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# A Comparative Assessment of Rheological Laws for Mud Flows

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## Abstract

Climate change has increased the frequency of prolonged and intense rainfall events. As a result, increasingly frequent slurry flows, channelised landslides that occur on mountain slopes, have manifested all over the world causing extensive damage. On 15-16 December 1999 the municipality of Cervinara was hit by several slurry flows and some authors simulated the events considering the mixture as an equivalent continuous fluid. These models have adopted depth-averaged approaches, in which both internal and basal flow resistance are described by the bottom shear stress. By investigating the various rheological models applied, the comparisons highlighted the differences in terms of shear stress values with the same flow depth and unit discharge width. Despite these differences, the models applied satisfactorily to simulating the same event. A further rheological law is introduced, derived from the characterization in the laboratory of a mud reconstituted from a soil sample taken at the Cervinara site, which models the slurry as a shear-thinning fluid. Whatever the flow depth and flow conditions, the low shear-thinning index values imply an almost constant shear stress. Future simulations of the full event could reveal the performance of this bottom-up approach in predicting the consequences of a field-scale landslide.

**Keywords:** Rheology; Debris flow; Mud flow; Shallow flow; Numerical modelling

## 1. INTRODUCTION

Extreme events with intense and prolonged rainfall have been increasingly frequent over the last decades due to the ongoing climate change. As a matter of fact, devastating landslides have caused huge damages, both in terms of human lives and economic losses. Among those phenomena, slurry flows, occurring along natural streams on mountain slopes after long or intense rainfalls, are very dangerous landslides.
In 15-16 December 1999, a series of destructive slurry flows hit the residential area of Cervinara in Campania, Italy, which developed from the liquefaction of the pyroclastic soils covering the carbonate bedrocks. Several authors have back-analysed these events based on the approximation of the water-sediment mixture as a continuous equivalent fluid. In these analyses, depth-averaged approaches are considered a convenient trade-off between descriptive capabilities and complexity of the adopted models. Within this approach, both internal and basal flow resistance are described by the bottom shear stress.

## 2. COMPARISON OF RHEOLOGICAL LAWS

This work focuses on the different rheology models applied to simulate the propagation of the Cervinara 1999 events. In the first modelling attempt, the basal shear stress $\tau_b$ at the interface between the sliding surface and the landslide mass has been described by Revellino et al. (2004) through a modified Voellmy law, consisting of the sum of a frictional and a turbulent term as shown in Eq. [1].

$$\tau_b = \rho g h \cdot \mu \left( \cos\theta + \frac{v^2}{gR} \right) + \rho g \frac{v^2}{\xi} \qquad [1]$$

The frictional term depends on slope incline $\theta$ and centripetal acceleration $v^2/R$, where $v$ is the flow velocity and $R$ is the curvature radius. The equivalent fluid density is assumed as $\rho = 1400\ \mathrm{kg/m^3}$, the friction coefficient $\mu = 0.07$, the curvature radius $R = 67\ \mathrm{m}$, and the turbulence coefficient $\xi = 200\ \mathrm{m/s^2}$.

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Later, assuming laminar flow conditions, a non-frictional rheology of the basal shear stress $\tau_b$ has been described by Cascini et al. (2011) with a generalised Bingham quadratic law as in Eq. [2].

$$\tau_b = \tau_y + \frac{FKv}{8h} + \gamma \frac{n_{td}^2 v^2}{\sqrt[3]{h}} \qquad [2]$$

first term represents yield shear stress $\tau_y$ ranging from 500 to 700 Pa, the second is expressed as function of the Bingham viscosity $K = 10 - 20\ Pa \cdot s$ and flow resistance parameter $F = 24$ (O'Brien et al. 1993), and the quadratic term depends on the equivalent Manning coefficient $n_{td} = 0.05$ for the turbulent and dispersive shear stress components with $\gamma = 12000\ \mathrm{N/m^3}$ (Cascini et al. 2011).
In what follows, the bottom shear stresses resulting from rheological laws reported in Eq. [1] and Eq. [2] are compared as a function of flow depth $h$ and unit width flow discharge $q = hv$. considering a variety of flow conditions covering a wide range of observed flow depths and unit width discharges. The flow conditions resulting from steady, uniform flows are also reported for four values of the bottom slope ($\theta = 5°, 15°, 25°, 35°$), to represent the flow conditions which may occur over relatively long reaches during the slurry propagation. By means of iso-contours of the bottom shear stress, the results are reported in Figures 1 and 2 for the Revellino et al. (2004) model, and in Figures 3 and 4 for the Cascini et al. (2011) model.

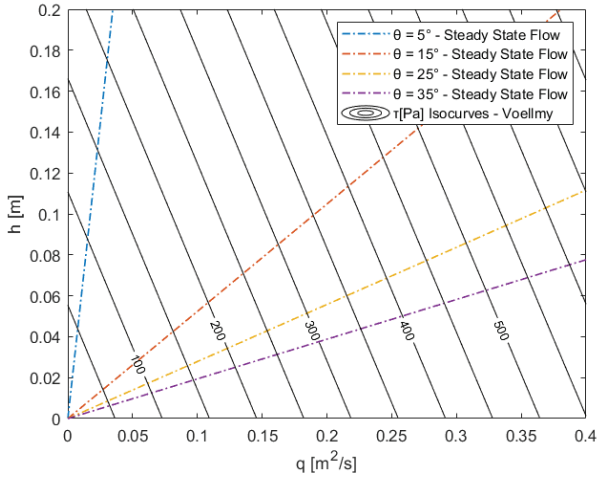


**Figure 1**. Map of the basal shear stresses $\tau_b$ considering the Voellmy law rheology adopted by Revellino et al. (2004) for different flow conditions (flow depth $h = 0 - 0.2$ m, unit width discharge $q = 0 - 0.4\ \mathrm{m^2/s}$)

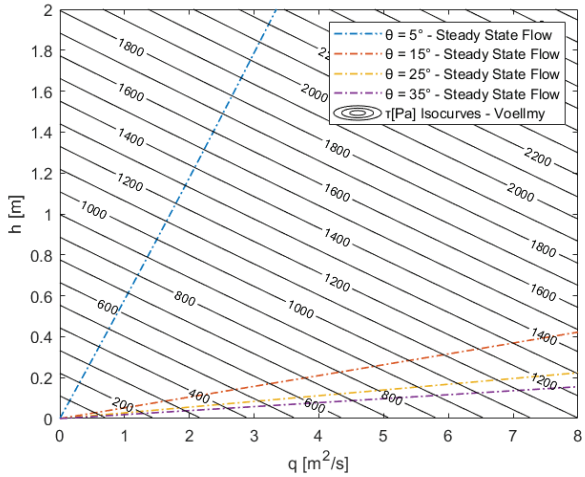


**Figure 2**. Map of the basal shear stresses $\tau_b$ considering the Voellmy law rheology adopted by Revellino et al. (2004) for different flow conditions (flow depth $h = 0 - 2$ m, unit width discharge $q = 0 - 8\ \mathrm{m^2/s}$)

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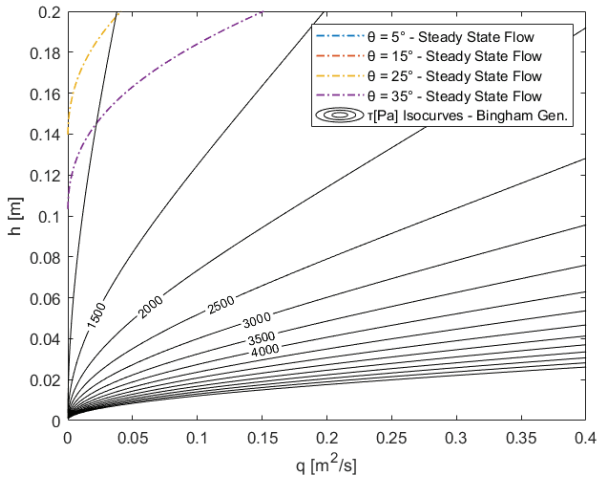


**Figure 3**. Map of the basal shear stresses $\tau_b$ considering the quadratic law rheology adopted by Cascini et al. (2011) for different flow conditions (flow depth $h = 0 - 0.2$ m, unit width discharge $q = 0 - 0.4$ m$^2$/s)

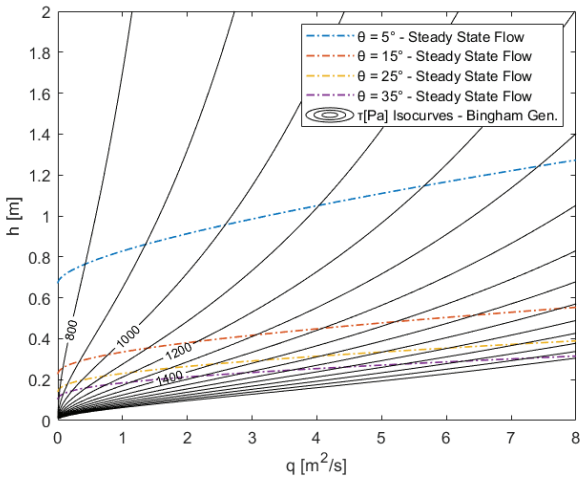


**Figure 4**. Map of the basal shear stresses $\tau_b$ considering the quadratic law rheology adopted by Cascini et al. (2011) for different flow conditions (flow depth $h = 0 - 2$ m, unit width discharge $q = 0 - 8$ m$^2$/s)

The comparison of the maps (Figures 1, 2, 3, and 4) reveals the apparent difference in the rheological models employed both under steady-uniform or unsteady flow conditions. In detail, the quadratic law Eq. [2] produces stresses way bigger than the previous one (Eq. [1]), which could, for instance, imply a slower propagation with respect to the model by Revellino et al. (2004).
Both rheological models were applied satisfactorily to the simulation of the same event. This circumstance reveals an inherent limit of the back-calculation approach. While it makes possible to reasonably reconstruct a given event, it provides poor performance in unambiguously identifying the rheological model of the slurry.
As a test of the preliminary feasibility of an alternative predictive approach, the comparison is extended to a further rheological model of the slurry as a shear-thinning fluid, derived by laboratory characterisation of a reconstituted mud from a soil sample taken at the Cervinara site (Carotenuto et al. 2015). The power-law rheology is expressed in Eq. [3].

$$\tau_b = K\left(\frac{2n+1}{n}\frac{u}{h}\right)^n \qquad [3]$$

The shear-thinning index is assumed to be $n = 0.01$, while the fluid consistency $K = 71.3$ Pa $\cdot$ s$^n$ as reported by Iervolino et al. (2017) for a raw soil sample from Cervinara. The map of shear stresses related to this rheology is illustrated in Figure 5 and Figure 6.

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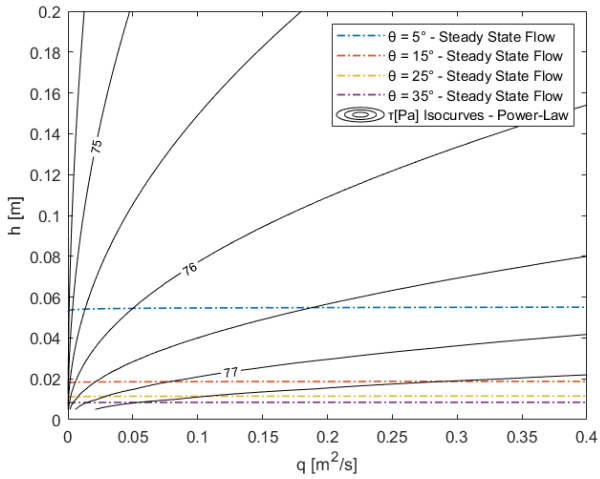


**Figure 5**. Map of the basal shear stresses $\tau_b$ considering the power law rheology for different flow conditions (flow depth $h = 0 - 0.2$ m, unit width discharge $q = 0 - 0.4$ m$^2$/s)

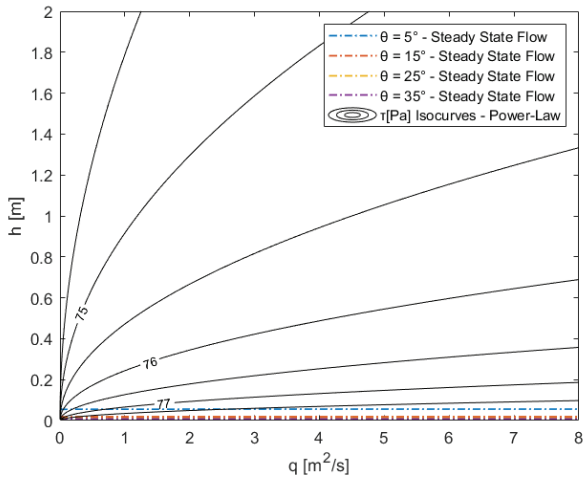


**Figure 6**. Map of the basal shear stresses $\tau_b$ considering the power law rheology for different flow conditions (flow depth $h = 0 - 2$ m, unit width discharge $q = 0 - 8$ m$^2$/s)

In this case, the values of the resistances are smaller than the ones represented by the previous rheologies investigated. Due to the little shear-thinning index the bottom shear stress $\tau_b$ results very little depending on the flow depth $h$ and discharge $q$.

## 3. CONCLUSIONS

By the comparison between the rheologies previously applied (Revellino et al. 2004, Cascini et al. 2011) to the simulation of the events of slurry flows occurred at Cervinara in 1999, large differences were detected in terms of bottom shear stress for the same flow depth and unit width discharge. On the contrary, the low value of the shear-thinning index, adopted to the power law rheology proposed by Iervolino et al. (2017), implies that the shear stress is almost constant whatever the conditions of flow depth and discharge are. Moreover, those resistances are smaller with respect to the ones expressed by the Voellmy law and the quadratic law. While this suggests a faster propagation of the slurry, only the simulation of the full event, which will be addressed in forthcoming research, could reveal the performance of this bottom-up approach in predicting the consequences of a landslide at the field scale.

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