

New Criteria for Self-Synchronization of Two Unbalanced Vibro-Exciters

Original

New Criteria for Self-Synchronization of Two Unbalanced Vibro-Exciters / Smirnova, Vera B.; Proskurnikov, Anton V.; Utina, Natalia V.. - ELETTRONICO. - (2022), pp. 1-4. (Intervento presentato al convegno 16th International Conference on Stability and Oscillations of Nonlinear Control Systems (Pyatnitskiy's Conference) tenutosi a Moscow, Russian Federation nel 01-03 June 2022) [10.1109/STAB54858.2022.9807478].

Availability:

This version is available at: 11583/2969052 since: 2022-06-30T11:11:18Z

Publisher:

IEEE

Published

DOI:10.1109/STAB54858.2022.9807478

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

IEEE postprint/Author's Accepted Manuscript

©2022 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

New Criteria for Self-Synchronization of Two Unbalanced Vibro-Exciters

Original

New Criteria for Self-Synchronization of Two Unbalanced Vibro-Exciters / Smirnova, Vera B.; Proskurnikov, Anton V.; Utina, Natalia V.. - ELETTRONICO. - (2022), pp. 1-4. (Intervento presentato al convegno 16th International Conference on Stability and Oscillations of Nonlinear Control Systems (Pyatnitskiy's Conference) tenutosi a Moscow, Russian Federation nel 01-03 June 2022) [10.1109/STAB54858.2022.9807478].

Availability:

This version is available at: 11583/2969052 since: 2022-06-30T11:11:18Z

Publisher:

IEEE

Published

DOI:10.1109/STAB54858.2022.9807478

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

IEEE postprint/Author's Accepted Manuscript

©2022 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

components of the motion. According to [6] we suppose that

$$\varphi_i(t) = \Omega t + \alpha_i(t) + \Psi_i(t, \Omega t) \quad (\Omega = \text{const}) \quad (4)$$

$$\frac{1}{2\pi} \int_0^{2\pi} \Psi_i(t, \Omega t) d\Omega t = 0. \quad (5)$$

Here α_i and Ψ_i are, respectively, the slow and the fast components of φ_i .

We also assume that Ψ_i is small and in the equation (2) we have

$$\dot{\varphi}_i \approx \Omega, \quad \ddot{\varphi}_i \approx 0. \quad (6)$$

By virtue of (3) equation (1) transforms into

$$I_i \ddot{\varphi}_i + k_i \dot{\varphi}_i = L_i^0 + m_i \varepsilon_i \ddot{x} \sin \varphi_i \quad (i = 1, 2). \quad (7)$$

Equation (2) is transformed by means of (6) and (4) where $\Psi_i(t, \Omega t)$ is assumed to be negligibly small. We have

$$M \ddot{x} + cx = \sum_{i=1}^2 f_i \cos(\Omega t + \alpha_i) \quad (f_i = m_i \varepsilon_i \Omega^2). \quad (8)$$

Following [6] we consider the solution of linear non-homogeneous equation (8) and substitute it into (7). Then by virtue of (5) and (4) we obtain from (7) the equations for slow components:

$$I_i \ddot{\alpha}_i + k_i \dot{\alpha}_i = k_i(\Omega_i - \Omega) + V_i \quad (i = 1, 2) \quad (9)$$

with

$$\begin{aligned} \Omega_i &:= \frac{L_i^0}{k_i}, \\ V_i &:= -\frac{A_{xx} f_i}{2\pi} \int_0^{2\pi} \sum_{s=1}^2 f_s \cos(\Omega t + \alpha_s) \sin(\Omega t + \alpha_i) d(\Omega t), \end{aligned} \quad (10)$$

where

$$A_{xx} = \frac{1}{M(\omega^2 - \Omega^2)}, \quad \omega^2 = \frac{c}{M}. \quad (11)$$

Since

$$\begin{aligned} \frac{1}{2\pi} \int_0^{2\pi} \cos(\Omega t + \alpha_s) \sin(\Omega t + \alpha_i) d(\Omega t) &= \\ &= \frac{1}{2} \sin(\alpha_i - \alpha_s), \end{aligned} \quad (12)$$

one has from (9)

$$\begin{cases} I_1 \ddot{\alpha}_1 + k_1 \dot{\alpha}_1 = k_1(\Omega_1 - \Omega) - \frac{1}{2} A_{xx} f_1 f_2 \sin(\alpha_1 - \alpha_2), \\ I_2 \ddot{\alpha}_2 + k_2 \dot{\alpha}_2 = k_2(\Omega_2 - \Omega) - \frac{1}{2} A_{xx} f_2 f_1 \sin(\alpha_2 - \alpha_1). \end{cases} \quad (13)$$

Note that the equilibrium $\alpha_i = \text{const}$ ($i = 1, 2$) may exist only if

$$\Omega = \frac{k_1 \Omega_1 + k_2 \Omega_2}{k_1 + k_2}. \quad (14)$$

So for slow components of φ_i we have obtained the system

$$\begin{cases} I_1 \dot{\alpha}_1 + k_1 \alpha_1 + A \Phi(\alpha_1 - \alpha_2) = 0, \\ I_2 \dot{\alpha}_2 + k_2 \alpha_2 - A \Phi(\alpha_1 - \alpha_2) = 0, \end{cases} \quad (15)$$

where

$$\Phi(\sigma) = \sin \sigma - \frac{\beta}{A}, \quad (16)$$

$$A = \frac{1}{2} A_{xx} f_1 f_2, \quad \beta = \frac{k_1 k_2 (\Omega_1 - \Omega_2)}{k_1 + k_2}. \quad (17)$$

We suppose that

$$\left| \frac{\beta}{A} \right| < 1. \quad (18)$$

The self-synchronization of the two vibro-exciter means that

$$\sigma(t) \triangleq \alpha_1(t) - \alpha_2(t) \rightarrow \text{const}, \quad \dot{\sigma}(t) \rightarrow 0 \text{ as } t \rightarrow +\infty. \quad (19)$$

The goal of this paper is to establish conditions on the parameters of (15) which guarantee the self-synchronization of the rotors.

For the purpose we reduce the system (15) to Volterra integro-differential equation with respect to $\sigma(t)$:

$$\dot{\sigma}(t) = \sigma_0(t) - \int_0^t \Gamma(t - \tau) \Phi(\sigma(\tau)) d\tau \quad (20)$$

with

$$\sigma_0(t) = \dot{\alpha}_1(0) e^{-\frac{k_1}{I_1} t} - \dot{\alpha}_2(0) e^{-\frac{k_2}{I_2} t}, \quad (21)$$

$$\Gamma(t) = A \left(\frac{1}{I_1} e^{-\frac{k_1}{I_1} t} - \frac{1}{I_2} e^{-\frac{k_2}{I_2} t} \right). \quad (22)$$

There exists a number of frequency-algebraic criteria which are destined for the investigation of asymptotic behavior of Volterra equation (20) (see [10] and references therein). The criteria have been proved by Popov method of a priori integral indices [7]. They are formulated in terms of the transfer function of the equation (20):

$$K_0(p) = A \left(\frac{1}{I_1 p + k_1} + \frac{1}{I_2 p + k_2} \right) \quad (p \in \mathbb{C}). \quad (23)$$

The specific character of equation (20) and of the nonlinear function $\Phi(\sigma)$ have brought about the employment of special Popov functionals, which have been generated exclusively, for systems with periodic nonlinearities (synchronization systems)

In the succeeding sections we demonstrate the conditions for self-synchronization obtained by the employment of various types of Popov functionals destined for synchronization systems.

III. THE BAKAEV-GUZH PROCEDURE

The main idea of Bakaev-Guzh procedure [8], [12] is to single out within a Popov functional a periodic function with zero mean integral value.

The advantage of the method for infinite dimensional MIMO system is described in detail in [10]. In [13] the results of [10] are simplified for SISO system. Consider the equation

$$\dot{\sigma}(t) = \alpha(t) - \int_0^t \gamma(t - \tau) \varphi(\sigma) d\tau \quad (24)$$

with

$$\alpha, \gamma : [0, \infty) \rightarrow \mathbb{R}, \varphi : \mathbb{R} \rightarrow \mathbb{R}. \quad (25)$$

Assume that $\alpha \in C[0, \infty)$, γ is piece-wise continuous and

$$|\gamma(t)|, |\alpha(t)| < Me^{-rt}, \quad (M, r > 0). \quad (26)$$

Assume also that

$$\begin{aligned} \varphi \in C^1(\mathbb{R}), \varphi(\sigma) = \varphi(\sigma + \Delta) \quad (\Delta > 0), \\ \mu_1 \leq \varphi'(\sigma) \leq \mu_2 \quad (\mu_1 \cdot \mu_2 < 0). \end{aligned} \quad (27)$$

The transfer function of (24) is as follows

$$K(p) = \int_0^{\infty} \gamma(t)e^{-pt} \quad (p \in \mathbb{C}). \quad (28)$$

Theorem 1: [13] Suppose there exist numbers $\varepsilon, \delta, \tau > 0$, $s_1 \leq \mu_1$, $s_2 \geq \mu_2$ such that

1) the frequency-domain inequality is valid:

$$\begin{aligned} \pi_0(\omega) \triangleq \operatorname{Re}\{K(i\omega) - \tau(K(i\omega) + s_1^{-1}i\omega)^* \cdot \\ \cdot (K(i\omega) + s_2^{-1}i\omega) - \varepsilon|K(i\omega)|^2 - \delta \geq 0, \quad \forall \omega \geq 0, \end{aligned} \quad (29)$$

where symbol $*$ stands for the complex conjugation;

2)

$$\delta > \frac{\nu_0^2 \nu^2}{4(\varepsilon \nu_0^2 + \tau \nu^2)}, \quad (30)$$

where

$$\begin{aligned} \nu &= \frac{\int_0^{\Delta} \varphi(\sigma) d\sigma}{\int_0^{\Delta} |\varphi(\sigma)| d\sigma}, \\ \nu_0 &= \frac{\int_0^{\Delta} \varphi(\sigma) d\sigma}{\int_0^{\Delta} |\varphi(\sigma)| \sqrt{(1 - s_1^{-1}\varphi'(\sigma))(1 - s_2^{-1}\varphi'(\sigma))} d\sigma}. \end{aligned} \quad (31)$$

Then for every solution of (24) it is true that

$$\sigma(t) \rightarrow q, \quad \varphi(q) = 0; \quad \dot{\sigma}(t) \rightarrow 0 \text{ as } t \rightarrow +\infty. \quad (32)$$

Note that the choice of different couples $\{s_1, s_2\}$ may result in essentially different conditions for self-synchronization.

In paper [14] Theorem 1 has been applied to system (15) in case $s_1^{-1} = -1, s_2^{-1} = 1$. Here we consider the case of $s_1^{-1} = 0, s_2^{-1} = 1$. In this case we have

$$\begin{aligned} \nu &= -\frac{\pi\beta}{2(\beta \arcsin \frac{\beta}{A} + \sqrt{A^2 - \beta^2})}, \\ \nu_0 &= -\frac{3\pi\beta}{2\sqrt{2}(3\beta + 2(A - \beta)^{\frac{3}{2}}A^{-\frac{1}{2}})}, \end{aligned} \quad (33)$$

$$\begin{aligned} \pi_0(\omega) &= \omega^4(-\delta I_1^2 I_2^2 + A\tau I_1 I_2 (I_1 + I_2)) + \\ &+ \omega^2(Ak_1 I_2^2 + Ak_2 I_1^2 + A\tau(I_1 k_2^2 + I_2 k_1^2) - \\ &- A^2(\varepsilon + \tau)(I_1 + I_2)^2 - \delta(I_1^2 k_2^2 + I_2^2 k_1^2)) + \\ &+ (Ak_1 k_2^2 + Ak_2 k_1^2 - A^2(\varepsilon + \tau)(k_1 + k_2)^2 - \delta k_1^2 k_2^2). \end{aligned} \quad (34)$$

The inequality (29) is valid for all $\omega \geq 0$ if the following inequalities are true:

$$\varepsilon + \tau \leq \frac{k_1 k_2}{2A(k_1 + k_2)}, \quad (35)$$

$$\delta \leq \frac{A(k_1 I_2^2 + k_2 I_1^2)}{2(k_1^2 I_2^2 + k_2^2 I_1^2)}, \quad (36)$$

$$\delta \leq \frac{A\tau(I_1 + I_2)}{I_1 I_2}. \quad (37)$$

Introduce the constant

$$P \triangleq \frac{k_1 k_2 (k_1^2 I_2^2 + k_2^2 I_1^2)}{2(k_1 + k_2)(k_1 I_2^2 + k_2 I_1^2)} \cdot \frac{I_1 + I_2}{I_1 I_2}. \quad (38)$$

Theorem 2: The limit relations (19) are true for every solution of (20) provided that either

$$A \leq P, \quad \frac{\nu_0^2 \nu^2}{(\nu_0^2 + \nu^2)} < \frac{k_1 k_2 (k_1 I_2^2 + k_2 I_1^2)}{2(k_1 + k_2)(k_1^2 I_2^2 + k_2^2 I_1^2)}, \quad (39)$$

or alternatively

$$A > P, \quad \frac{\nu_0^2 \nu^2}{(\nu_0^2 + \nu^2)} < \frac{k_1^2 k_2^2 (I_2 + I_1)}{4A I_1 I_2 (k_1 + k_2)^2}. \quad (40)$$

Proof: We choose

$$\varepsilon = \tau = \frac{k_1 k_2}{4A(k_1 + k_2)}. \quad (41)$$

In the first case, let

$$\delta = \frac{A(k_1 I_2^2 + k_2 I_1^2)}{2(k_1^2 I_2^2 + k_2^2 I_1^2)}. \quad (42)$$

Then due to (39) we obtain that

$$\delta \leq \frac{A\tau(I_1 + I_2)}{I_1 I_2}. \quad (43)$$

So the condition 1) of Theorem 1 is fulfilled. The condition 2) of Theorem 1 takes the form

$$4\varepsilon\delta = 4\tau\delta > \frac{\nu_0^2 \nu^2}{(\nu_0^2 + \nu^2)} \quad (44)$$

which follows from (39).

In the second case let

$$\delta = \frac{A\tau(I_1 + I_2)}{I_1 I_2} \quad (45)$$

Then it follows from (40) that (29) and (36) are true. Thus the Theorem is proved. ■

IV. LEONOV'S METHOD OF NONLOCAL REDUCTION

The idea of nonlocal reduction [8], [11] is to “inject” in Popov functionals of infinite dimensional system the trajectories of stable system of low order.

Consider the equation

$$\ddot{\sigma} + a\dot{\sigma} + \varphi(\sigma) = 0 \quad (46)$$

with $\varphi(\sigma)$ described in previous section.

This equation has been exhaustively investigated (see [8] and references therein). It is well known that it has a bifurcational value $a_{cr}(\varphi)$ such that for $a > a_{cr}$ the limit relations (32) are true for its every solution.

The frequency-algebraic stability criterion exploiting for (24) the nonlocal reduction technique is as follows.

Theorem 3: [8] Suppose there exist numbers

$s_1 \leq \mu_1, s_2 \geq \mu_2, \varepsilon, \tau > 0, \lambda \in (0, r)$ such that the conditions are valid:

1) the frequency-domain inequality

$$\pi(\omega, \lambda) \triangleq \operatorname{Re}\{K(i\omega - \lambda) - \tau(K(i\omega - \lambda) + s_1^{-1}(i\omega - \lambda))^* \cdot (K(i\omega - \lambda) + s_2^{-1}(i\omega - \lambda))\} - \varepsilon|K(i\omega - \lambda)|^2 \geq 0 \quad (47)$$

is true for all $\omega \geq 0$,

2)

$$2\sqrt{\varepsilon\lambda} > a_{cr}(\varphi). \quad (48)$$

Then for every solution of (24) the limit relations (32) are true.

Next assertion describes the application of Theorem 3 to equation (20) in case $s_2 = -s_1 = 1$.

Introduce the function

$$f(x) = \frac{4x(k_1 - I_1x)(k_2 - I_2x)}{A((k_1 + k_2) - (I_1 + I_2)x)}. \quad (49)$$

Let

$$M \triangleq \sup_{x \in [0, r]} f(x) \quad (50)$$

where

$$r = \min\left\{\frac{k_1}{I_1}, \frac{k_2}{I_2}\right\} \quad (51)$$

Theorem 4:

If

$$M > a_{cr}^2(\Phi) \quad (52)$$

then for every solution of (20) the limits (19) are valid.

Proof: Note that

$$K(i\omega - \lambda) = A\left(\frac{1}{\varkappa_1 + iI_1\omega} + \frac{1}{\varkappa_2 + iI_2\omega}\right), \quad (53)$$

where

$$\varkappa_i = k_i - \lambda I_i \quad (i = 1, 2). \quad (54)$$

It is not difficult to establish by direct computation that if

$$\varepsilon + \tau = \frac{\varkappa_1 \varkappa_2}{A(\varkappa_1 + \varkappa_2)}, \quad (55)$$

the condition 1) of Theorem 3 is satisfied.

We choose the value of λ such that

$$f(\lambda) = M \quad (56)$$

Then

$$4\lambda(\varepsilon + \tau) = M. \quad (57)$$

It is sufficient to choose

$$\tau < \frac{M - a_{cr}^2}{4\lambda} \quad (58)$$

so that condition 2) of Theorem is fulfilled. Theorem 4 is proved. ■

V. CONCLUSION

In this paper the problem of synchronization between two vibro-excitors (eccentric rotors) installed on a common oscillating platform is considered. By means of stability theory for “pendulum-like” systems we establish analytic criteria for self-synchronization of two rotors.

ACKNOWLEDGMENT

This work was supported by Russian Foundation for Basic Research, grant 20-01-00619.

REFERENCES

- [1] I.I. Blekhan, “Self-synchronization of vibrators in some types of vibrational machines (in Russian),” *Inzhenerny Sbornik*, vol. 15, pp. 49–72, 1953.
- [2] I.I. Blekhan, *Vibrational mechanics* World Scientific Publ. Co., 2000.
- [3] Xiaozhe Chen and Xiangxi Kong and Xueliang Zhang and Lingxuan Li and Bangchun Wen, “On the Synchronization of Two Eccentric Rotors with Common Rotational Axis: Theory and Experiment,” *Shock and Vibration*, vol. 2016, p. 6973597, 2016.
- [4] Pan Fang and Qiming Yang and Yongjun Hou and Ye Chen, “Theoretical study on self-synchronization of two homodromy rotors coupled with a pendulum rod in a far-resonant vibrating system,” *Journal of Vibroengineering*, vol. 16, no. 5, pp. 2188–2204, 2014.
- [5] Pan Fang and Min Zou and Huan Peng and Mingjun Du and Gang Hu and Yongjun Hou, “Spatial synchronization of unbalanced rotors excited with paralleled and counterrotating motors in a far resonance system,” *Journal of Theoretical and Applied Mechanics*, vol. 57, no. 3, pp. 723–738, 2019.
- [6] Sperling L. and Merten F. and Duckstein H., *Rotation und Vibration in Beispilen zur Methode der direkten Bewegungsteilung*, *Technische Mechanik*, vol. 17, no. 3, pp. 231–243, 1977.
- [7] V. Rasvan, “Four lectures on stability. Lecture 3. The frequency domain criterion for absolute stability,” *Control Engineering and Applied Informatics*, vol. 8, no. 2, pp. 13–20, 2006.
- [8] G. A. Leonov, D. Ponomarenko, and V. B. Smirnova, *Frequency-Domain Methods for Nonlinear Analysis. Theory and Applications*, Singapore–New Jersey–London–Hong Kong: World Scientific, 1996.
- [9] G. A. Leonov, V. Reitmann, and V. B. Smirnova, *Non-local methods for pendulum-like feedback systems*, Stuttgart–Leipzig, Teubner, 1992.
- [10] V. B. Smirnova and A. V. Proskurnikov, “Volterra equations with periodic nonlinearities: multistability, oscillations and cycle slipping,” *Int. J. Bifurcation and Chaos*, vol. 29, no. 5, p. 1950068 (26p.), 2019.
- [11] G. A. Leonov, “The nonlocal reduction method in the theory of absolute stability of nonlinear systems,” *Automation and Remote Control*, vol. 45, no. 3, pp. 315–323, 1984.
- [12] Ju.N. Bakaev and A.A. Guzh, “Optimal reception of frequency modulated signals under Doppler effect conditions (in Russian),” *Radiotekhnika i Elektronika*, vol. 10(1), pp. 175–196, 1965.
- [13] A. V. Proskurnikov and V. B. Smirnova, “Constructive estimates of the pull-in range for synchronization circuit described by integro-differential equations,” *Proc. of IEEE Int. Symposium on Circuits and Systems (ISCAS)*, pp. 09180519 (5p.), Seville, Spain, 2020.
- [14] V.B Smirnova and A.V. Proskurnikov, “Self-synchronization of unbalanced rotors and the swing equation,” *IFAC-PapersOnLine*, vol. 54, no. 17, pp. 71–76, 2021.