \gamma^3} = \frac{F_v}{F_{st} \cdot F_i} \quad (30)$$

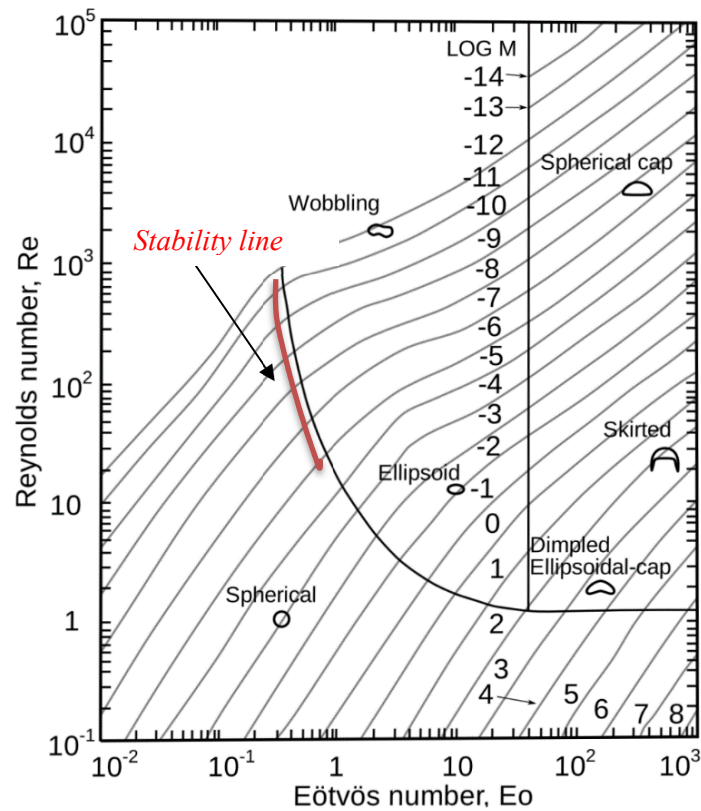


Figure 9 " Grace diagram.(Acrivos, 1979)"

417

418

419 The line tends to be steeper in low Morton regions, such as the liquid metal region (Figure 9). The Stability
 420 line becomes more and more independent of the Morton number, and the Eötvös number at which instabilities
 421 occur becomes a constant. The viscous effect is negligible in this region, and the stability is only controlled by
 422 the balance between the inertia and the surface tension forces (Morton number). Since the Weber and Eötvös
 423 numbers are connected, the stability can also be expressed as a function of the Eötvös number.

424

$$Stability = f\left(\frac{F_i}{F_{sf}}\right) = f(We(Eo)) = g(Eo) \quad (31)$$

425

426 The region in which the bubble passes from a spherical-like body to a wobbling disk for different material
 427 properties has been investigated. Figure 10 suggests that the transition happens almost at the same Eötvös
 428 number for all three metals ($Eo = 1.7$). This is further proof that this process is not influenced by the Morton
 number, because the viscosity effects are negligible, compared to the inertia and superficial ones.

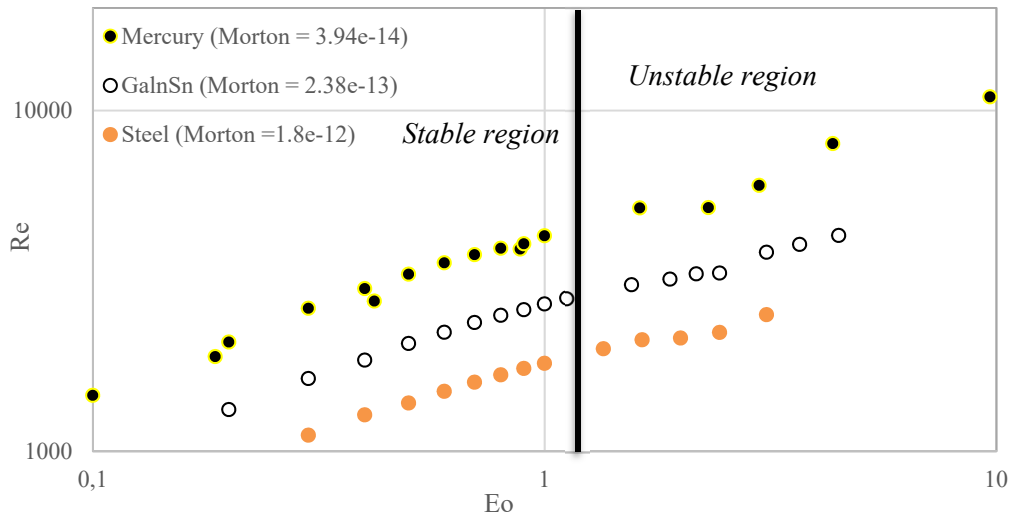


Figure 10 " Reynolds vs. Eötvös for bubbles rising in different liquid metals."

Novel correlation for bubble aspect ratio

At the beginning of the simulation the bubble is into a quiescent liquid, and it has a spherical shape. The bubble starts to rise, due to an unbalance of the gravity and Archimedes force.

As the bubble rises with increasing velocity, hydrodynamic pressure due to metal flow at the top of the bubble increases. Due to the incompressibility constraint, bubble compression at the top and bottom leads to elongation at the sides, and the bubble shape becomes elliptic.

Moore[12] proposed a simple correlation for the bubble aspect ratio X , which is valid for spherical-like bubbles, but loses its applicability for higher Weber numbers:

$$X = 1 + \frac{9}{64} \cdot We + O(We^2) \quad (32)$$

In the following section, we propose a new semi-analytical method for the aspect ratio of a bubble rising in liquid metals. The bubble aspect ratio depends only on the total pressure field on the bubble surface. Since $Re \gg 1$, the viscous pressure is neglected, with respect to the kinetic pressure, and the inviscid approximation thus becomes reasonable.

Analytical theory

Force balance and curvature

Once the bubble has reached its terminal velocity, the acceleration of the system is zero and the bubble and all its components are in force equilibrium ($\sum F = 0$). The force balance of a bubble interface can be calculated if the inner and the outer pressure are known, as illustrated in Figure 11.

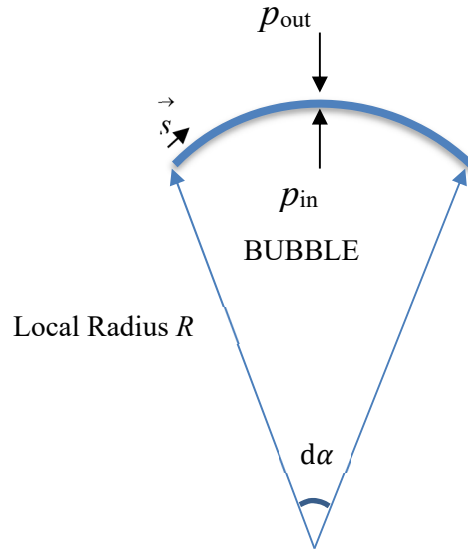


Figure 11 "Force balance on the bubble interface."

451

452

453

The force balance for each moment ($\forall t$) can be expressed by Eq.(33).

454

$$p_{in} = p_{out} + \gamma\chi \quad (33)$$

455

Since the velocity field is zero at the beginning of the rise ($t=0$), Eq.(34) and Eq.(35) can be asserted.

456

$$p_{in} = \gamma\chi_o + \rho gh \quad (34)$$

457

$$p_{in} - \rho gh = \gamma\chi_o = \frac{4\gamma}{D} \quad (35)$$

458

The outer pressure is found by applying the Navier-Stokes equations along a streamline from the non-perturbed region to the interface, Eq. (36). Since the viscous effect is negligible, the flow field can therefore be

459

460

approximated as an irrotational field ($\nabla \times \vec{u} = 0$). The energy inside the fluid is conserved.. The total outer

461

pressure is the sum of the gravitational pressure, ρgh and the kinetic pressure, $\frac{1}{2}\rho(u_\infty^2 - u^2)$. The gravitation

462

force is not technically constant, since it depends on the depth of the liquid, but for simplification reasons it

463

was set constant to ρgh , with h the depth of the bubble center of the mass. Assuming that the fluid is almost

464

perfectly inviscid, there are no interface boundary layers and the interface tangential liquid velocity is non-

465

zero ($u_{//}(s) \neq 0, u_r(s) = 0$).

466

$$p_{out}(s) = \rho gh + \frac{1}{2}\rho(u_\infty^2 - u^2(s)) \quad (36)$$

467

The inner pressure can easily be computed by applying the Navier-Stokes equations to the inner gas. Basically,

468

the pressure field inside the bubble is spatially almost constant, that is, $\nabla p = 0$.

469

Once the bubble has reached its terminal velocity, the shape remains constant, which means that the bubble

470

curvature is kept constant at each point. In the same way, the velocity field across the bubble does not change.

471

The gravitational term linearly decreases during rising of the bubble, since the depth is decreasing

472

$\rho g[h - u_\infty \cdot t]$. The inner gas pressure also decreases to keep the system stable and to provide force balance

473

at each point in time. Therefore, the $p_{in}(t) - \rho gh(t)$ term is a constant during bubble rising.

474

$$\chi(s) = \frac{\frac{1}{2}\rho u^2(s) + \rho gh - p_{in} - \frac{1}{2}\rho u_\infty^2}{\gamma} \quad (37)$$

475 Eq. (37) suggests that the local interface curvature is linearly dependent on the kinetic pressure and inversely
 476 proportional to the surface tension coefficient. Furthermore, $\rho gh - p_{in} - \frac{1}{2}\rho u_{\infty}^2$ is constant for each point of
 477 the interface. Therefore, the driving factor to compute the curvature is the interface tangential velocity field,
 478 $u_{//}(s)$. In order to obtain the ideal resolution of the bubble shape problem, inviscid Euler equations have to be
 479 solved around the bubble to find the tangential velocity, $u_{//}$, to the surface, as this is needed to calculate the
 480 bubble curvature, $\chi(s)$, according Eq. (37).

481 Here, an equivalent method, which avoids the need to solve the Euler equations for the ellipsoidal coordinates
 482 and provides an accurate correlation for stable ellipsoidal bubbles in inviscid liquid, is proposed.

483 Let us now consider the bubble at the first instant in which it is quiescent and has a perfect spherical shape. In
 484 this situation, the velocity field is zero, and the main driven force is the Archimedes force

485 Nevertheless, for simplicity, the hydrostatic pressure is set equal to ρgh for the bubble aspect ratio calculation
 486 on the entire interface, since $h \gg D$, and the relative difference between p_{up} and p_{down} is negligible. Setting the
 487 hydrostatic pressure constant is a reasonable approximation to compute the bubble shape. However, it cannot
 488 be assumed in the momentum balance equation because the difference between the upper and lower hydrostatic
 489 pressures is the driving factor behind the rise in the bubble. If the hydrostatic pressure is assumed to be
 490 constant, the bubble is in static equilibrium. It can be established, from the balance of the forces acting on the
 491 surface at $t = 0$ (Figure 11), that $p_{out} - \rho gh$, which was previously demonstrated to remain constant during
 492 bubble rising, is equal to the initial curvature over-pressure, $\gamma\chi_0$.

$$493 \quad p_{in} - \rho gh = \gamma\chi_0 \quad (38)$$

494 Using Eq. (37) and Eq.(38), the final balance equation on interface Eq. (39) shows how the local curvature
 495 behaves. It is equal to an initial constant value plus a kinetic pressure dependent value. It is clear that the
 496 velocity field is the only factor that influences the local bubble curvature. Regions with high kinetic pressure
 497 result in a low curvature interface, and vice-versa. Ideally, the velocity field $u(s)$ can be found by solving the
 498 Euler equations for the ellipsoidal shape and it clearly depends on the local curvature $u(s) \rightarrow u(s, \chi(s))$ of
 499 the bubble. This process tends to become complex and long.

$$500 \quad \gamma\chi(s) = \gamma\chi_0 + \frac{1}{2}\rho(u^2(s) - u_{\infty}^2) \quad (39)$$

501 ***Curvature at the top of the bubble***

502 In our analysis, the bubble shape was considered to be a perfect ellipsoid $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\right)$. Since the flow
 503 is axisymmetric, the bubble shape in the x and y directions is the same ($a = b$) and the ellipsoid can be
 504 perfectly characterized by only the a and c parameters. To characterize the problem, it is only necessary to
 505 solve the curvature equation at two different points on the bubble, since there are only 2 unknowns. The first
 506 point considered is the bubble stagnation point in which the kinetic pressure is the highest, Eq. (40). Here, the
 507 fluid stops, and all the kinetic energy is transmuted into pressure. This is a convenient solution point since here
 508 the velocity is zero for any aspect ratio.

$$509 \quad \chi_{\min} = \chi(\theta = 0) = \chi_0 - \frac{1}{2}\frac{\rho \cdot u_{\infty}^2}{\gamma} \quad (40)$$

510 ***Curvature at the side of the bubble***

511 The second point that was considered is the one with the highest curvature, which occurs at an angle of 90° ,
 512 with the curvature being defined as:

$$513 \quad \chi_{\max} = \chi\left(\theta = \frac{\pi}{2}\right) = \chi_o + \frac{1}{2} \frac{\rho}{\gamma} \cdot \left(u^2\left(\frac{\pi}{2}\right) - u_\infty^2\right). \quad (41)$$

514 Here, the fluid velocity, to satisfy conservation of the mass ($\nabla \cdot u = 0$), increases, with a consequent production
 515 of a negative kinetic pressure on the interface, which, in turn, stretches the bubble. How the tangential velocity
 516 behaves at each other point is not of interest since only the largest and the smallest curvatures are needed. The
 517 major problem is to find the tangential velocity at 90° .

518 ***Tangential velocity field***

519 First, from the solution of the Euler equations in the spherical coordinates, only the tangential component of
 520 the velocity is present on the interface, and not the radial one. In a perfect spherical case, $u_{//}\left(\frac{\pi}{2}\right) = \frac{3}{2}u_\infty$.

521

522 Eq (42) and Eq.(43) are Euler incompressible equations for mass and momentum conservation, respectively.

$$523 \quad \nabla \cdot u = 0 \quad (42)$$

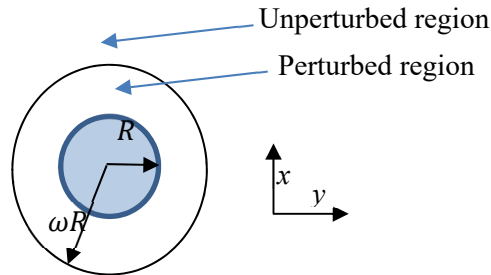
$$524 \quad \rho \cdot \frac{\partial u}{\partial t} + \rho \cdot (u \cdot \nabla)u = -\nabla p + \rho g \quad (43)$$

525 Eq. (44) and Eq.(45) are the radial and tangential velocity field solutions of the Euler incompressible equations.

$$526 \quad u_r(r, \theta) = u_\infty \left(1 - \frac{R^3}{r^3}\right) \cos(\theta) \quad (44)$$

$$527 \quad u_{//}(r, \theta) = -u_\infty \left(1 + \frac{R^3}{2r^3}\right) \sin(\theta) \quad (45)$$

528 In an ideal case, the tangential velocity decreases ($\propto \frac{1}{r^3}$) from $\frac{3}{2}u_\infty$ to u_∞ , moving away from the sphere. An
 529 equivalent model is used for the ellipsoidal bubble in which, for a certain range, the velocity is uniformly
 530 constant and equal to the interface tangential velocity (Figure 13). We assume that, after a certain distance, the
 531 flow is no longer perturbed by the presence of the sphere (Figure 13). Nevertheless, the velocity is considered
 532 spatially uniform in the perturbed region and equal to the tangential velocity at the interface (Eq.(46)). The
 533 key point is to find the distance beyond which the velocity drops from the tangential velocity value to zero.
 534 The distance has been decided imposing that all liquid momentum of the real case is confined only in the
 535 perturbed range in the modeled case. In view of integral volume flowrate conservation (Figure 12), the ideal
 536 “perturbed range” can be found for a spherical bubble (Eq.(47)) and it is equal to $\sqrt{3} \cdot R$.
 537 ω is defined as the ratio between the distance from the center of the bubble at which the flow starts to become
 538 unperturbed to the side radius of the bubble.



539

540

Figure 12 "Top view of a spherical bubble."

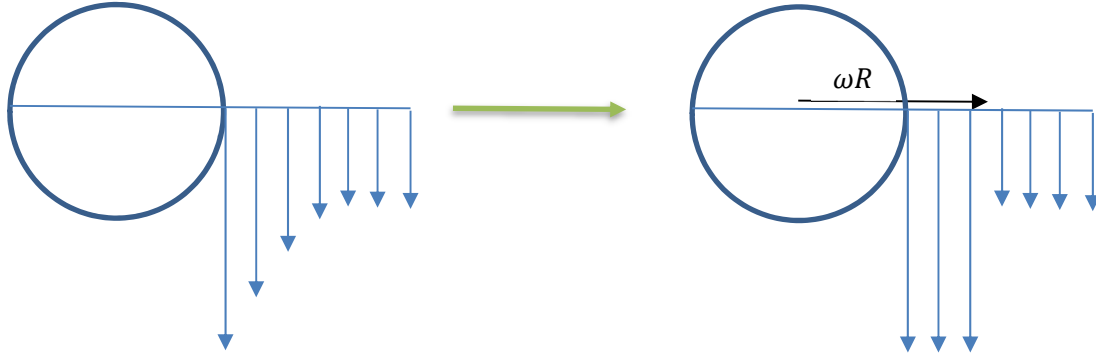
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542

$$\pi(\omega R)^2 \cdot u_\infty = \pi[(\omega R)^2 - R^2] \cdot \frac{3}{2} u_\infty \quad (46)$$

543

$$\omega = \sqrt{3} \quad (47)$$



544

545 Figure 13 "Difference between real side velocity field and modeled one for a spherical body (x-z or y-z)."

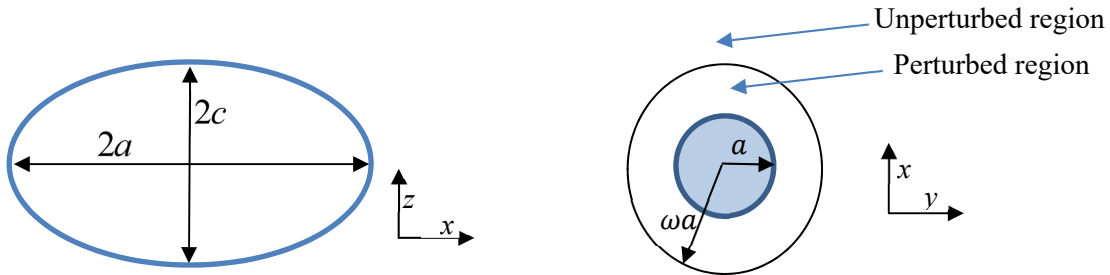
546

547 This approach can also be used for ellipsoidal bubbles, but with some geometrical corrections. V_{tot} is the total
 548 volume of the bubble, and it is kept constant during the rise since no break-up phenomena take place. The
 549 width of the bubble is dependent on its characteristic diameter $\left(D = \sqrt[3]{\frac{6 \cdot V_{tot}}{\pi}}\right)$ and on the bubble aspect ratio
 550 $\left(\chi = \frac{a}{c}\right)$. Using the current definitions, and assuming that the bubble volume does not change under
 551 compression, the width is found as a function of the initial diameter and the aspect ratio (Eq.(48)).

552

$$a = D \cdot \sqrt[3]{\frac{\chi}{8}} \quad (48)$$

553 Volume flowrate conservation Eq. (49) is applied to an ellipsoidal bubble (Figure 14) to determine the liquid
 554 side velocity $u\left(\frac{\pi}{2}\right)$, Eq. (50).



555

556 Figure 14 "On the left, side representation of the ellipsoidal bubble, on the right top view of the ellipsoidal
557 bubble."

558

$$\pi(\omega a)^2 \cdot u_\infty = \pi[(\omega a)^2 - a^2] \cdot u\left(\frac{\pi}{2}\right) \quad (49)$$

559

$$u\left(\frac{\pi}{2}\right) = u_\infty \cdot \frac{\omega^2}{\omega^2 - 1} \quad (50)$$

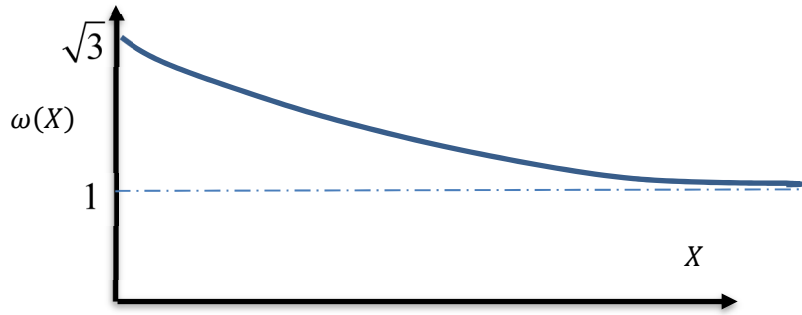
560 In the ellipsoidal case, finding ω is the key point since it is only known a priori in the spherical case. We
 561 assume that ω is a function of aspect ratio X , which is always greater than unity, and $\omega(X = 1) = \sqrt{3}$. It is
 562 generally known that $u_{//}\left(\frac{\pi}{2}\right)$ is higher in highly distorted bubbles, since a larger amount of fluid is deflected

563 as a result of the presence of the bubble and this fluid flows entirely on the side of the bubble, $\frac{d u(\pi/2)}{d X} > 0$.
 564 Terminal bubble velocity u_{∞} decreases slightly as the aspect ratio increases, since the drag coefficient is higher
 565 for highly distorted bubbles: $\frac{d u_{\infty}}{d X} < 0$. Therefore, according to Eq. (51), $\omega^2/(\omega^2 - 1)$ increases with X . By
 566 solving Eq. (52), it is possible to prove that $\omega(X)$ monotonously decreases with X , Eq. (53). Furthermore,
 567 $\omega(X)$ only reaches 1 asymptotically (Figure 15):

$$568 \quad \frac{d\left(\frac{\omega^2}{\omega^2-1}\right)}{dX} = \frac{d\left(\frac{u_{90}}{u_{inf}}\right)}{dX} = \frac{\left(\frac{du_{90}}{dX}\right) * u_{inf} - \left(\frac{du_{inf}}{dX}\right) * u_{90}}{u_{inf}^2} > 0 \quad (51)$$

$$569 \quad \frac{d\frac{\omega^2}{\omega^2-1}}{dX} = \frac{2\omega \cdot \frac{d\omega}{dX} (\omega^2-1) - \omega^2 \cdot 2\omega \cdot \frac{d\omega}{dX}}{(\omega^2-1)^2} = \frac{d\omega \cdot (-2\omega)}{(\omega^2-1)^2} > 0 \quad (52)$$

$$570 \quad \frac{d\omega}{dX} < 0 \quad (53)$$



571
 572 Figure 15 “Possible omega function (the ratio between the distance from the center of the bubble at which
 573 the flow starts to become unperturbed to the side radius of the bubble).”

574 For practical calculations, it was hypothesized that $\omega(X)$ follows a decrease that is proportional to $\frac{1}{X^n}$, as can
 575 be seen in Eq. (54). Under this assumption, the function is controlled by the n parameter, which is an unknown,
 576 while n represents how fast the function tends to 1. The latter is a free parameter that is guessed by comparing
 577 the theory with the PSI-BOIL simulation.

$$578 \quad \omega(X) \approx \frac{(\sqrt{3}-1)}{X^n} + 1 \quad (54)$$

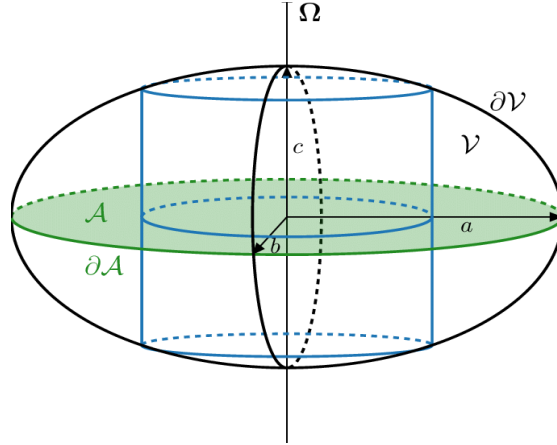
579 **Curvature Calculation.**

580 According to the previous consideration, higher curvature Eq. (55) and lower curvature Eq. (56) can be
 581 calculated, and correspond to $\frac{\pi}{2}$ and the 0 angle, respectively.

$$582 \quad \chi_{\max} = \chi\left(\frac{\pi}{2}\right) = \frac{4}{D} - \frac{1}{2} \cdot \frac{\rho \cdot u_{\infty}^2}{\gamma} \cdot \left(1 - \left(\frac{\omega^2(X)}{\omega^2(X)-1}\right)^2\right) \quad (55)$$

$$583 \quad \chi_{\min} = \chi(0) = \frac{4}{D} - \frac{1}{2} \cdot \frac{\rho \cdot u_{\infty}^2}{\gamma} \quad (56)$$

584 The analytical expression of the aspect ratio of an ellipsoid can be found from the different calculations (
 585 Figure 16). In our case, the velocity field and the bubble shape are axisymmetric. In fact, there is no dependence
 586 of any variable on $0 < \varphi < 2\pi$ ($a = b$).



587
 588 Figure 16 " ellipsoidal representation of the bubble."
 589

590 The side (Eq.(57)) and the top (Eq.(58)) curvatures of a 3D ellipsoid can be analytically expressed as a function
 591 of the ellipsoidal aspect ratio (X) and the semi-major axis (a). Eq.(59) is obtained from the ratio of Eq.(58) to
 592 Eq.(57).

$$593 \quad \chi_{\max} = \frac{1}{R_s} + \frac{1}{R_t} = \frac{a}{c^2} + \frac{1}{a} = \frac{\left(\frac{a}{c}\right)^2 + 1}{a} = \frac{X^2 + 1}{a} \quad (57)$$

$$594 \quad \chi_{\min} = \frac{1}{R_s} + \frac{1}{R_t} = \frac{c}{a^2} + \frac{c}{a^2} = \frac{2}{a} \cdot \frac{c}{a} = \frac{2}{a \cdot X} \quad (58)$$

$$595 \quad \frac{\chi_{\min}}{\chi_{\max}} = \frac{\left(\frac{2}{a \cdot X}\right)}{\left(\frac{X^2 + 1}{a}\right)} = \frac{2}{X(X^2 + 1)} \quad (59)$$

596 Substituting the definition of the max. and min. curvature for bubbles (Eq.(55) and Eq.(56)) in Eq. (58), an
 597 explicit correlation between the Weber number and the bubble aspect ratio (Eq.(63)) is found. This a proof that
 598 the bubble deformation in inviscid fluids is only controlled by the ratio of the inertia to the surface tension
 599 forces.

600 Let us define $F(X)$ as Eq.(60) for compactness reasons.

$$601 \quad F(X) = \frac{\omega^2(X)}{\omega^2(X) - 1} \quad (60)$$

$$602 \quad \frac{\chi_{\min}}{\chi_{\max}} = \frac{2}{X \cdot (X^2 + 1)} = \frac{4 - \frac{1}{2} \frac{\rho \cdot u_{\infty}^2 \cdot D}{\gamma}}{4 - \frac{1}{2} \frac{\rho \cdot u_{\infty}^2 \cdot D}{\gamma} \cdot (1 - F^2(X))} \quad (61)$$

$$603 \quad \frac{\chi_{\min}}{\chi_{\max}} = \frac{2}{X \cdot (X^2 + 1)} = \frac{4 - \frac{1}{2} We}{4 - \frac{1}{2} We \cdot (1 - F^2(X))} \quad (62)$$

$$604 \quad We = \frac{16 - 8X \cdot (X^2 + 1)}{2 \cdot (1 - F^2(X)) - X \cdot (X^2 + 1)} \quad (63)$$

605 Once $\omega(X)$ has been estimated, the correlation can successfully be used in Eq. (63). If we linearize the function
 606 in the surroundings of $X=1$, the Moore linear correlation is found.

607 Moreover, the omega function cannot be known a priori, unless the Euler equations are analytically solved in
 608 ellipsoidal coordinates. This formula is valid as long as the bubble shape can be considered an ellipsoid. When

609 the top curvature becomes zero, the ellipsoidal aspect ratio mathematically diverges to infinity. In order to
 610 keep this theory valid, the top curvature has to be greater than zero, Eq. (64). This condition is achieved when
 611 the Weber number of a bubble is less than 8, Eq. (65). However, this limit is practically never reached because
 612 the bubble becomes unstable much sooner ($We \sim 4$).

$$613 \quad \chi_{TOP} = \frac{4}{D} - \frac{1}{2} \cdot \frac{\rho \cdot u_{\infty}^2}{\gamma} \gg 0 \quad (64)$$

$$614 \quad We \ll 8 \quad (65)$$

615

616 ***Comparison of the aspect ratio of the analytical solution to PSI-BOIL***

617 The following analytical solution has been compared with the CFD results to determine the n parameter that
 618 fully characterizes the omega function, $\omega(X)$ (Eq.(66)). Aspect ratio X was calculated for 30 simulations for 3
 619 different sets of materials (Argon+Steel, Argon+GalSn and Nitrogen+Mercury) within the $0.1 < Eo < 1.2$
 620 range.

621 Unfortunately, the bubble shape is more sensitive than the terminal velocity to the grid size. The relative error
 622 of the aspect ratio clearly depends on the $\frac{\Delta x}{R_{min}} = \Delta x \cdot \chi_{max}$ ratio, which represents how well the grid manages
 623 to represent the bubble interface at the point with the highest curvature on the side of the bubble.

624 Grid sensitivity studies were conducted for an $Eo = 0.5$ bubble, and the relative error pertaining to the aspect
 625 ratio was almost 10%, compared to an ultrafine grid simulation where a spatial resolution of 50 cells on the
 626 bubble diameter has been adopted ($D/dx=50$). Furthermore, it was observed that the lack of accuracy of the
 627 grid always leads to an underestimation of the aspect ratio. Even though the relative error is lower for small
 628 bubbles, since they are less stretched, a conservative analysis was adopted, and a 10% relative error was set
 629 for all the points.

630 The best interpolation was found for $n = 1.34$. Exponential fitting has been performed with $R2 = 0.92$.

$$631 \quad \omega(X) \approx \frac{(\sqrt{3}-1)}{X^{1.34}} + 1 \quad (66)$$

$$632 \quad F(X) = \frac{\omega^2(X)}{\omega^2(X)-1} \quad (67)$$

$$633 \quad We = \frac{16-8X \cdot (X^2+1)}{2 \cdot (1+F^2(X))-X \cdot (X^2+1)} \quad (68)$$

634 Eq.(68) precisely predicts the bubble aspect ratio in an inviscid regime. It is applicable to bubble rising in
 635 liquid metal for $0 < Eo < 1.2$. However, above $Eo = 1.2$, the bubble becomes unstable, the shape is no longer
 636 ellipsoidal and the theory is no longer valid.

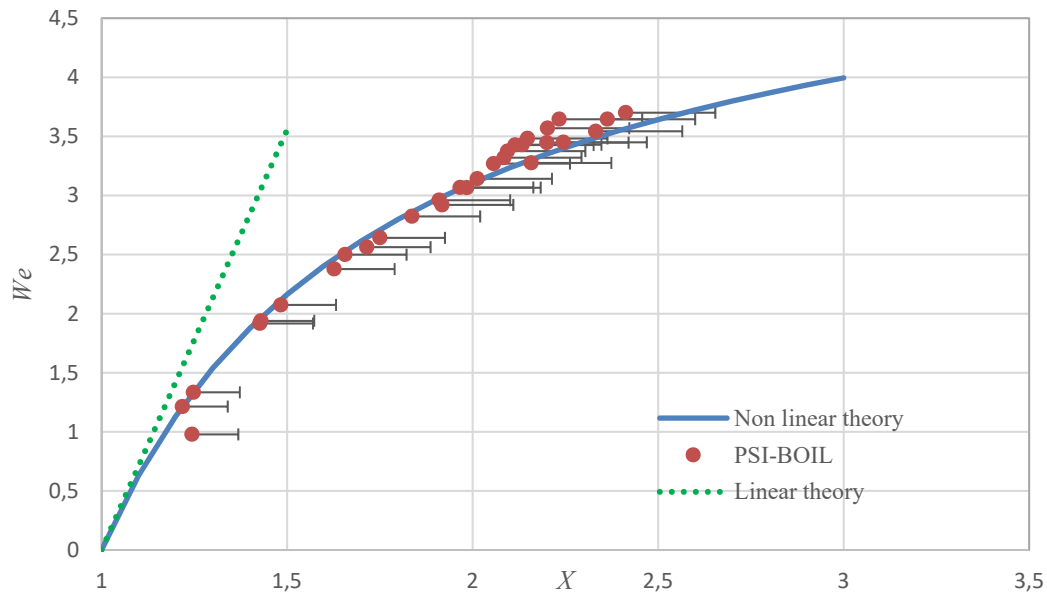


Figure 17 "Comparison between linear and nonlinear theory for inviscid liquid."

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Conclusions

641

In this study, Computational Fluid Dynamics (CFD) simulation has been used to improve knowledge of a bubble rising in liquid metals. More precisely, the bubble rising velocity and the bubble deformation were investigated using the CFD code, that is, PSI-BOIL.

644

A rising bubble in liquid metal was studied, and the predicted terminal rising velocity was compared with Tomiyama's correlation. The simulation results showed that the drag coefficient is only controlled by the Eötvös number for large bubbles and shows the same tendency as Tomiyama's correlation. A semi-analytical model for bubble shape deformation was then derived for liquid metal and compared with the simulation results. The result shows that inviscid approximation works well for bubbles in liquid metal, and that the bubble aspect ratio is only controlled by the Weber number. Furthermore, a new stability criterion for bubbles in a liquid metal has been proposed. In the proposed criterion, bubble stability in the considered liquid metal was not controlled by the Morton number, but only by the Eötvös number. Transition from an ellipsoidal to a wobbling disk shape took place for the fixed Eötvös number ($Eo = 1.7$). The novelty is the formulation of a stability criterion for high Reynold ($1000 < Re < 10000$) bubble stability for which experimental correlation are hardly achievable.

655

As a follow-up of this article, further studies on bubble aspect ratio are going on. Simulations on bubble rising in metals under magnetic field (horizontal and vertical) to study bubble shape, trajectory and velocity, are also planned.

658

659

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660

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661

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