PEC Wedge in Anisotropic Media: Generalized Wiener-Hopf Equations

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The study of PEC wedge in anisotropic media is of great interest e.g. in GPR and aerospace applications. The extension of the Wiener-Hopf (WH) technique in angular regions (Daniele, Lombardi, Scattering and Diffraction by Wedges 1 and 2, Wiley-ISTE, 2020 and references therein) demonstrated its efficacy to solve wave scattering problems in presence of geometries containing angular regions and/or stratified planar regions, see for instance (Daniele, et al., IEEE TAP, 66:12, pp. 6482–6499, 2018) and (Daniele, et al., Radio Sci, 52, pp. 1472–1491 and pp. 1492–1509, 2017). The general procedure, first, deduce the functional equations in spectral domain of angular regions; second, impose the boundary conditions to get the Generalized Wiener-Hopf Equations (GWHEs) of the problem and; finally, calculate the solution of the system of the WH equations using exact or semi-analytical/approximate techniques of factorization (Daniele, Lombardi, Radio Sci, 42, pp. 1–19, 2007, RS6S01). In this work, we extend the possibility to analyze the scattering problem by wedges to a general linear medium, in particular, we start from the problem constituted of a PEC wedge immersed in a simple anisotropic dielectric medium with tensor permittivity:

$$\boldsymbol{\varepsilon} = \begin{vmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix} \tag{1}$$

The novel derivation of functional equations is obtained in angular regions using trasversalization of Maxwell's equations (Bresler, Marcuvitz, Report R-495,56, PIB-425, MRI Polytechnic Institute of Brooklyn, 1956) in oblique Cartesian coordinates (Proc. R. Soc. A., Proc. R. Soc. A., 2021, submitted). The solution of these oblique transverse differential equations of first order in Laplace domain are projected on the reciprocal eigenvectors of the algebraic operator matrix. It yields the functional equations of an arbitrary angular region. Once obtained the functional equations we impose boundary conditions, obtaining the GWHEs of practical problems. We note that the functional equations, in case of arbitrary linear media, are more complex due to the constitutive parameters of the medium that are tensor in this case. Isotropic case has been extensively reported in the cited papers. We recall that GWHEs differ from the Classical Wiener-Hopf equations (CWHEs) for the definitions of the unknowns in spectral domain. While CWHEs introduce plus and minus functions that are always defined in the same complex plane, the GWHEs present plus and minus functions that are defined in different complex planes but related together. In several important practical cases, suitable mappings allow to redefine the plus and the minus functions of GWHEs in the same complex plane, in particular for isotropic medium. In general the mapping is always available when only a propagation constant is present, instead in media with two different propagation constants the mapping is not available and we need to work on the original multiple complex planes defined in terms of the propagation constants. The selection of anisotropies as (1) forbids the reduction to CWHEs but it allows to work in multiple complex planes with Cauchy decomposition procedures with spectral propagation constants along wedge face directions of simple forms. Further details on the formulation, numerical validations and results will be shown during the presentation. This work is supported by the PRIN Grant 2017NT5W7Z GREEN TAGS.