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Flutter analysis of laminated composite structures using Carrera Unified Formulation

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Abstract

In present work, the flutter analysis of laminated composite structures has been performed using the p - k method in Carrera Unified Formulation (CUF). In the framework of CUF, a hierarchical kinematic finite element model is used to compute the flutter condition of laminated composite plate and box-beam structures as it is very accurate and computationally efficient. The CUF refined theories are based on the Lagrange and Taylor-like cross-sectional displacement fields. In CUF, the order of the expansion can be chosen arbitrary, which is an independent parameter in the formulation. The governing equation is based on the principle of virtual displacement and defined in the form of “fundamental nuclei” using CUF. Theodorsens theory was used to define the aerodynamics loading conditions and the p - k method was used to compute the flutter conditions. Flutter conditions of different types of laminated composite structures with Lagrange and Taylor expansion were performed. A similar model was developed in MSC-Nastran and computed results were compared with literature and CUF model. The results indicate that the analyzed model has good agreement with reference

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and MSC-Nastran. The study suggests that the CUF models can produce accurate results with a low computational cost.

Keywords: Flutter, Unified Formulation, p - k method, Composites

1. Introduction

Flutter is a dynamic instability found in a different kind of flexible structure like air-vehicles, bridges, and blades. It can introduce the catastrophic failure of the structure. It is a devastating reason for failure, and most anxiety subject to the designer. So, the flutter should be considered and thoroughly analyzed during the design process of the aircraft structure. Several authors have analyzed the aeroelastic behavior of aircraft structure since Theodorsen developed a mechanism for the flutter analysis and demonstrated the flutter problem theoretically and experimentally [1, 2]. Later many methods have been developed to solve the flutter problem and improve the solution methodology such as v - g method, k -method and p - k method [3, 4].

Now-a-days, the laminated composite structures are broadly used in the design process of modern air vehicles, bridges and blades, owing to their high structural efficiency, specific strength and potential benefits. The wings can be modeled as the isotropic and thin-walled composite beam [5]. It can also be modeled as the laminated composite beam for the finite element solution to performed the free vibration analysis [6]. For further extension, the laminated composite beam can be considered as the thin- and thick-walled box-beam model for experimental, analytical and numerical solutions in order to determine the elastic stiffness, tailoring effect and torsional warping in finite element approach [7, 8, 9].

The laminated composite structures are very much flexible and aeroelastic behavior of these flexible structures should be analyzed to avoid flutter failure. For the prediction of flutter condition and divergence behavior of these structures, the combination of strip theory with simplified structural box-beam models was analyzed [10]. Various parameters such as aspect ratio, stacking sequences and sweep angle are considered for aeroelastic tailoring [11]. The analytical approach has been used to find the flutter condition for the wing made of the composite material [12]. In the aeroelastic investigation of a structure made with anisotropic composite, directionality property played a complex role in predicting the flutter and divergence behavior [13]. The analytical approach also used for the box-beam model and aeroelas-

33 tic optimization have been studied [14, 15]. The aeroelastic characteristics of
34 laminated plate have been observed for various lamination parameters, aspect
35 ratio, material properties and sweep angle and found different aeroelastic pa-
36 rameters (flutter/divergence) at the same speed of air (free stream) [16]. The
37 Theodorsen theory can be used for quasi-steady and unsteady aerodynamics
38 to compute the flutter. Several authors predicted the flutter condition using
39 the Theodorsen theory along with various approaches (such as strip theory
40 and panel method) [17, 18, 19, 20]. To improve the flutter solution, Hassig
41 [4] proposed the p - k method that can provide better approximation than the
42 other methods [21]. This method also can be used for the smart laminated
43 plate in hygrothermal environment [22].

44 The aim of this work is to develop a finite element model of laminated
45 composite structures (plate and box-beam) using Carrera Unified Formulation
46 (CUF) and to perform flutter analysis by the p - k method. The dif-
47 ferent kinds of complex laminated composite structural models have been
48 considered and those types of the model required proper description of the
49 kinematics, which are defined accurately and computationally efficient man-
50 ner in CUF. Initially, CUF was developed to deal with a plate and shells
51 [23, 24, 25] later, it was extended and to deal with the beam model [26, 27].
52 The CUF has a unique capability such as the order of Taylor expansion can
53 be chosen arbitrarily to define the cross-sectional displacement fields. These
54 capabilities give us freedom to choose the order of structural models with-
55 out making changes in matrices or equations and without the need of any
56 ad hoc formulation, it can deal with arbitrary geometry, different material
57 characteristics, and boundary conditions. This present approach is compu-
58 tationally efficient for different kinds of structures such as thin-walled [28],
59 laminated and sandwich structure [29]. It can be used for rotating blades
60 [30] and spinning blades [31] in the rotor dynamics fields. In the field of fluid
61 interaction, CUF has been used for vortex [32, 33, 34] and double lattice
62 method [35, 36], and piston theory [37, 38] for supersonic flows. Recently,
63 [39] the coupling of CUF with the Theodorsen theory has been done to pre-
64 dict the flutter condition by the v-g method. In this work, the p - k method
65 has been implemented in CUF framework to analyze the flutter condition.

66 2. Structural Model: Carrera Unified Formulation

67 The present structural model formulated within the popular Carrera Uni-
68 fied Formulation (CUF) framework [40, 41, 42, 43, 44]. In accordance with

69 CUF, u is defined as the displacement fields which can be stated as a com-
 70 bination of the $F_\tau(x, z)$ (function of cross-section) and $u_\tau(y)$ displacement
 71 vector (generalized), and can unify as:

$$72 \quad u(x, y, z, t) = F_\tau(x, z)u_\tau(y), \quad \tau = 1, 2, \dots, T \quad (1)$$

73 where subscript τ and T stands for summation (i.e. Einstein notation) and
 74 the number of terms in the expansion, respectively. The details of Taylor
 75 expansion (TE) and displacement fields are given in Table 1.

Table 1: Compact form and displacement fields of Taylor expansion.

Order	T	F_τ	Displacement fields
0	1	$F_1 = 1$	Second-order (TE2)
1	3	$F_2 = x \ F_3 = z$	$u_x = u_{x1} + x \ u_{x2} + z \ u_{x3} + x^2 \ u_{x4} + xz \ u_{x5} + z^2 \ u_{x6}$
2	6	$F_4 = x^2 \ F_5 = xz \ F_6 = z^2$	$u_y = u_{y1} + x \ u_{y2} + z \ u_{y3} + x^2 \ u_{y4} + xz \ u_{y5} + z^2 \ u_{y6}$
3	10	$F_7 = x^3 \ F_8 = x^2z \ F_9 = xz^2 \ F_{10} = z^3$	$u_z = u_{z1} + x \ u_{z2} + z \ u_{z3} + x^2 \ u_{z4} + xz \ u_{z5} + z^2 \ u_{z6}$
\vdots	\vdots	\vdots	
N	$(N+1) \ (N+2) / 2$	$F_{(N+1)(N+2)/2} = x^N \ F_{(N+1)(N+2)/2} = z^N$	

76 Two types of CUF models are used in this present work: Taylor expansion
 77 (TE) and Lagrange expansion (LE). The TE model is refined by increasing
 78 the order of expansion from second-order (TE2) to fourth-order (TE4). Sim-
 79 ilarly, LE is refined by adding the elements in the cross-section, i.e., one
 80 nine-noded (1L9) and two nine-noded (2L9) along the chord. The details
 81 of the displacement fields of the LE model can be found in literature [44].
 82 Cross-sectional elements are extended along the y-axis with four-noded beam
 83 elements (B4) to create the structure (Figure 1).

84 3. Aeroelastic Model: Steady and Unsteady Theories

85 A complete solution of a thin airfoil in the incompressible fluid, which is
 86 associated with the harmonic oscillations laterally, Theodorsen's presented
 87 the lift distribution function by going beyond the quasi-steady model. He
 88 considered a control surface of a plate that was assumed as flat, which can
 89 rotate with regard to an axis at distance $x = b_c a$ via the angle of attack $\Lambda(t)$
 90 and move vertically $h(t)$. Theodorsen's unsteady lift prediction expression is

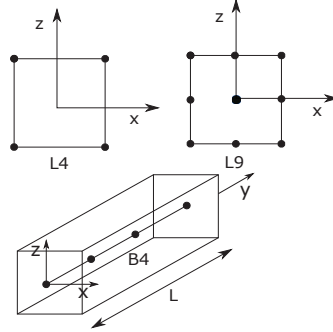


Figure 1: Four- and nine-nodes Lagrange elements and four-nodes beam element.

91 [45]:

$$L_a = \pi\rho_a b_c^2 \left[\ddot{h} + V_\infty \dot{\alpha} - b_c a \ddot{\alpha} \right] + 2\pi\rho_a b_c C(k) V_\infty \left[\dot{h} + V_\infty \alpha + b_c \left(\frac{1}{2} - a \right) \dot{\alpha} \right] \quad (2)$$

92 where ρ_a is air density, b_c is semi-chord, V_∞ is free-stream velocity and
 93 Theodorsen function $C(k)$ is the relating the reduced frequency $k = \frac{\omega b_c}{V_\infty}$.
 94 The position of the axis of rotation with regard to the center of section denoted as a , which depends on the load applied, used lamination scheme and support condition. The first term of equation (2) is the non-circulatory (i.e., added mass term), whereas the second term is the circulatory. The second term is called “quasi-steady” model when Theodorsen function $\{C(k) = 1\}$ and “unsteady aerodynamics” when Theodorsen function is a complex function $C(k) = F(k) + iG(k)$. The simplified expression of $C(k)$ has been presented by Jones [46] by considering the solution of Wagners indicial (1925), which was concerned to exponential approximation:

$$104 \quad C(k) \equiv 1 - \frac{0.165}{1 - \left(\frac{0.0455}{k} \right) i} - \frac{0.335}{1 - \left(\frac{0.3}{k} \right) i} \quad (3)$$

105 The first term of equation (2) can be neglected due to mass properties of the
 106 structure are small and it is related to single and double differentiated terms
 107 and can be written as:

$$108 \quad L_a \equiv 2\pi\rho_a V_\infty b_c \left[\dot{h} + V_\infty \alpha \right] \quad (4)$$

109 For correcting the coefficient of C_L (sectional lift) associated with an aspect
 110 ratio of the wing (AR_w) and sweep angle (Λ) effects, by using Diederich's
 111 approximation, the expression becomes:

$$112 \quad C_{l\alpha} = \frac{dC_L}{d\alpha} = \frac{\pi AR_w}{\pi AR_w + C_{l\alpha 0} \cos(\Lambda)} C_{l\alpha 0} \cos(\Lambda) \quad (5)$$

113 where, $(AR_w) = \frac{2 L_w}{C_m}$, (L_w = wing length and C_m = mean chord) [Figure 2],
 114 and $C_{l\alpha 0}$ is stated as slope of lift-curve ($\approx 2\pi$). In order to reproduce the
 115 pressure distribution on atop of airfoil, which is thin, slightly inclined and
 116 uncambered. For capturing the effect of pressure distribution on concerned
 117 geometry of the wing model, the quantity of $b_c \pi$ has been approximated by
 118 $\int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx$, and equations (2) and (4) can be rewritten as:

$$119 \quad L_a = \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx \rho_a V_\infty c(k) \left[\dot{h} + V_\infty \alpha + b_c \left(\frac{1}{2} - a \right) \dot{\alpha} \right] \quad (6)$$

$$121 \quad L_a \equiv \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx \rho_a V_\infty c(k) [\dot{h} + V_\infty \alpha] \quad (7)$$

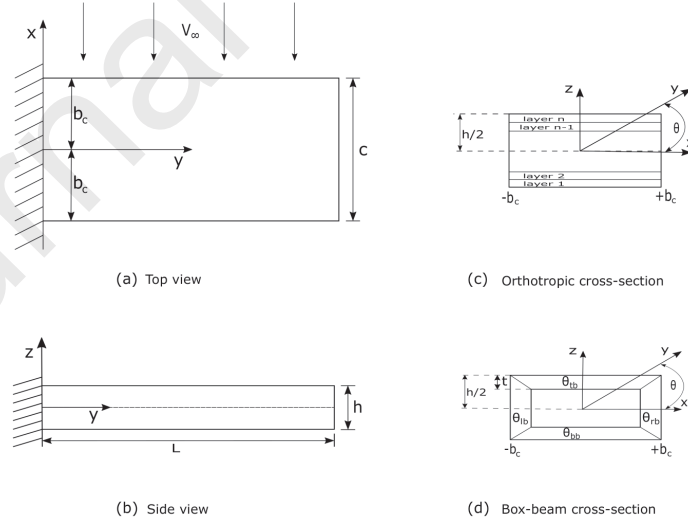


Figure 2: Sketch of beam model and coordinates reference system.

122

123 4. Formulation of equation of motion: CUF framework

124 The equation of motion for aeroelastic model has been derived using the
125 principle of virtual displacement (PVD), stated as below:

$$126 \quad \delta L_{int} = \delta L_{ext} + \delta L_{ine} \quad (8)$$

127 where δL_{int} (internal work), δL_{ext} (external work), δL_{ine} (inertial work) and
128 δ stands for virtual variation.

129 The strain and kinetic energy can be write in the form of fundamental nuclei
130 as follow:

$$131 \quad \delta L_{int} = \delta q_{\tau i}^T K_s^{ij\tau s} q_{sj}, \quad (9)$$

$$132 \quad \delta L_{ine} = \delta q_{\tau i}^T M_s^{ij\tau s} \ddot{q}_{sj} \quad (10)$$

134 where $K_s^{ij\tau s}$ and $M_s^{ij\tau s}$ defined as a fundamental nucleus for the stiffness and
135 mass matrix, respectively and components are found in literature [47].

136 The generalized form of the work produced by the lift is:

$$137 \quad \delta L_{ext} = \int_y \int_x \delta u_z(x, y, z_{top}) L_a(x, y, z_{top}) dx dy \quad (11)$$

138 where z_{top} is associated with upper coordinate of a cross-section in z and
139 L is lift given in equation (7). In Carrera Unified Formulation framework,
140 external work can be written as:

$$141 \quad \delta L_{ext} = \delta q_{\tau i}^T D_L^{ij\tau s} \dot{q}_{sj} + \delta q_{\tau i}^T K_L^{ij\tau s} q_{sj} \quad (12)$$

142 where $D_L^{ij\tau s}$ (damping) and $K_L^{ij\tau s}$ (stiffness) are contributions due to the
143 aerodynamic force, which are defined in fundamental nuclei form:

$$144 \quad D_L^{ij\tau s} = cost I_l^{ij} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} F_{\tau}(x, z_{top}) I_L F_s(x, z_{top}) dx \quad (13)$$

$$K_L^{ij\tau s} = cost I_l^{ij} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} F_{\tau,x}(x, z_{top}) I_L F_s(x, z_{top}) dx$$

145 where

$$146 \quad cost = \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \rho_a V_{\infty} c(k) \quad (14)$$

147

148

$$I_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15)$$

149

150

$$I_l^{ij} = \int_l N_i N_j dy \quad (16)$$

151 After assembly of global finite element matrices by assuming the periodic
152 solution $a_q = \bar{a}_q e^{i\omega t}$ in the form of quadratic eigenvalue problem (QEP) and
153 transforming into classical liner system of $2 \times R$ can we write as:

$$\begin{cases} [M_s]\ddot{a}_q + [D_L]\dot{a}_q + ([K_s] + [K_L])a_q = 0 \\ -\dot{a}_q + \dot{a}_q = 0 \end{cases} \quad (17)$$

155 and by presenting:

156

$$a = \begin{Bmatrix} a_q \\ \dot{a}_q \end{Bmatrix}, \quad \dot{a} = \begin{Bmatrix} \dot{a}_q \\ \ddot{a}_q \end{Bmatrix} \quad (18)$$

157 equation of motion assumed in the following form:

$$\frac{R}{T} - \frac{1}{i\omega} I = 0, \quad (19)$$

159 where

160

$$I^{-1}R = \begin{bmatrix} (K + K_L)^{-1}D_L & (K + K_L)^{-1}M \\ -I & 0 \end{bmatrix} \quad (20)$$

161 An iterative procedure was required to calculate the flutter condition because
162 of aerodynamic contribution matrices, which have a dependency on the re-
163 duced frequency (k). The p - k method proposed by Hassig [4] is used for
164 solving the flutter problem. The General form of a proposed equation given
165 as:

$$\left[\left(\frac{V_\infty}{c_m} \right)^2 [M_s]p^2 + [K_s] - \frac{1}{2}\rho V_\infty^2 [A(p)] \right] \{a_q\} = 0 \quad (21)$$

167 where, $[A(p)] =$ unsteady aerodynamic forces. The Simplified fundamental
168 equation for modal flutter analysis presented in [48]:

$$\left[[[M_s]p^2 + \left([B_s] - \frac{1}{4}\rho c V_\infty \frac{[Q_a^I]}{k} \right) p + [K_s] - \frac{1}{2}\rho V_\infty^2 [Q_a^R] \right] \{a_q\} = 0 \quad (22)$$

169

170 where, $[Q_a^I], [Q_a^R]$ are imaginary and real part of aerodynamic force matrix, c
 171 is reference length, p is eigenvalue, k reduced frequency and $[B_s] = 0$ because
 172 structural damping is not considered here. The circular frequency ω and
 173 reduced frequency k are not independent since $k = \frac{\omega b_c}{V_\infty}$,

$$174 \quad k = \frac{b}{V_\infty} im(p) \quad (23)$$

175 and

$$176 \quad p = \gamma\omega + i\omega \quad (24)$$

177 where, γ is transient decay rate coefficient ($2\gamma = g$).

178 5. Results And Discussion

179 Based on the above formulation, a FORTRAN finite element model is
 180 developed to analyze the flutter condition of laminated composite structures
 181 (plate and box-beam) model. Carrera Unified Formulation (CUF) has a
 182 hierarchical finite element model, which is very accurate and economically
 183 efficient. In CUF, we can choose any order of Taylor's expansion with-
 184 out changing in the formulation. Plate elements (CQUAD4) based on the
 185 Mindlin-Reissner shell theory are used to develop the MSC-Nastran model
 186 (i.e., isotropic, orthotropic and box-beam) for the comparison of results ob-
 187 tained by CUF. Numerical examples of isotropic and laminated composite
 188 structures are presented in subsequent sections. The flutter condition of both
 189 models is computed by using the p - k method and flutter condition (flutter ve-
 190 locity) is defined at the point where the real part of the eigenvalue (damping)
 191 is null.

192 5.1. Isotropic plate

193 The first numerical example is based on the work of Petrolo [49]. Where
 194 a cantilever straight plate with the geometrical data: length $L = 0.305$ m,
 195 thickness $t = 0.001$ m, and chord $c = 0.076$ m, having isotropic material prop-
 196 erties: $E = 73.8$ GPa, $\nu = 0.3$ and $\rho = 2768$ Kg/m³ has been analyzed. The
 197 flutter analysis of the plate model using CUF based finite element method
 198 (FEM) and MSC-Nastran model with sweep angle $\lambda = 0^\circ$ has been performed.
 199 The different orders of Taylor expansion (TE) and Lagrange expansion (LE)
 200 with nine-noded (L9) cross-sectional elements have been considered for CUF

201 model to compute the results. The first five modes (i.e., bending and tor-
 202 sion) of natural frequency (f_n), flutter velocity (V_F), and flutter frequency
 203 (f_F) of CUF and MSC-Nastran model are reported in Table 2. MSC-Nastran
 204 model was created using the plate elements (CQUAD4) with 8x20 grids. The
 205 computed flutter velocity and frequency using the p - k method are compared
 206 with MSC-Nastran and results obtained using the doubled-lattice method of
 207 literature. It is observed that the modal frequencies are matching well with
 208 MSC-Nastran. Also, observed that modal frequencies and flutter conditions
 209 of third-order Taylor expansion (TE3), fourth-order (TE4), and two nine-
 210 noded LE model are more accurate for the present isotropic plate and the
 desired result is achieved with a very low computational cost.

Table 2: Natural frequencies (Hz) and flutter conditions.

	Model	Mesh	DOFs	f_{n_1}	f_{n_2}	f_{n_3}	f_{n_4}	f_{n_5}	V_F (m/s)	f_F (Hz)
Reference [49]	TE4	20B4		9.40	58.84	74.21	165.10	230.81	72.75	59.77
Present	TE2	12B4	666	9.14	57.19	73.71	160.60	227.94	71.60	42.05
	TE3	12B4	1110	9.14	57.16	73.70	160.52	227.77	69.80	39.15
	TE4	12B4	1665	9.14	57.17	73.72	160.54	227.97	74.90	40.47
	LE (1L9)	12B4	999	9.14	57.17	73.70	160.53	227.74	70.70	39.41
	LE (2L9)	12B4	1665	9.14	57.17	73.70	160.53	227.74	70.70	39.41
	MSC-Nastran	8x20	945	9.09	56.74	72.12	158.95	222.07	66.51	39.52

211

212 5.2. Orthotropic plate

213 A six-layered laminated composite plate is considered in this section with
 214 symmetric laminate $[30_2/0]_s$. The cantilever plate with length ($L = 0.305$
 215 m), chord ($c = 0.0762$ m) and thickness ($t = 0.000804$ m) having material
 216 properties $E_L = 98.00$ GPa, $E_T = 7.90$ GPa, $G_{LT} = 5.60$ GPa, $\nu = 0.280$, and
 217 $\rho = 1520.00$ kg/m³ has been analyzed to find flutter conditions. The different
 218 orders of Taylor expansions, e.g., second-order (TE2), third-order (TE3) and
 219 fourth-order (TE4) used to describe the cross-section of a plate. Similarly,
 220 two types of Lagrange elements, one nine-noded (1L9) and two nine-noded
 221 (2L9), along with the chord used to defined the cross-section of a single layer.
 222 The first five modes of natural frequency and flutter condition for the $[30_2/0]_s$
 223 plate are reported in Table 3. The mode shapes of bending and torsion for
 224 LE (2L9) model are shown in Figure 3. Computed results indicate that TE3,

225 TE4 and 2L9 produce more accurate results as compared to TE2 and 1L9.
 226 The first five modes of frequencies and damping with respect to functions
 227 of speed for the present CUF (TE3) and MSC-Nastran model are plots in
 228 Figure 4. The various stacking sequence $[0_2/90]_s$, $[45/-45/0]_s$ and $[45_2/0]_s$
 229 are used to compute flutter conditions for TE3 and 2L9 models, which are
 230 reported in Table 4. Here results show that different stacking sequences can
 influence both natural frequency and flutter conditions.

Table 3: Natural frequencies (Hz) and flutter conditions for a laminated plate (6-layers) $[30_2/0]_s$.

	Model	Mesh	DOFs	f_{n_1}	f_{n_2}	f_{n_3}	f_{n_4}	f_{n_5}	V_F (m/s)	f_F (Hz)
Reference [36]	TE4	15B4	2070	6.05	35.91	56.51	100.03	172.23	25.86	26.66
Present	TE2	10B4	558	6.34	37.91	69.43	107.43	213.96	31.90	28.38
	TE3	10B4	930	6.31	37.49	57.73	104.65	178.90	27.80	27.90
	TE4	10B4	1395	6.21	37.25	56.94	103.76	173.82	27.60	27.22
	LE (1L9)	10B4	3627	6.31	37.52	57.77	104.72	179.11	29.50	27.59
	LE (2L9)	10B4	6045	6.30	37.33	57.11	104.04	174.68	28.10	27.10
	MSC-Nastran	10x30	1705	6.26	37.05	55.97	103.15	170.42	25.40	27.39

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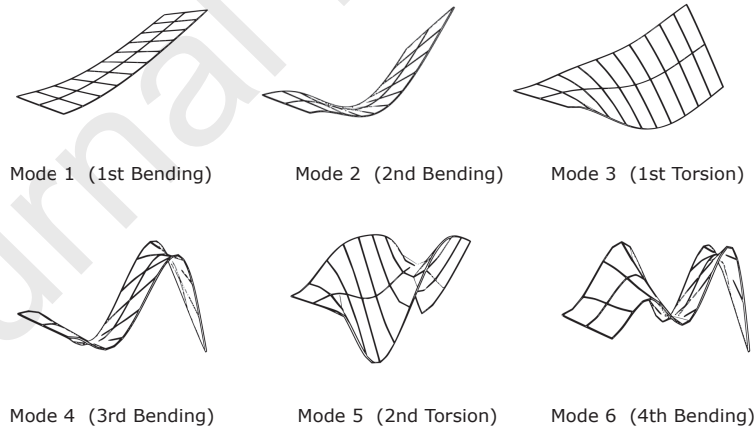


Figure 3: Mode shapes of six-layer plate $[30_2/0]_s$ for LE (2L9) model using CUF.

232 Another plate model was considered from the Kameyama *et al.* [16] work
 233 having eight-layer symmetric laminates $[-22.5/67.5/22.5/-67.5]_s$ with a total
 234 thickness of the laminate 0.804 mm. The thicknesses of the plies were 0.037,

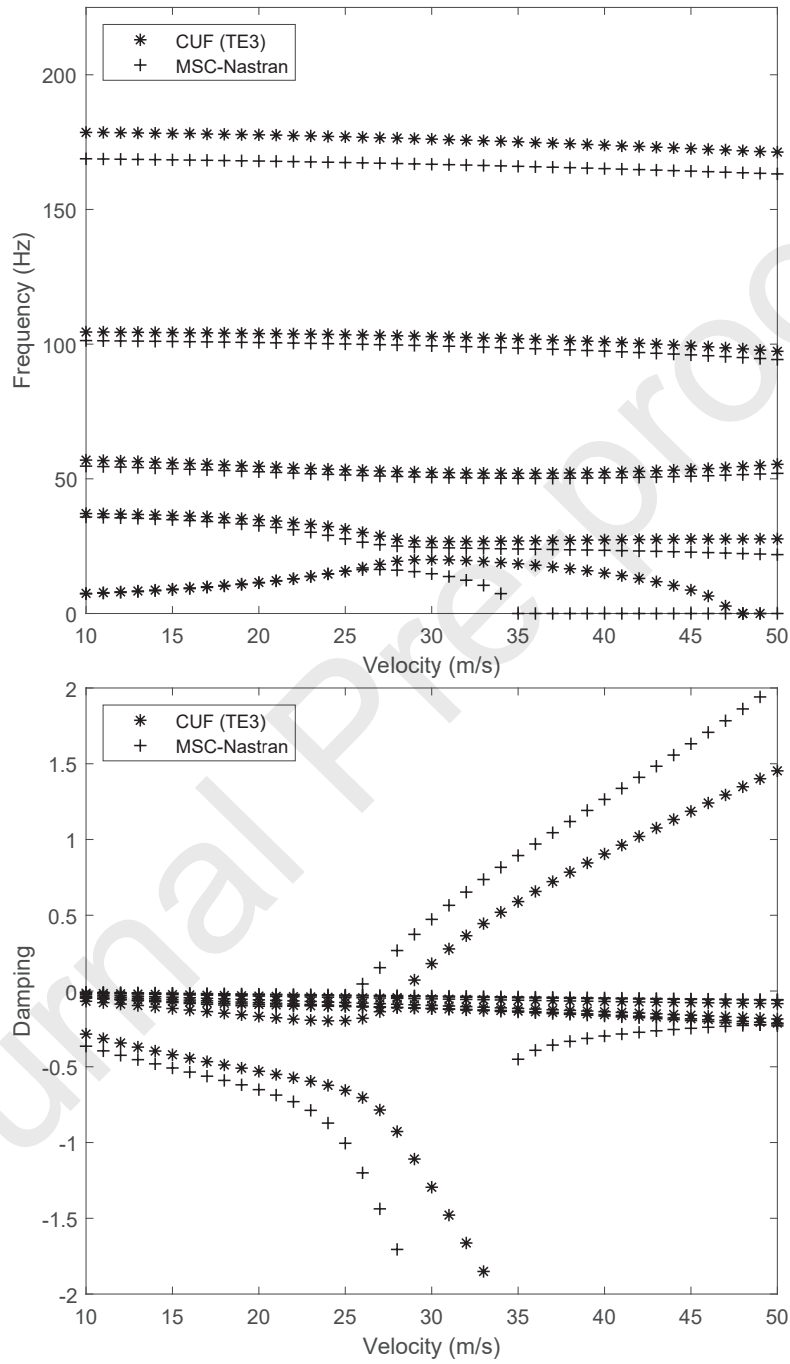


Figure 4: Flutter diagram of orthotropic six-layer plate $[30_2/0]_s$.

Table 4: Natural frequencies (Hz) and flutter conditions for 6-layers plate.

Lamination	Model	Present						Reference		
		f_{n_1}	f_{n_2}	f_{n_3}	f_{n_4}	f_{n_5}	V_F (m/s)	f_F (Hz)	[16] (CLT)	[50] (Exp)
[0 ₂ /90] _s	TE3	11.04	39.55	69.16	133.08	193.62	23.20	27.33	23.0	25
	LE (2L9)	11.04	39.51	69.16	132.50	193.62	22.90	27.23		
[45/−45/0] _s	TE3	4.88	30.22	50.74	84.59	157.83	47.30	31.48	40.1	>32
	LE (2L9)	4.88	30.10	49.93	84.09	152.84	46.80	31.44		
[45 ₂ /0] _s	TE3	5.78	36.60	69.08	103.38	207.51	29.60	25.04	27.5	28
	LE (2L9)	5.78	35.89	68.98	102.78	206.20	29.00	24.93		
[30 ₂ /0] _s	TE3	6.31	37.49	57.73	104.65	178.90	27.60	27.22	27.1	27
	LE (2L9)	6.30	37.33	57.11	104.04	174.68	28.10	27.10		

235 0.048, 0.064 and 0.253 mm, respectively. The first five modes associated
 236 with bending and torsion of natural frequency and flutter velocities for CUF
 237 model and MSC-Nastran are reported in Table 5. The present results have
 238 a good agreement with reference. Similar trends observed like the previous
 239 6-layer plate models. All the TE2, TE3, TE4, 1L9, and 2L9 are suitable, but
 240 TE4 and 2L9 are most accurate. Flutter velocity has been found in order
 241 with reference results obtained by the generic algorithm for flutter solution,
 whereas in the present analysis p - k method has been used.

Table 5: Natural frequencies (Hz) and flutter condition (velocity [m/s]) for 8-layers plate.

	Model	Mesh	DOFs	f_{n_1}	f_{n_2}	f_{n_3}	f_{n_4}	f_{n_5}	V_f (m/s)
Reference [16]	CLT			7.2	45.4	59.1	127.7	182.3	38.8
Present	TE2	10B4	558	7.3	46.2	59.1	129.8	182.7	38.60
	TE3	10B4	930	7.2	45.1	59.0	126.8	182.5	37.20
	TE4	10B4	1395	7.2	45.1	59.0	126.7	182.3	36.30
	LE (1L9)	10B4	4743	7.2	45.1	59.0	126.8	182.5	38.90
	LE (2L9)	10B4	7905	7.2	45.1	59.0	126.8	182.2	36.80
	MSC-Nastran	10x30	1705	7.2	45.0	58.1	126.2	179.0	35.01

242

243 5.3. Box-beam structure

244 A prismatic thin-walled beam wing model considered for flutter anal-
 245 ysis as the main frame of the wing was a box-type structure. A similar

246 type of analysis on the box-beam was discussed in the introduction section.
 247 The dimensions and material properties associated with the composite beam
 248 (box-beam) structure are listed in Table 6. The box-beam configuration is
 249 shown in Figure 2, where the lamination sequence of four walls, i.e., bottom
 250 wall, right wall, top wall and left wall are defined as $[\theta_{bb}/\theta_{rb}/\theta_{tb}/\theta_{lb}]$. The
 251 arbitrary lamination sequence $[\theta/0.1\theta/2\theta/10\theta]$ of the walls is considered to
 252 avoid the bending-torsion coupling of the structure. In previous sections, it
 253 is already observed that TE4 and 2L9 are the more accurate model and the
 254 same model in general, is taken here for flutter analysis. In the first case
 255 $\theta = 30^\circ$ is considered and the flutter conditions have been computed using
 256 CUF and MSC-Nastran model. The first five modes of natural frequencies
 257 and flutter conditions (frequency and velocity) are reported in Table 7. The
 258 present TE4 and 8L9 results are matching well with the MSC-Nastran. First
 259 four modes of frequencies and damping with respect to the function of speed
 260 for present TE4 and MSC-Nastran models are shown in Figure 5. The fre-
 261 quency and damping variation with velocity also matching well but with a
 262 low computational cost. So, it can be commented that the present models
 263 are efficient and accurate and CUF models can be used for flutter analysis
 264 of laminated composite box-beam structure. Now, the detailed flutter anal-
 265 ysis of various composite box-beam model has been performed by the p - k
 266 method. Flutter velocities are computed with variations in θ for different
 267 cross-sectional elements and are listed in Table 8. The polar plot for these
 268 models is also shown in Figure 6. The polar plot shows that the flutter veloc-
 269 ity is increasing between $\theta = 0^\circ$ to $\theta = 30^\circ$ for LE but in TE model case up
 270 to $\theta = 60^\circ$. The flutter velocities are reaching a minimum for all the models
 271 at $\theta = 90^\circ$. The lower order TE models (i.e., TE2 and TE3) are less effec-
 272 tive in predicting flutter conditions compared to the LE models. In addition,
 273 the variations in the computed flutter velocity of the TE model indicate that
 274 the higher-order model (TE4) is more effective than lower-order models in
 275 determining flutter conditions. It can be observed from results that the TE4
 276 and 8L9 models predict the most accurate flutter conditions as compared to
 277 other models.

278 6. Conclusion

279 Flutter condition of plates and box-beam structures made of isotropic and
 280 orthotropic materials has been analyzed in this work. Carrera Unified Formu-
 281 lation (CUF) has been used to define the structural model, one-dimensional

Table 6: Material and dimensional parameter for composite box beam structure.

Material Properties	GPa, kg/m ³
E_{11}	206.8
E_{22}	5.17
$G_{23} = G_{31}$	2.55
G_{12}	3.10
ν_{12}	0.25
ρ	1528.5
Dimensions	m
chord (c)	0.5
height (h)	$c / 15$
thickness of wall (t)	$c / 150$
length (L)	3.5

Table 7: Natural frequencies (Hz) and flutter conditions for box-beam ($\theta = 30^\circ$).

Model	Mesh	DOFs	f_{n_1}	f_{n_2}	f_{n_3}	f_{n_4}	f_{n_5}	V_F (m/s)	f_F (Hz)
TE4	10B4	1395	1.80	11.16	23.84	28.46	30.83	105.09	17.52
LE (8L9)	10B4	4464	1.75	10.62	21.55	25.40	28.81	109.10	15.84
MSC-Nastran	5x30(Flange) 3x30(Web)	2480	1.74	10.41	20.73	24.38	26.03	95.09	14.82

Table 8: Flutter condition {velocity (m/s)} for box-beam model.

Model	DOFs	$\theta=0^\circ$	$\theta=30^\circ$	$\theta=60^\circ$	$\theta=90^\circ$	$\theta=120^\circ$	$\theta=150^\circ$	$\theta=180^\circ$
TE2	558	105.4	120.3	122.8	98.9	121.8	118.9	104.5
TE3	930	100.6	114.7	116.2	89.4	105.0	113.0	100.7
TE4	1395	93.0	105.0	109.9	90.2	106.6	110.7	96.6
LE (4L9)	1584	109.5	115.6	110.8	90.8	112.3	114.8	111.0
LE (8L9)	4464	95.4	109.1	100.2	91.9	95.9	106.1	100.5

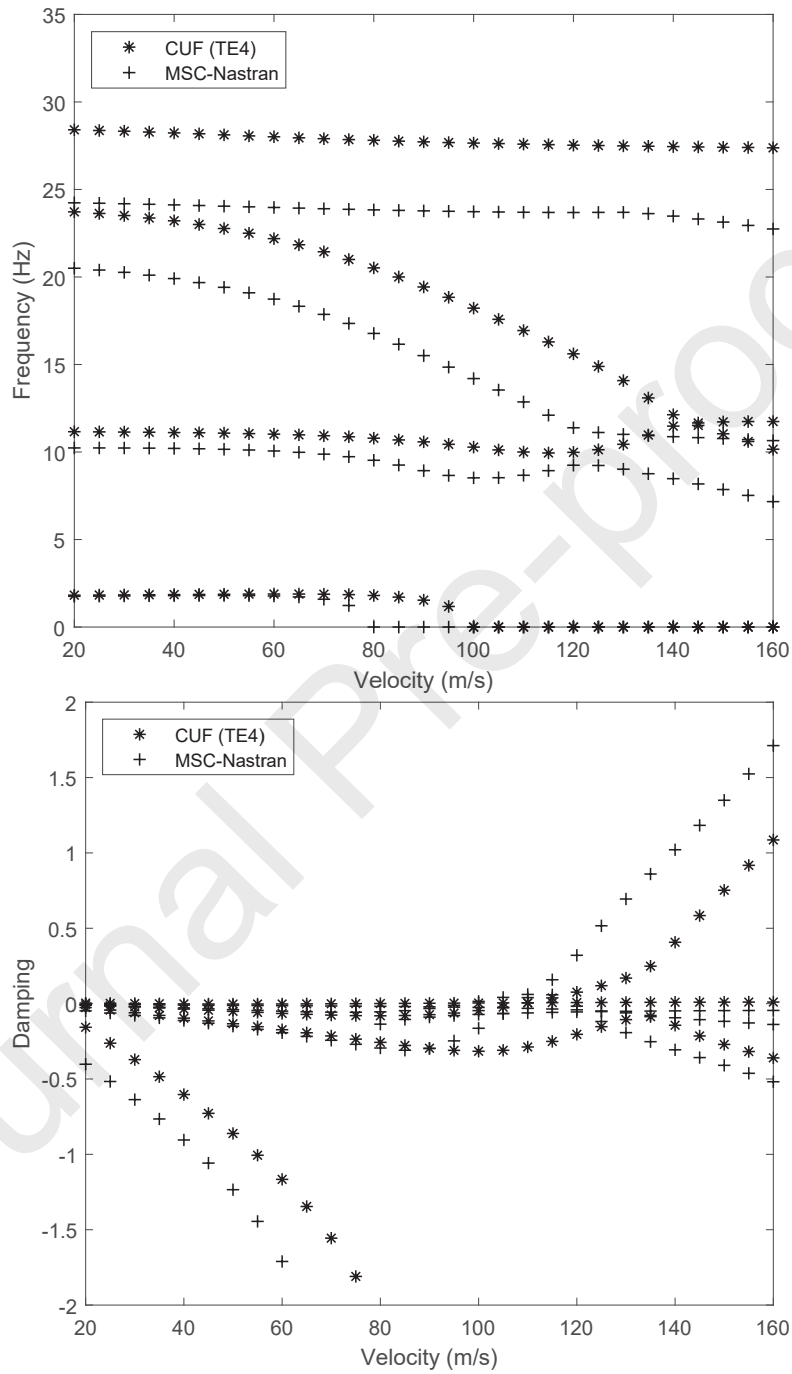


Figure 5: Flutter diagram of box-beam with $\theta = 30^\circ$.

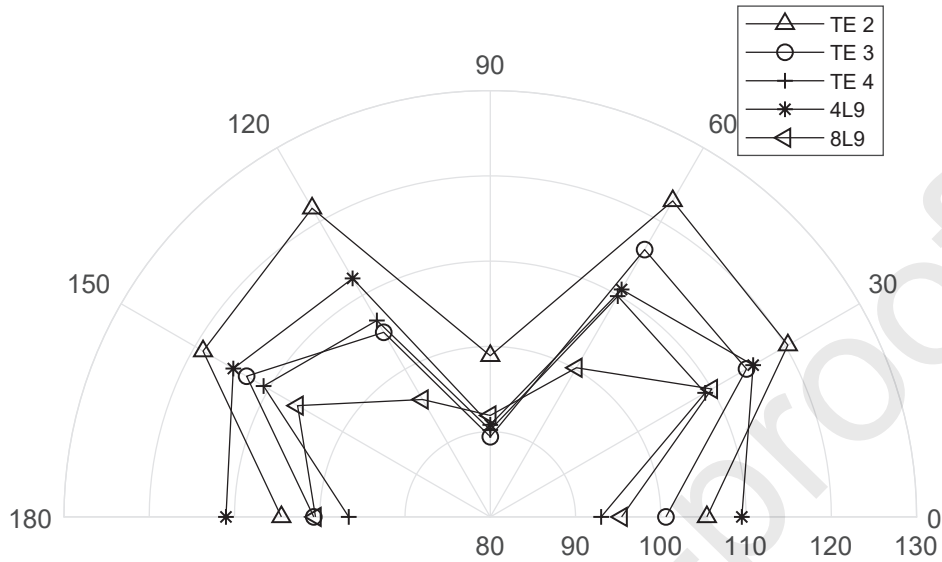


Figure 6: Polar plot of flutter velocities for angle ply of box-beam.

282 models exploit with Lagrange- and Taylor-like expansions to describe the
 283 displacement field of cross-section accurately. The order of Taylor expansion
 284 can be chosen arbitrarily in input; it is an independent parameter of the
 285 formulation. The governing equation is derived using the principle of virtual
 286 displacement and aerodynamic loading conditions are defined in the form of
 287 fundamental nuclei in CUF. Furthermore, the p - k method has been imple-
 288 mented and flutter conditions are computed for the unsteady aerodynamics
 289 in CUF framework and results have been compared with literature and MSC-
 290 Nastran; they are in good agreement. The computed results indicate that for
 291 isotropic case, all the TE and LE model can produce good results. Similarly,
 292 for the orthotropic plate the TE3, TE4, and 2L9 are preferable to find the
 293 flutter conditions. For the composite box-beam, TE4 and 8L9 are the most
 294 accurate models. It is also observed that the present TE and LE aeroelastic
 295 1D models are very accurate, efficient with a low computational cost. In
 296 the future investigation, aeroelastic analysis of the fixed and rotating-beam
 297 models with a quasi-steady and unsteady aerodynamics and gust analysis
 298 can be performed in the CUF framework.

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304 **References**

- 305 [1] T. Theodorsen, G. I. E., Mechanism of flutter, a theoretical and exper-
306 imental investigation of the flutter problem., NACA Report 689, 1940.
- 307 [2] T. Theodorsen, G. I. E., General theory of aerodynamic instability and
308 the mechanics of flutter., NACA Report 496, 1949.
- 309 [3] D. H. Hodges, G. A. Pierce, Introduction to structural dynamics and
310 aeroelasticity, Vol. 15, cambridge university press, 2011.
- 311 [4] H. J. Hassig, An approximate true damping solution of the flutter equa-
312 tion by determinant iteration., J. Aircraft 8 (11) (1971) 885–889.
- 313 [5] L. W. Rehfield, A. R. Atigan, D. H. Hodges, Nonclassical behavior of
314 thinwalled composite beams with closed cross sections., J. Amer. Heli-
315 copter Soc. 35 (2) (1990) 42–50(9).
- 316 [6] X. X. Wu, C. T. Sun, Vibration analysis of laminated composite thin-
317 walled beams using finite elements, AIAA Journal 29 (5) (1991) 736–742.
- 318 [7] E. Smith, I. Chopra, Formulation and evaluation of an analytical model
319 for composite box-beams, in: 31st Structures, Structural Dynamics and
320 Materials Conference, 1990.
- 321 [8] C. Kim, S. R. White, Thick-walled composite beam theory including
322 3-d elastic effects and torsional warping, Int. J. Solids Struct. 34 (31)
323 (1997) 4237 – 4259.
- 324 [9] V. Berdichevsky, E. Armanios, A. Badir, Theory of anisotropic thin-
325 walled closed-cross-section beams, Compos. Eng. 2 (5) (1992) 411 – 432.
- 326 [10] T. A. Weisshaar, Aeroelastic tailoring of forward swept composite wings
327 divergence, Journal of Aircraft 17 (6) (1980) 442–448.

- 328 [11] T. A. Weisshaar, Aeroelastic tailoring of forward swept composite wings,
329 Journal of Aircraft 18 (8) (1981) 669–676.
- 330 [12] I. Lottati, Flutter and divergence aeroelastic characteristics for compos-
331 ite forward swept cantilevered wing, Journal of Aircraft 22 (11) (1985)
332 1001–1007.
- 333 [13] Z. Qin, L. Librescu, Aeroelastic instability of aircraft wings modelled as
334 anisotropic composite thin-walled beams in incompressible flow, Journal
335 of Fluids and Structures 18 (1) (2003) 43 – 61.
- 336 [14] S. Guo, J. Banerjee, C. Cheung, The effect of laminate lay-up on the
337 flutter speed of composite wings, Proceedings of The Institution of Me-
338 chanical Engineers Part G-journal of Aerospace Engineering - Proc. Inst.
339 Mech. Eng. G-J A E 217 (2003) 115–122.
- 340 [15] S. Guo, W. Cheng, D. Cui, Aeroelastic tailoring of composite wing struc-
341 tures by laminate layup optimization, AIAA Journal 44 (12) (2006)
342 3146–3150.
- 343 [16] M. Kameyama, H. Fukunaga, Optimum design of composite plate wings
344 for aeroelastic characteristics using lamination parameters, Computer
345 Struct. 85 (3) (2007) 213 – 224.
- 346 [17] H. Haddadpour, M. Kouchakzadeh, F. Shadmehri, Aeroelastic insta-
347 bility of aircraft composite wings in an incompressible flow, Compos.
348 Struct. 83 (1) (2008) 93 – 99.
- 349 [18] R. Firouz-Abadi, A. Askarian, P. Zarifian, Effect of thrust on the aeroe-
350 lastic instability of a composite swept wing with two engines in subsonic
351 compressible flow, Journal of Fluids and Structures 36 (2013) 18 – 31.
- 352 [19] M. R. Catlett, J. M. Anderson, C. Badrya, J. D. Baeder, Unsteady re-
353 sponse of airfoils due to small-scale pitching motion with considerations
354 for foil thickness and wake motion, Journal of Fluids and Structures (94)
355 (2020) 102889.
- 356 [20] R. Yurkovich, Status of unsteady aerodynamic prediction for flutter of
357 high-performance aircraft, Journal of Aircraft 40 (5) (2003) 832–842.
- 358 [21] D. Pitt, A new non-iterative P-K match point flutter solution, 1999.

- 359 [22] P. Mahato, D. Maiti, Aeroelastic analysis of smart composite structures
360 in hygro-thermal environment, *Compos. Struct.* 92 (4) (2010) 1027 –
361 1038.
- 362 [23] E. Carrera, Theories and finite elements for multilayered, anisotropic,
363 composite plates and shells, *Archives of Computational Methods in En-*
364 *gineering* 9 (2) (2002) 87–140.
- 365 [24] E. Carrera, Theories and finite elements for multilayered plates and
366 shells:a unified compact formulation with numerical assessment and
367 benchmarking, *Archives of Computational Methods in Engineering*
368 10 (3) (2003) 215–296.
- 369 [25] E. Carrera, Assessment of theories for free vibration analysis of homoge-
370 neous and multilayered plates, *Shock and Vibration* 11 (2004) 261–270.
- 371 [26] E. Carrera, G. Giunta, Refined beam theories based on a unified formu-
372 lation, *International Journal of Applied Mechanics* 2 (2010) 117–143.
- 373 [27] E. Carrera, G. Giunta, P. Nali, M. Petrolo, Refined beam elements with
374 arbitrary cross-section geometries, *Computers Struct.* 88 (5) (2010) 283
375 – 293.
- 376 [28] E. Carrera, M. Filippi, E. Zappino, Free vibration analysis of laminated
377 beam by polynomial, trigonometric, exponential and zig-zag theories,
378 *Journal of Composite Materials* 48 (19) (2014) 2299–2316.
- 379 [29] E. Carrera, M. Filippi, E. Zappino, Laminated beam analysis by polyno-
380 mial, trigonometric, exponential and zig-zag theories, *European Journal*
381 *of Mechanics - A/Solids* 41 (2013) 58 – 69.
- 382 [30] E. Carrera, M. Filippi, E. Zappino, Free vibration analysis of rotating
383 composite blades via carrera unified formulation, *Composite Structures*
384 106 (2013) 317 – 325.
- 385 [31] E. Carrera, E. M. Filippi, E. Zappino, Analysis of Rotor Dynamic by
386 One-Dimensional Variable Kinematic Theories, *Journal of Engineering*
387 *for Gas Turbines and Power* 135 (9), 092501.
- 388 [32] A. Varello, E. Carrera, L. Demasi, Vortex lattice method coupled with
389 advanced one-dimensional structural models, *Journal of Aeroelasticity*
390 *and Structural Dynamics* 2 (2011) 53–78.

- 391 [33] A. Varello, A. Lamberti, E. Carrera, Static aeroelastic response of wing-
392 structures accounting for in-plane cross-section deformation, *International*
393 *Journal of Aeronautical and Space Sciences* 14 (2013) 310–323.
- 394 [34] E. Carrera, A. Varello, L. Demasi, A refined structural model for static
395 aeroelastic response and divergence of metallic and composite wings,
396 *CEAS Aeronautical Journal* 4 (2) (2013) 175–189.
- 397 [35] M. Petrolo, Advanced 1d structural models for flutter analysis of lifting
398 surfaces, *International Journal of Aeronautical and Space Sciences* 13
399 (2012) 199–209.
- 400 [36] M. Petrolo, Flutter analysis of composite lifting surfaces by the 1d car-
401 rera unified formulation and the doublet lattice method, *Composite*
402 *Structures* 95 (2013) 539 – 546.
- 403 [37] A. Pagani, M. Petrolo, E. Carrera, Flutter analysis by refined 1d dy-
404 namic stiffness elements and doublet lattice method, *Advances in Air-*
405 *craft and Spacecraft Science* 1 (2014) 291–310.
- 406 [38] E. Carrera, E. Zappino, Aeroelastic analysis of pinched panels in su-
407 per-sonic flow changing with altitude, *Journal of Spacecraft and Rockets*
408 51 (1) (2014) 187–199.
- 409 [39] M. Filippi, E. Carrera, Aerodynamic and mechanical hierarchical aeroe-
410 lastic analysis of composite wings, *Mechanics of Advanced Materials and*
411 *Structures* 23 (2015) 1–31.
- 412 [40] E. Carrera, M. Petrolo, Refined one-dimensional formulations for lami-
413 nated structure analysis, *AIAA Journal* 50 (1) (2012) 176–189.
- 414 [41] E. Carrera, M. Filippi, P. Mahato, A. Pagani, Free-vibration tailoring of
415 single- and multi-bay laminated box structures by refined beam theories,
416 *Thin-Walled Structures* 109 (2016) 40 – 49.
- 417 [42] I. Kaleel, M. Petrolo, A. Waas, E. Carrera, Computationally efficient,
418 high-fidelity micromechanics framework using refined 1d models, *Com-*
419 *posite Structures* 181 (2017) 358 – 367.
- 420 [43] E. Carrera, E. Zappino, One-dimensional finite element formulation with
421 node-dependent kinematics, *Computer Struct.* 192 (2017) 114 – 125.

- 422 [44] R. B. Bharati, P. K. Mahato, E. Carrera, M. Filippi, A. Pagani, Free
423 Vibration and Stress Analysis of Laminated Box Beam with and With-
424 out Cut-Off, *Recent Advances in Theoretical, Applied, Computational*
425 *and Experimental Mechanics*, 2020, pp. 185–196.
- 426 [45] A. H. Bisplinghoff R. L., H. R. L., *Aeroelasticity.*, Dover Publication
427 Inc, New York, 1996.
- 428 [46] R. T. Jones, *Operational treatment of the nonuniform-lift theory in*
429 *airplane dynamics.*, Technical Report 667, NASA, 1938.
- 430 [47] E. Carrera, M. Filippi, P. K. Mahato, A. Pagani, Advanced models for
431 free vibration analysis of laminated beams with compact and thin-walled
432 open/closed sections, *J. Compos. Mater.* 49 (17) (2015) 2085–2101.
- 433 [48] SIEMENS: *Aeroelastic Analysis User’s Guide* (2014).
- 434 [49] M. Petrolo, *Advanced aeroelastic models for the analysis of lifting sur-*
435 *faces made of composite materials*, Ph.D. thesis, PhD. Dissertation, Po-
436 litecnico di Torino, Turin, Italy (2011).
- 437 [50] S. J. Hollowell, J. Dugundji, *Aeroelastic flutter and divergence of stiff-*
438 *ness coupled, graphite/epoxy cantilevered plates*, *Journal of Aircraft*
439 21 (1) (1984) 69–76.