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## Article

# Thermodynamic Definition of Time: Considerations on the EPR Paradox

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**Abstract:** Causality is the relationship between causes and effects. Following Relativity, any cause of an event must always be in the past light cone of the event itself, but causes and effects must always be related to some interactions. In this paper, causality is developed as a consequence of the analysis of the Einstein, Podolsky, and Rosen paradox. Causality is interpreted as the result of time generation, due to irreversible interactions of real systems among them. Time results as a consequence of irreversibility; so, any state function of a system in its space cone, when affected by an interaction with an observer, moves into a light cone or within it, with the consequence that any cause must precede its effect in a common light cone.

**Keywords:** causality; irreversibility; EPR; time; entropy

**MSC:** 80A23; 80A19; 76M55; 4A55



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## 1. Introduction

The problem of the link between entropy and time has a long story and many viewpoints [1]. In relation to irreversibility, Franklin analysed some processes at their steady states, evaluated their entropy variation, and pointed out that they are related only to energy transformations [2].

Thermodynamics is the physical science that develops the study of the energy transformations, allowing scientists and engineers to obtain fundamental results [3–5] in physics, chemistry, biophysics, biomedicine, engineering, and information theory. In the history of thermodynamics, entropy has generally been recognised as one of the most important physical quantities, because of its interdisciplinary applications [1,6–8].

Entropy variation is always related to the evolution of the state of any system, between its initial and its final conditions [1,7]. Many different approaches have been introduced in order to describe irreversibility in any process [3–5], but entropy remains the fundamental physical quantity to quantify irreversibility [1,7,9,10].

Classical thermodynamics has been developed by using a small number of state variables [3–5], while, in nonequilibrium thermodynamics, the system is described by considering its subsystems [11,12], under the assumption that each of them is in local equilibrium [6,11,12]. This last hypothesis is required in order to introduce the concept of temperature as a measurable quantity [6,11,12]. So,  $T\sigma = \sum_i J_i X_i > 0$  represents a measure of the dissipation [13], where  $T$  is the absolute thermodynamic temperature;  $\sigma$  is the entropy production density, i.e., the time rate of the entropy density, generated by an irreversible process;  $J_i$  is the  $i$ -th heat or mass flow; and  $X_i$  is the  $i$ -th generalised driving force for vector transport processes or for chemical reactions [14–16].

Recently, an increasing interest has been growing in the analysis of the fundamental role of fluxes in thermodynamics and natural systems [17–25]. Moreover, a scientific interest is growing in establishing a comprehensive approach to irreversible processes based on microscopic analysis [1,6,7,11,12,26–29].

In this context, a possible link has been proposed between the macroscopic approach to irreversibility and the microscopic behaviour of a system [30], suggesting the consumption of free energy as the principle cause of far from equilibrium states. Consequently, the related entropy production is generated by the redistribution of energy, momentum, mass, and charge [31].

The effort in developing a thermodynamic approach to real systems has led to a growing interest towards the concepts of potentials and availability, in order to consider time in the development of theoretical models [32]. Honig [33] developed the relation between time and entropy in irreversible processes by considering the heat transfer through the system's border. His approach considers time  $t$  only as a parameter to represent the path that systems follow from their initial to final configurations. The concept of path, in a state-space, leads to considering a set of consequent and related events, one followed by another, each caused by the previous one [34].

However, this concept can also represent the thermodynamic expression of causality. Indeed, causality is the relationship between causes and effects [35]. To discuss this concept from a physical viewpoint, we consider that in Special and General Relativity, a light cone is the path that a flash of light, emanating from a single event and travelling in all directions, would take through spacetime. So, in physics, causality represents the flow of events such that the causes of an event must always be in the past light cone of the event itself, and always related to some interactions of the system [36]. In Special and General Relativity, the light cone is the path that a photon of light, emitted by a single source, takes through space-time [37,38]. In Special and General Relativity, an effect cannot occur from a cause that is not in the past light cone of that event [37]. Moreover, a cause cannot have an effect outside its future light cone [38]. Consequently, an event cannot produce any effect if it is outside of the future light cone of another event [39].

In relation to the second law of thermodynamics, the arrow of time is introduced [40]. However, causality requires a direction of time [41] too because of its connection between a cause and its effect: this property is just a feature of this physical theory. The connection between causality and entropy has recently been pointed out [41], with the consequence of defining time as the metric of causality. Moreover, time has been highlighted to be discrete in Nature [41].

In this context, the definition and understanding of the nature of time must be highlighted to be difficult, because any process is usually described in relation to the time flow. In Newtonian physics, time is a mathematical variable, but, without being something real [42]. Simultaneity and duration of phenomena are absolute: duration is an abstract property of the whole. On the contrary, in Relativity, Einstein introduced a completely new approach to time, which makes instants and durations dependent on the observer. The physical universe is space-time, a mathematical space continuously filled with ideal clocks [42]. These clocks are all synchronised with respect to a given observer, which measures the duration of a phenomenon by using two clocks located in the places where the phenomenon starts and ends. So, another inertial observer, in motion with respect to the first one, does not agree on the space-time coordinates of the same events. Consequently, the same phenomenon has a different duration in relation to any observer. As a consequence of this distinction, the requirement of the definition of physical time emerges [42].

Following Bell [43] and Pauli [44], Brown [45] pointed out that clocks do not measure time, but their behaviour in relation to some aspects of space-time [42]. Consequently, physical systems evolve in the phase space following the path that increases entropy! However, all these results require a physical mathematical definition of time.

In summary, it seems plausible to study how the concepts of time, irreversibility, and causality could be linked together. The problem to link the macroscopic approach to the microscopic one is of interest in the various physical problems; indeed, macroscopic and microscopic approaches have been pointed out to be complementary tools [46] in relation to the study of complex problems, where both approaches coexist [1,6,7], such as, for

example, the relations between quantum mechanics and classical physics, matter–radiation interaction, electrodynamics of the Wheeler–Feynman model, and nano-thermodynamics.

The aim of this paper is to propose a possible thermodynamic approach to link the concepts of time with the ones of irreversibility and causality. The result obtained will bring light to the analytical definition of time. Time is a quantity useful to analyse some considerations on the Einstein, Podolsky, and Rosen (EPR) paradox due to its consequences related to causality. Thus, a possible solution of the EPR paradox is suggested, introducing a link to Heisenberg’s uncertainty principle. To do so, the analysis of time will be developed in relation to its thermodynamic definition related to an atomic footprint of irreversibility, related to the interaction between atomic electrons and environmental electromagnetic fields (photons). Consequently, a hypothesis emerges: only duration has sense in relation to the physical time originated from this footprint, related to the entropy production due to this interaction. So, if the system is subjected to reversible processes, the irreversibility footprint disappears and time (as a duration of the interaction) becomes null. Thus, in completely reversible processes, the systems move only in the space component of space-time, without having any movement in its time components. Any completely reversible system seems to be able also to disappear in a space position and to appear in any other space position, without spending time. Furthermore, two reversible systems seem to interact with an infinite space range, because time cannot be measured until the entropy production is fully generated. Starting from these results, the uncertainty principle is introduced in relation to time and, consequently, some considerations on the EPR paradox are developed, with particular regards to causality.

## 2. Materials and Methods

Since 1927, Bohr developed the principle of complementarity, which is a fundamental theory of quantum mechanics, based on observation and measurement [47]. This principle points out that, considering two quantum systems, the measurement on one of them involves a physical interaction with the experimental setup that affects both systems. This phenomenon is uncontrollable, even if it can be predicted statistically [48].

Since 1935, Einstein, Podolsky, and Rosen developed some criticisms on the Bohr results [49], known as EPR paradox. This paradox presents some consequences on the foundation of quantum mechanics, even if many other interesting problems were originated by this criticism—in particular, the problem of the collapse of the wave function, which represents a current open problem.

In 1935, Schrödinger introduced the definition of entangled states—as the quantum pure states,  $|\psi\rangle$ , from an ensemble of systems—that cannot be represented by tensorial products of eigenstates of the states themselves, which analytically results [50]

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle \quad (1)$$

where  $\otimes$  is the tensorial product and  $|\psi\rangle$  are the states in the Hilbert space  $\mathcal{H}$ . A state is entangled if and only if it cannot be factorised (See Appendix A).

During a measurement, a full wave function  $|\psi\rangle$  collapses into an eigenstate,  $|\psi_i\rangle$ , of the state bases, such that [51]

$$\begin{aligned} \langle\psi| |\psi_i\rangle &= 1 \\ \langle\psi| |\psi_j\rangle &= 0, \forall j \neq i \end{aligned} \quad (2)$$

These relations express analytically the effect of the interaction between the system and the experimental set up.

Recently, the definition of time [30,52] has been introduced by considering an analysis of photon–atomic electron interaction, in relation to irreversibility [53–55], based on an engineering thermodynamic viewpoint [56–59]. Time is conjectured to be related both to the entropy production and to the entropy production rate. This result agrees with the approach of Planck and Einstein, who pointed out that the law of system evolution is precisely the law of evolution of entropy [60,61].

The starting considerations of our approach can be summarised as follows:

- The atom, without interaction, can be considered an isolated system, and any process inside it is completely reversible.
- The atom, in interaction with a photon, is an open system, where fluxes occur: in this case, a photon can be absorbed, and the atomic electron can have an energy level transition. Then, the electron can jump down in its fundamental energy state, with a related photon emission: the system is subjected to inflow and outflow of photons.
- The atom in interaction is subjected to the irreversible process of the perturbation of its centre of mass: this open atom is irreversible, just because it is in interaction with the environment, and it is subjected to fluxes.

At the atomic level, photons can be absorbed by the atomic or molecule electrons, and an electronic energy transition occurs between the energy levels of two atomic stationary states. Then, photons can also be emitted by the excited electrons, when the electrons jump down into the energy level of the original stationary state. During this phenomenon, the electrons seem to follow a reversible energetic path, because they come back to the original stationary state of low energy level [62–66]. Indeed, when we consider a single atom or molecule, the energy perturbation of the centre of mass is of the order of  $10^{-13}$  J, while a usual energy for the electron transition, between two atomic or molecule levels, is of the order of  $10^{-8}$  J, with an excited state lifetime of the order of  $10^{-8}$  s [65]. Consequently, this approximation (not considering the effect of the atomic nucleus) is usually introduced. However, we stress that it is only an approximation [62–66], which cannot be introduced in the analysis of irreversibility, because it requires some considerations just on the role of the nucleus, during the photon–atomic electron interaction [56–58,67], in accordance with some recent experimental results [68]. As a consequence of the interaction between the atomic or molecule electron and the photon, a footprint occurs in the atom or molecule [56]. The results obtained in Refs. [30,56,57] point out that the interaction between a photon and an atomic electron affects the energy level both of the electron and of the centre of mass of the atom, in accordance with the theoretical and experimental results summarised in Refs. [53–55,66,69]. Consequently, the macroscopic irreversibility is the result of the microscopic irreversibility, due to the photon–electron interaction, which is the interaction between environmental electromagnetic waves and matter.

Following the results obtained in the thermodynamic analysis of electromagnetic fields [70], this interaction can be expressed in terms of entropy production, and of the entropy production rate. However, the ratio between the entropy production and the entropy production rate can represent a time. In analogy with analytical mechanics, where position and velocity can be used as independent variables for the state space [71], we introduce entropy production  $\sigma$  and entropy production rate  $\Sigma$  as independent variables of the state space  $\Omega = \{(\sigma, \Sigma)\}$  [72]. We can use this space to study the behaviour of the photon–atomic electron interaction. So, following the dimensional analysis in thermodynamics [73], we can introduce the definition of time,  $\tau$ , as follows [52,74]:

$$\tau = \frac{\sigma}{\Sigma} \quad (3)$$

Here, the entropy production rate can be written in relation to the electromagnetic waves, by considering the Gouy–Stodola theorem, as follows [70,72]:

$$T_0 \Sigma = \frac{A}{2} \varepsilon_0 c E_{el}^2 + \frac{A}{2\mu_0} c B_m^2 \quad (4)$$

where  $E_{el}$  is the electric field,  $B_m$  is the magnetic field,  $c = 299,792,458 \text{ m s}^{-1}$  is the speed of light,  $\varepsilon_0 = 8.8541878128(13) \times 10^{-12} \text{ F m}^{-1}$  is the electric permittivity in vacuum and  $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$  is the magnetic permeability in vacuum,  $A$  is the area of the border of the thermodynamic control volume, and  $T_0$  is the environmental temperature.

The entropy production can be related to the analysis of irreversibility [53–55] of the interaction between a photon and an atomic electron [52,74]. Here, the fundamental results are summarised in order to be used in the thermodynamic approach to EPR paradox. To do so, we consider a photon, which incomes to an atomic electron. In accordance with the usual approach of physical chemistry to quantum chemistry and physics [65,75,76], the study of spectroscopic phenomena is developed by analysing the simpler representative system and then generalising it. Indeed, the spectroscopic analysis of a system is usually developed analytically by considering a Hydrogen-like atom, and then generalising the results by comparing them with the experimental ones: the conclusion is that the results obtained for the simplest quantum system (Hydrogen-like atom) can be correctly generalised for more complex atomic and molecular systems [65,75,76]. Following this usual approach, in order to develop a simple analytical analysis of irreversible phenomena in quantum systems, without using numerical simulation [65,66,75], we consider a Hydrogen-like atom, which is an open system, from a thermodynamic viewpoint. The incoming photon has an energy  $E_\gamma = h\nu$  and a momentum  $\mathbf{p}_\gamma = h\nu\mathbf{u}_c/c$ , where  $h = 6.62607004 \times 10^{-34}$  J s is the Planck constant,  $\nu$  is the frequency of the electromagnetic wave,  $\mathbf{u}_c$  is the versor of the speed of light [77]. If the frequency of the photon is just  $\nu = (E_f - E_i)/h$ , where  $E_i$  is the energy of the ground state of the electron, and  $E_f$  is the energy of an accessible excited level of the electron, then the electron can absorb the photon. So, the electron can jump from its ground state into the accessible excited energy state. After the lifetime of this state ( $10^{-15}$  s [65]), the electron jumps down into the fundamental state, emitting a new photon.

Usually, this phenomenon is treated by introducing the Franck–Condon principle [62,69], i.e., a semiclassical interpretation of the electronic transition intensity [62], which states that an electronic transition is most likely to occur without changes in the positions of the nuclei in the molecular entity and its environment [62,63]. Here, we wish to stress that the Franck–Condon principle is an approximation, useful in spectroscopy, quantum physics, and quantum chemistry. However, in relation to the aim of this paper, this approximation does not allow us to consider all the phenomena related to irreversibility, because we wish to consider the physical behaviour of the whole system. Thus, here, we analyse a simple quantum system (the Hydrogen-like atom) without considering the Franck–Condon approximation during a photon–electron interaction [56].

Consequently, during photon–atomic electron interaction, a change in the kinetic energy of the centre of mass of the atom is considered [56,78]. So, the final energy of the atom, after the photon absorption, results [56,65,78]

$$E_f = E_i + h\nu - \frac{h^2\nu^2}{2Mc^2} \quad (5)$$

where  $E_f$  is the energy of the electron in the excited state;  $E_i$  is its energy in the initial state;  $\nu$  is the frequency of the incoming photon;  $M$  is the mass of the atom; and, in an analogous way, when the photon is emitted, it results [56,65,78]

$$E_i = E_f - h\nu' - \frac{h^2\nu'^2}{2Mc^2} \quad (6)$$

where  $\nu'$  is the frequency of the outgoing photon. Consequently, we expect an energy footprint in the atom [56,57,78], because, considering the effect on the centre of mass, the interaction between a photon and an electron in an atom affects both the energy level of the electron and the energy level of the centre of mass of the atom. Here, we stress that this effect is well-known in quantum physics [65], even if the approximation of neglecting it is usually accepted, because of the small energy contribution of the centre of mass. The quantum state function—after this interaction, the solution of the Schrödinger equation—can be obtained by the usual quantum mechanical approach [77,79].

So, these considerations represent an analytical result for explaining macroscopic irreversibility by means of microscopic irreversibility, in accordance with Einstein. The



irreversibility is generated by the photon–electron interaction, i.e., the interaction between electromagnetic waves and matter. Consequently, an energy footprint emerges; it can be evaluated as follows [30,56,57]:

$$E_{ftp} = \Delta E_\gamma = \Delta E_{CM} = \langle \psi(\mathbf{r}, \mathbf{R}) | \mathcal{H} | \psi_f(\mathbf{r}, \mathbf{R}) \rangle = \frac{m_e}{M} E_\gamma \quad (7)$$

where  $\mathcal{H}$  is the Hamiltonian of the photon–atomic electron interaction,  $m_e = 9.1093837 \times 10^{-31}$  kg is the mass of electron,  $E_\gamma = h\nu$  is the energy of the incoming photon, and CM means centre of mass. So, the related entropy production can be evaluated by considering the Gouy–Stodola theorem [30,56,57,72]:

$$T_0 \sigma = \frac{m_e}{M} E_\gamma \quad (8)$$

where  $T_0$  is the environmental temperature.

In this way, the definition of time, in relation to irreversibility, can be explicitly evaluated in relation to measurable physical quantities:

$$\tau = \frac{2 m_e}{M c} \frac{E_\gamma}{\varepsilon_0 E_{el}^2 + \mu_0^{-1} B_m^2} \quad (9)$$

Now, a relation with the EPR paradox can be highlighted. In order to show it, we consider the original EPR gedanken experiment [80], in which two spin-1/2 particles, generated by the same source—A and B—move in opposite directions. These particles can be detected only if they interact with an experimental setup. Until their interaction, they are only in the spatial component of the space-time, because the time component is generated only from their interaction with an observer. So, independently by their distance, the two systems *are not aware* of being separate, because they are not subjected to the time dimension. Before the interaction with the observer, their state function is [81,82]

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( \left| \frac{1}{2} \right\rangle_A \left| -\frac{1}{2} \right\rangle_B - \left| -\frac{1}{2} \right\rangle_A \left| \frac{1}{2} \right\rangle_B \right) \quad (10)$$

So, as a consequence of the interaction of one of the particles with an observer, the state function collapses into a particular value, determined by the interaction itself. As a consequence of the interaction, time also begins to flow. When the particle interacts with the setup of the observer, its energy  $E$  is known, with a small value of uncertainty  $\delta E$  related to the setup used. However, in relation to Heisenberg's uncertainty principle,

$$\delta E \delta \tau \geq \hbar \quad (11)$$

where  $\hbar = h/2\pi = 1.0545910^{-34}$  J s; time cannot be determined with the same accuracy [79], being  $\delta E = \sqrt{E - \langle E \rangle^2}$  and  $\delta \tau = \sqrt{\tau - \langle \tau \rangle^2}$ . The different values of time are distributed around the origin of time, following the probability [79]

$$|\psi|^2 = \frac{1}{\sqrt{2\pi} c \delta \tau} \exp \left( -\frac{\tau^2}{2(\delta \tau)^2} \right) \quad (12)$$

being  $\langle \tau \rangle = 0$ , in our discussion. So, during the collapse, the particles can influence each other, because they are still outside of time until we can detect one of them, up to the end of the interaction with the setup. So, for example, it can occur that [82]

$$|\psi\rangle_A = \left| \frac{1}{2} \right\rangle \Rightarrow |\psi\rangle_B = \left| -\frac{1}{2} \right\rangle \quad (13)$$

### 3. Results

In this paper, some considerations on the relation between a thermodynamic definition of time and the EPR paradox have been developed. To do so, first, the definition of time was introduced by considering the irreversibility in the interaction between a photon and an atomic electron.

When a photon interacts with the electrons of an atom or of a molecule, this interaction changes the energy level of the electrons, as a consequence of the energy absorbed or lost by the electron during the interactions. This atomic process can be well-described by using the Bohr approximation [65], as John Clarke Slater has highlighted [66]. Indeed, Bohr extended the Planck's results, describing the absorption of a photon by an electron as a resonant process, at a maximum rate of absorption energy [65].

This means that the time of absorption of the incoming photons flux is minimum. In this context, we can consider the photons flux as a flux of energy absorbed by the open system (in this case, the atomic system) in the least time. In this way, an atom can be considered as an open system with incoming and outgoing fluxes of photon. The consequence is that there exists a change in the kinetic energy of the centre of mass of the atom or molecule. The behaviour of the centre of mass is not usually considered, because of the useful effectiveness of the Franck–Condon approximation. However, in relation to thermodynamic analysis of irreversibility, the centre of mass behaviour must be taken into account. From a thermodynamic viewpoint, this approach brings us to consider macroscopic irreversibility as the global superposition of the microscopic irreversibility of a large number of atoms and molecules in matter. Time is the measurable result of this irreversibility [56,74,83,84].

Here, by using this approach, the results obtained can be summarised as follows:

- Time is the result of irreversibility, generated by the continuous interaction between electromagnetic waves and matter;
- Locally, entropy can decrease, but the entropy production (due to irreversibility) must always increase, with the consequence that time can only increase.

In relation to our result, we highlight that Einstein [85] pointed out that the definition of time must be based on the clock measure. So, a time–clock relation must be considered [86]: this hypothesis states that there is a conceptually necessary relation between time and a physical process that functions as the core of a clock, with the consequence that time and the physical process cannot be defined independently [86]. In all the critical analyses of the physical foundation of time, the concept of duration has always been highlighted as a fundamental quantity [37,85,87–91]. A fundamental requirement is the time–clock relation, and we can improve it by considering that a global time coordinate  $t$  is the time measured by clocks of these observers; so,  $t$  is the proper time measured by an observer at rest with respect to the local matter distribution [92]. In relation to our approach, it is possible to consider the following [83]

- Our definition of time is related to the entropy: the entropy variation is extended to any epoch and domain, because any process generates entropy variation.
- Our definition of time is linked to entropy variation and fluxes: this definition satisfies that the physical basis for the time scale might break down in the very early universe due to phase transitions;
- The extrapolation of the present physical scales into the past can be introduced by considering the entropy and the temperature of any epoch of the Universe, its formation included.
- Local time flow rate is different in relation to global Universe time flow rate, because global Universe flow rate is the global effect of the entropy rate generation, while the local time follows the distribution of the local entropy variation and rate [83], in accordance with the Theory of Relativity [93].

In summary, we consider that our Universe is always in a disequilibrium state; so, continuous fluxes of energy occur, and, in relation to the structure of the Universe, it can



occur only in the form of electromagnetic waves (fluxes of photons from a microscopic point of view). Consequently, the analysis of the interactions between these waves and matter is fundamental in the definition both of irreversibility and of time itself. The photon–atomic electron interaction generates an energy footprint, in accordance with the experimental results obtained by Doyle [53–55]. In the analysis of this quantum process, time never appears. Thus, it can be defined by introducing the relation between the energy footprint and the power that generates the process itself, i.e., the power of the electromagnetic wave, related to the inflow of photons. Consequently, time results as the footprint of irreversibility.

In this context, a relation with the EPR paradox emerges. Indeed, the results point out that, without any interaction, time cannot exist, due to its definition related to the interaction between electromagnetic waves and matter. In our opinion, time, as we usually measure it, can be detected only after the interval  $\delta\tau$ , as defined in Heisenberg’s uncertainty principle, from our reference frame. So,  $\delta\tau$  is what an observer senses in his/her reference frame, which corresponds to the entropy production  $\sigma$ , required to generate the time interval itself. During this entropy production, the particles analysed in the original EPR gedanken-experiment influence each other. In the interval  $\delta\tau$ , the particles are only in the space component of space-time, but without any time component. Thus, EPR paradox can find a possible interpretation: the time component of space-time appears only after interaction; so, any event, in the only space component of the space-time, seems immediate.

#### 4. Discussion

In Newtonian physics [94], an effect cannot occur before its cause [95]. In Special and General Relativity, this statement has been improved by stressing that an effect cannot occur from a cause that is not in the back light cone of that event [85]. These results are the consequence of finite speed of light, which results the maximum velocity in our Universe too. Consequently, no information can be transferred at a velocity higher than the speed of light.

The concept of causality has deeply been improved by its link to the meaning of the simultaneous observer-dependent [93]: the cause must always precede its effect, in accordance with all inertial observers. Consequently, the cause and its effect are separated by a timelike interval in the space-time [85,93]. Moreover, these two related events can change any signal at a velocity smaller than the speed of light.

Then, in quantum field theory, causality is closely related to the principle of locality. This concept is still under study, because it is strictly related to the interpretation of quantum mechanics, with particular regards to quantum entanglement and Bell’s Theorem [96]. Recently, in causal dynamical triangulation [97], causality has been related to the foundation of space-time geometry [98].

All these points of view can be summarised by the results obtained in this paper. Indeed, in relation to the link of the thermodynamic statistical analysis of the irreversible paths with their stochastic order [99], the energy flows—between system and environment—have been shown to select and shape the paths [18,20–25,100–103]. As a consequence of this interaction, irreversibility occurs also at the atomic dimension. However, this result allows us to approach the problem of causality by starting from the analysis of the EPR paradox. In this way, time is the consequence of causality, which results a sequence of ordered events, and of irreversibility. Indeed, the continuous interactions between the atomic electrons and the electromagnetic waves in the environment cause the nonequilibrium state of our Universe [31,58,67,74,83,84,104].

The approach, here used, agrees with the study [105] of the relation between nonequilibrium thermodynamics and stochastic thermodynamics, based on the evaluation of the entropy production. Indeed, Giordano analysed systems in a nonequilibrium stationary state and their relaxation to equilibrium. During this relaxation, he was able to show the linear relation between fluxes and forces, satisfying the Onsager reciprocity principle. He obtained also a nonlinear expression for the entropy production of such systems. Our approach [52,56,78] to irreversibility, based on the analysis of the photon–electron interaction

in an atom without the Franck–Condon principle, is developed by using the thermodynamic analysis of the fluxes occurring in an open system. Moreover, Giordano proved that his results are valid for any physical linear system, which can also be found by linearisation of an arbitrarily nonlinear system around a given work point [105]. In analogy with this statement, also the simple model analysed in our paper (the Hydrogen-like atom) can be considered representative of the spontaneous behaviour of quantum systems, because we use a semiclassical model of quantum interaction between a photon and an atomic electron, which is described by a linearised approximation of the energy transition, where no quantum numbers and selection rules are considered [65].

Moreover, our results agree also with those of McCulloch and Giné [106]. Indeed, they proved that the EPR paradox presents a simple resolution, in which the energy uncertainty of the two-particle system is very small when it does not interact with the environment between measurements, forcing an increase in the uncertainty of time; consequently, time results greater than the time they have been separating [106].

## 5. Conclusions

The results obtained confirm the Bohr approach, but highlight also the fundamental role of space-time, obtained by Einstein, and the recent relation between time, causality, and space-time [97]. Indeed, the approach suggested points out that the conditioning of the measurements is due to the zero value of the time component of the space-time, unless one of the system interacts with the experimental setup. Consequently, it follows that irreversibility represents a constraint in the interactions, conditioning the behaviour of correlated systems.

Thus, causality, as found by an observer, is related to time generation, due to the irreversible interactions of real systems among them. If the events are correlated at the beginning, independently from their location, they maintain their correlation. On the contrary, if the events are not correlated, they could be correlated only after an interaction. In particular, Equation (9) points out that, without interaction, time does not exist; consequently, it would be impossible to observe an order in the events. However, after any interaction, time occurs, and the order of the events emerges. Consequently, also the relations between two events can be identified, with particular regards to their causal relations.

Moreover, in the theory of Special Relativity, causality is related to simultaneous observer-dependent [107]. Consequently, following Special Relativity, the cause must precede its effect, in accordance with all inertial observers. Furthermore, in General Relativity, the effect must belong to the future light cone of its cause, even if the space-time is curved [37]. These statements mean that the cause and its effect are separated by a timelike interval, and the effect belongs to the future of its cause [37,108].

Here, we conjecture that time interval is originated only during the interaction between a system and an observer; so, a timelike interval occurs only after this interaction. Consequently, before this interaction, all the systems are in a ‘contemporary’ state.

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## Appendix A

In this section, some statements from mathematics of quantum physics are summarised [109]:

- Let  $\mathcal{X}$  be a vector space. Any finite linear combination of vectors  $x \in \mathcal{X}$ ,  $\sum_i \alpha_i x_i$ , with  $\alpha \in \mathbb{R}$ , is named *convex*, if  $i \in [0, 1]$  and  $\sum_i \alpha_i = 1$ .
- If  $\mathcal{C} \subseteq \mathcal{X}$  is convex, an element  $x_e \in \mathcal{C}$  is called *extreme* if it cannot be obtained as  $x_e = \lambda x + (1 - \lambda) y$ , with  $\lambda \in [0, 1]$ ,  $\forall x, y \in \mathcal{C} - \{x_e\}$ .
- Let  $\mathcal{H}$  be a separable Hilbert space. Let  $\mathcal{S}(\mathcal{H})$  be a convex subset of  $\mathcal{H}$ . The extreme elements in  $\mathcal{S}(\mathcal{H})$  are called pure states, while nonextreme states are named mixed states or nonpure states.
- Schmidt's decomposition theorem [110]. Any pure state  $|\psi\rangle$  can be written as a sum of orthonormal unit vectors  $|u_i\rangle$  and  $|v_i\rangle$ , which span the space of possible state vectors for the system, and  $i$  runs up to the smaller of the dimensions of the two subsystem Hilbert spaces:

$$|\psi\rangle = \sum_i c_i |u_i\rangle \otimes |v_i\rangle \quad (\text{A1})$$

where  $c_i \in \mathbb{C}$ .

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