# Worst-Case Optimization of a Digital Link for Wearable Electronics in a Stochastic Framework

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***Abstract*—This paper demonstrates an optimization strategy for systems affected by uncertainties in the case of a textile interconnect line. Rather than simply conducting stochastic analysis at the end of the design process, tolerances are accounted for from the early stages of the flow. An unsupervised approach, used to describe the stochastic behavior of the line, is integrated within a heuristic optimization algorithm with the aim of selecting the optimal parameters of a passive equalizer.**

***Keywords—stochastic optimization, stochastic analysis, metamodeling, textile transmission line, wearable electronics***

## I. Introduction

In engineering, the term "uncertainties" designates tolerances and imperfections but also random environment variables such as temperature or humidity. Accounting for such uncertainties is a very important aspect of any design flow. For decades, Monte Carlo analysis has been a common strategy for a posteriori validation of a design [1]. Monte Carlo is the simplest form of stochastic analysis: system parameters vary within specific tolerance margins and the designer evaluates the adequacy of the system under test according to a chosen metric. Monte Carlo is computationally costly and in recent years more efficient techniques have been developed (see [2-3] and the references therein).

However, the common strategy is still to carry out design optimizations for a nominal case and use stochastic analysis as a posteriori validation. In some cases, yield optimization may take place, again, a posteriori, using the nominal design as a starting point.

In this paper we propose an alternative and less conventional approach. Uncertainties are embedded in the design flow from the early stages and parameter optimization algorithms become uncertainty-aware.

The feasibility and strength of the approach is demonstrated on a flexible differential link including a passive equalizer described in Fig. 1.

## II. Scenario description

Wearable electronics, sometimes referred to as "smart textiles", have been attracting a lot of interest in the past two decades [4-7] There is particular interest for the field in the defense industry [7] but many civilian applications also exist. The idea of creating apparel featuring sensor hubs, processors and various interfaces brings new challenges in terms of signal and power integrity. Transmission lines are characterized by much higher fabrication imperfections than in the case of regular PCBs and suffer greater impact from environment parameters such as temperature and humidity. Bending may also be an issue.

In the framework described above, the example in Fig. 1 is considered. It consists of a differential serial link implemented by conductive copper wires weaved in a textile structure. The link features a differential driver at the near end and its paired receiver at the far end. A passive equalizer is inserted between the driver and the line. It is assumed that the equalizer and drivers are implemented either using conventional electronics or semi-flexible substrates and are affected by uncertainties that are negligible with respect to those affecting the textile transmission line. This assumption is reasonable if one assumes the link to be part of a military body armor, or specialized garment, featuring mixed materials.

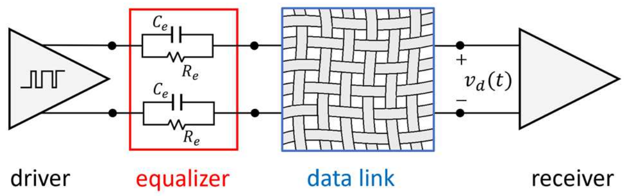


Fig. 1. System under test: digital link for wearable applications.

The objective is to optimize the equalizer by properly choosing the values of the passive components $R_e, C_e$, accounting for various uncertainties of the channel by observing the eye diagram on the receiver side.

## III. Stochastic Optimization Strategy

### *A. General Mathematical Framework*

Consider a cost function $f(q,x)$. If $q$ is a constant and $x$ is a deterministic variable, finding the minimum of $f(q,x)$ is a classic problem. Local minima may in some cases pose a challenge, but it is an issue which has been addressed successfully in many ways. Methods such as genetic algorithms, particle swarm, ant colony and many others are now mainstream.

Now assume that $q$ is a "hidden" random variable representing an uncertain parameter the designer cannot control directly; $x$ remains deterministic at this stage. Along the $x$ axis, multiple realizations of $f(q,x)$ exist as $q$ varies according to a probabilistic law. When conducting optimization in this framework, the very objective of the operation needs to be redefined.

Two scenarios appear obvious to the authors, although arguably many other interesting ones can be imagined. The first scenario is an average optimization which can be expressed as finding $x_{opt}$ such that:

$$x_{opt} = \underset{x \in X}{\mathrm{Argmin}}\left(E_q[f(q,x)]\right)$$

(i.e., the expected values computed for all the possible realizations of the cost function due to the random variable $q$ is minimal). The second is worst-case optimization which can be defined as follows.

Let $g_\alpha^{inf}(x)$ be a new deterministic cost function such that:

$$g_\alpha^{inf}(x) = inf\{g_\alpha(x) : P_q(f(q,x) < g_\alpha(x)) \geq \alpha\}, x \in \mathcal{X} \quad (1)$$

where $P_q$ denotes the probability computed on the stochastic variable $q$ for any values of the deterministic variable $x \in \mathcal{X}$ and $\alpha \in [0,1]$ is a parameter defining the level of confidence. Naively put, if one runs a very large number of Monte Carlo simulations for every value of $x$ and collects the maxima, then these maxima form $g_\alpha^{inf}(x)$ for $\alpha \to 1$. The worst-case optimum is $x_{opt}$ such that:

$$x_{opt} = \underset{x \in \mathcal{X}}{\text{Argmin}} \left( g_\alpha^{inf}(x) \right)$$

Note that the optimization process does not require analytical knowledge of $g_\alpha^{inf}(x)$ but simply a reduced number of pointwise estimations.

Once the criterion has been defined, let us assume that the designer attempts to solve the problem with an iterative algorithm of their choice. As the algorithm progresses, for every value of $x$, the probability distribution of $q$ needs to be explored. This can quickly become computationally challenging.

Moreover, in many practical cases one will have to deal with multivariable cost functions, $f(q_1, .., q_N, x_1, .., x_M)$, yielding a problem that is practically unsolvable by mere brute force and requires the use of computationally efficient methods. A solution is provided in paragraph D. Also, for practical reasons, confidence level $\alpha$ will be equated to 0.99 in this implementation.

Throughout the paper random variables $q_1, q_2, .., q_N$ will be referred to as subject variables. The designer has no direct control over them. Variables $x_1, x_2, .., x_M$ will be referred to as object variables. We consider object variables deterministic at this stage although paths to relax this assumption are being investigated.

### *B. Particular case of textile interconnect line*

For the test case considered in this paper we have two object variables characterizing the equalizer ($x_1 = R_e, x_2 = C_e$) and three subject variables characterizing the link, $q_{1,2,3}$. Coefficient $q_1$ translates the variation of the characteristic impedance according to the tolerances available in [4]. It is caused by imperfections in the fabrication process. Coefficient $q_2$ represents temperature and $q_3$ the variation of the line length.

A metric derived from the eye diagram, known as eye SNR and subsequently denoted as $E_{SNR}$ is used to evaluate the quality of the link, thus playing the role of the cost function $f(q_1, q_2, q_3, R_e, C_e)$ in the previous section. It is defined as:

$$E_{SNR} = \frac{L_1 - L_0}{\sigma_1 + \sigma_0} \quad (2)$$

where $L_1$ and $L_0$ are the logical "1" and respectively "0" NRZ levels and $\sigma_1$ and $\sigma_0$ are the variances of the distributions of all samples interpreted as either "1" or "0". Basically $E_{SNR}$ is a measure of the eye vertical opening weighted by the "smudginess" of the eye. In our implementation the cost function will be $-E_{SNR}$ since the common convention in optimization is to seek minima rather than maxima.

In this conference paper the authors focus on worst-case optimization essentially seeking an equalizer defined by $\{Re, Ce\}$ which guarantees a level of $E_{SNR}$ in the most unfavorable conditions defined by $\{q_1, q_2, q_3\}$.

### *C. Algorithm*

Modern optimization algorithms typically use multiple evaluations of the cost function per iteration in an attempt to converge on the global minimum and avoid being trapped in local minima. The authors have successfully experimented with both particle swarm and genetic algorithms for a variety of examples in the stochastic framework. The results presented in this paper are based on particle swarm optimization (PSO).

PSO is a heuristic optimization method inspired by the behavior of birds [8]. Particles are "virtual agents" meant to explore a manifold space and described by their position and their velocity. Let point $r$ defined by $x_1 = x_{1r}, .., x_M = x_{Mr}$ be the position of a particle. As the algorithm iterates $r$ follows the law:

$$r(k+1) = r(k) + v(k+1) \quad (3)$$

where $v$ denotes the "velocity" of the particle (a displacement in the manifold space in fact) and $k$ denotes the iteration. [8,9]. Once the cost function is evaluated in point $r$, the velocity is updated according to an equation that takes into account the best position occupied by the particle, the overall best position ever occupied by any particle in the swarm, but also includes an "inertia" – a tendency to maintain a trajectory. The particle thus "learns" from its own experience but also from the collective experience of the swarm. In our implementation velocity and inertia are updated according to [9].

The following steps provide a general description of the optimization strategy, without specific implementation details which are beyond the scope of this conference paper.

1. Select the number of particles in the swarm; initialize positions, velocities, inertias. For our test case the term "positions" designates various pairs $\{Re, Ce\}$ (following a Latin hypercube method for example).
2. For every pair $\{Re, Ce\}$ compute a closed form of the probability density function of $f(q_1, q_2, q_3, R_e, C_e)$ using the method described in paragraph D.
3. Use probability density functions to evaluate metric (in our case $g_\alpha^{inf}(x)$).
4. Select best case among the explored pairs $\{Re, Ce\}$.
5. Update inertia, velocities, then positions.
6. If maximum number of iterations or convergence tolerance is reached end. Else go to step 2 and repeat.

Note that what makes the optimization stochastic is the presence of step 2 and the nature of the metric in step 3. If step 2 is suppressed and a deterministic metric is used in step 3 the algorithm reverts to traditional, deterministic PSO.

### *D. Kernel Density Estimation*

Let $\varphi(f_r)$ denote the probability density function of $f(q_1, .., q_N, x_1, .., x_M)$ in a given point $r$ defined by $x_1 =$

$x_{1r}, .., x_M = x_{Mr}$ . The shape of this probability density function depends on random variables $q_1, .., q_N$. It is possible to derive a non-parametric, unsupervised, model of $\varphi(f_r)$ via what is known as kernel density estimation [10] according to the expression:

$$\varphi(f_r) \cong \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{f_r - f_{ri}}{h}\right)$$

where $f_{ri}$ is a realization of $f_r(q_1, .., q_N, x_1, .., x_M)$ for $q_1 = q_{1i}, .., q_N = q_{Ni}$ with $x_1 = x_{1r}, .., x_M = x_{Mr}$ . $K$ is the so-called kernel; the Gaussian function is a common choice of kernel. Finally, $h > 0$ is a smoothing parameter called window or bandwidth.

The main advantage of this approach is that it may be used even when $N$, the number of subject variables, is very large. Also, it is readily available in Matlab as the *ksdensity* function. For the results obtained in the present paper the authors used this standard implementation, albeit with a narrow windowing more adequate for capturing the "tail" of the probability density function.

## IV. Results

The context is the one of the link of Fig. 1. An NRZ encoding is assumed at a data rate of 2 Gbs. For the sake of simplicity linear models (similar to the ones implemented in IBIS-AMI simulations were used to model the buffers). If needed enhanced models of such devices can be adopted [11-13].

The nominal per unit line parameters of the textile interconnect are $L_{nom} = 0.479\ \mu H/m$, $C_{nom} = 25.5\ pF/m$ and $R_{nom} = 5.63\ \Omega/m$. The nominal line length is $l_{nom} = 30\ cm$. Uncertainties are taken into account according to the following relations

$$L = L_{nom} q_1$$

$$C = C_{nom}/q_1$$

$$R = R_{nom} q_1 [1 + (q_2 - 20)0.00393]$$

$$l = l_{nom} q_3$$

with $q_1 \in [0.912, 1.074]$ , $q_2 \in [-5, 35]$ and $q_3 \in [0.85, 1.15]$. One may note that variable $q_1$ impacts line parameters in correlated manner. It translates the variation in impedance due to imperfect manufacturing and matches tolerances indicated in [4]. Variable $q_2$ is simply the ambient temperature and affects the resistance of the copper wire. Variable $q_3$ translates the variation in line length due to garment sizing as well as fabrication (cutting and weaving).

All computations of $E_{SNR}$ used a pre-recorded random stream of 3000 bits and were performed using Matlab function *comm.EyeDiagram*. Future implementations might take advantage of recent advances in fast eye diagram scoring (see [14] and the references therein).

The algorithm presented in section III.C is used to compute an equalizer optimized for a worst-case scenario. The results highlight the difference between deterministic (nominal) optimization and stochastic optimization. A large simulation involving a systematic sweep of parameters $R_e$ and $C_e$ was also conducted in order to validate the optimization results. It consisted in a grid search investigation of 100 $\{Re, Ce\}$ points and 100 Monte Carlo runs per point. A worst-case optimum was in $\{Re, Ce\} = \{90\ \Omega, 2.5\ pF\}$ (see Fig. 3), which turns out to be different from the nominal case optimum illustrated in Fig. 2, $\{Re, Ce\} = \{77\ \Omega, 3.1\ pF\}$.

Figures 2 and 3 illustrate the third iteration of a PSO process with 15 particles in a deterministic case (figure 2) and, respectively, in a stochastic case (figure 3). Yellow contour lines allow the identification of the zones where the value of $E_{SNR}$ is large. The optima obtained by systematic inspection are indicated by the crossing of dashed lines. Crosses mark the position of the particles. The global best obtained by the swarm is indicated by a black dot. Note that the simulations are distinct (i.e. the results of the nominal optimization are not used to initialize the stochastic one).

The figures illustrate that in one case the algorithm is simply faced with a deterministic problem (a single contour map) whereas in the second case multiple realizations of the contour map are possible (only a selection was actually plotted).

As far as the efficiency is concerned, it is important to point out that the stochastic optimization process reaches the optimum after only 3 iterations with 15 particles, each iteration requiring 25 Monte Carlo runs per particle.

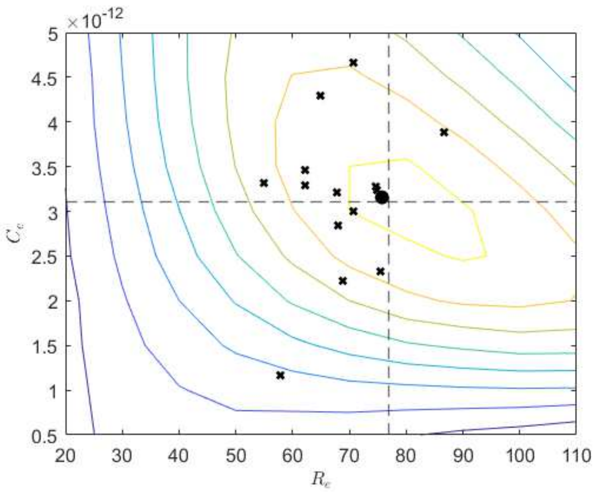


Fig. 2. Traditional optimization (nominal case). Third iteration of PSO.

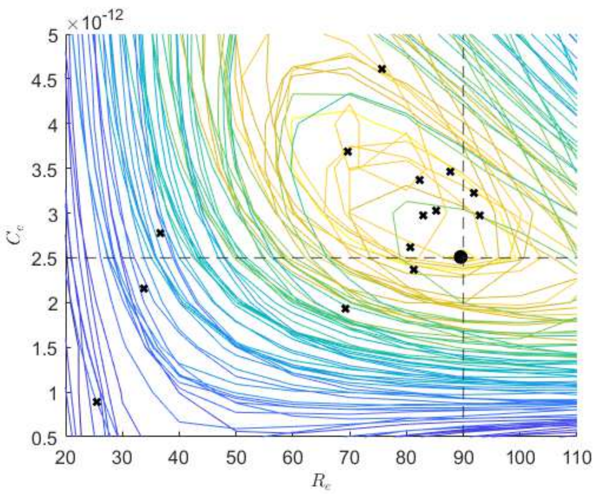


Fig. 3. Stochastic optimization. Third iteration of PSO. Multiple realizations of the cost function are displayed.

Without equalization, the value of $E_{SNR}$ is 2.3 in a worst-case scenario. Introducing the stochastically optimized shunt equalizer increases $E_{SNR}$ to 4.1. The respective eyes are compared in Fig. 4.

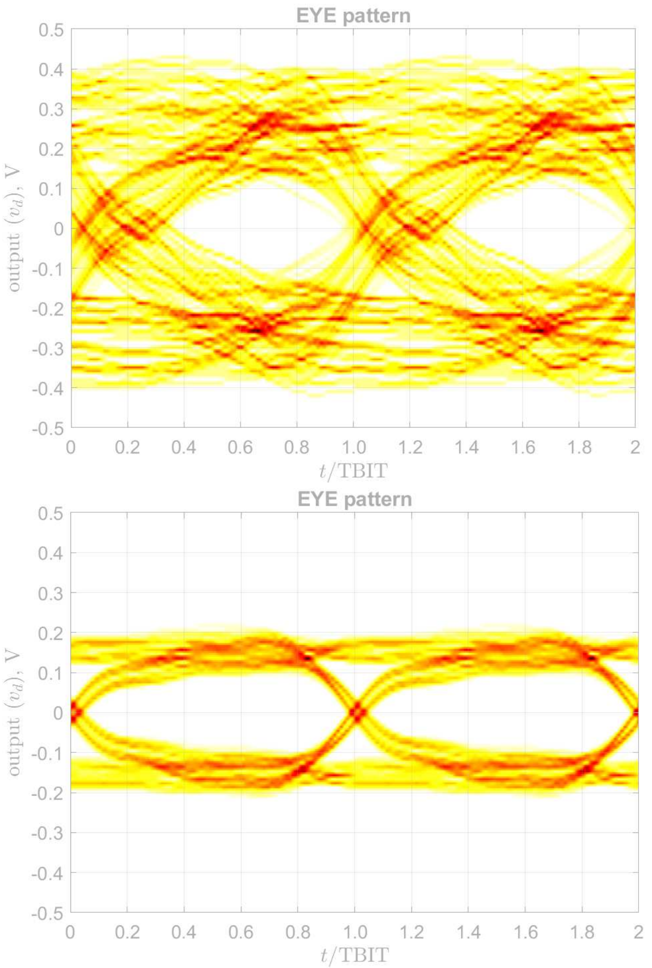


Fig. 4. Eye patterns computed for the data link of Fig. 1 from the received differential voltage $v_d(t)$ in a worst-case scenario without equalizer (upper panel) and with stochastically optimized equalizer (lower panel).

## V. Concluding Remarks

This paper investigates the feasibility of an alternative worst-case optimization scheme for textile interconnects. Specifically, an equalizer was designed by an optimization process which considers system uncertainties from the very beginning. The proposed approach combines an unsupervised methodology, which is used to deal with the stochastic nature of the cost function in an efficient way, and a custom implementation of a particle swarm optimizer. The results clearly demonstrate the effectiveness of the proposed strategy. Uncertainty-aware optimization is an interesting approach which can be a very powerful tool for designers who need to meet ambitious performance boundaries while having to work with high tolerances or with models where deterministic assumptions no longer hold.

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