Machine Learning for the Uncertainty Quantification of Power Networks

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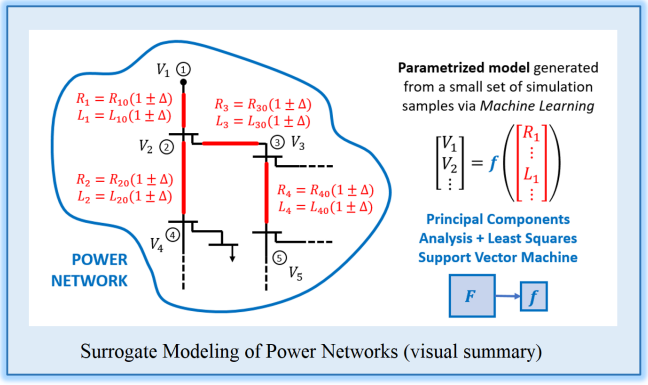
***Abstract*—This paper addresses the uncertainty quantification of a power network and is based on surrogate models built via Machine Learning techniques. Specifically, the least-square support vector machine regression is combined with the principal component analysis to generate a compressed surrogate model capable of predicting all the nodal voltages of the network as a function of variations of electrical parameters of the transmission lines. The surrogate model is built from a limited number of system responses provided by the computational model. The power flow analysis of the benchmark IEEE-118 bus system with 250 parameters is considered as a test case. The performance of the proposed modeling strategy in terms of accuracy, efficiency and convergence are assessed and compared with those of an alternative surrogate model based on a sparse implementation of the polynomial chaos expansion. The results of a Monte Carlo simulation are used as reference in the above comparison.**

***Index Terms*—Power systems, smart power grids, uncertainty quantification, power-flow analysis, surrogate models, least-squares support vector machine, high-dimensional problems**



Surrogate Modeling of Power Networks (visual summary)

I. Introduction

The reliability assessment of a modern electrical power system must incorporate the effects of unavoidable fluctuations of uncertain parameters in the generation, transmission and distribution networks. For the latter two, typical examples include the number, location and strengths of renewable generators with possible impact of weather effects [1], [2]. As an alternative to uncertain renewables, the physical parameters of transmission lines and other equipment in the power system might change due to aging effects or extreme weather conditions. These changes result in the modification of the electrical parameters of the lines, possibly leading to a non-optimal behavior of the power network or even to its failure.

In this framework, the availability of tools for Uncertainty Quantification (UQ) is helpful to address the inherent uncertain nature of the problem at hand. From a statistical perspective, the classical solution is offered by the Monte Carlo (MC) approach with a clever selection of simulation samples. The execution of a large set of deterministic simulations helps to extract, the statistical responses of the system in terms of the probability density function (PDF) of the branch power flowing through the lines or the voltage profile of the network, i.e. the magnitude and phase of the steady-state nodal voltages along the network. However, MC suffers from slow convergence rate, which means that usually a large number of simulations (in the order of several thousands) with the expensive computational model is usually required, yielding unaffordable CPU time for realistic networks.

The above problem has driven the research towards the development of efficient surrogate modeling techniques for both the worst-case and the statistical assessments of complex dynamical systems [2]– [6]. Among the available tools, Polynomial Chaos Expansion (PCE) and its advanced im-

Take-Home Messages:

- Flexible and powerful Machine Learning regressions can be considered as a viable and effective solution for the construction of accurate models for the uncertainty quantification in energy applications (e.g., electrical power distribution or transmission systems)
- Principal component analysis is combined with the least squares support vector machine regression to construct a compact surrogate model
- The surrogate model is able of predicting the nodal voltages of the network as a function of its parameters
- The proposed method offers a solution for both uncertainty quantification and the generation of a parametrized model
- High model scalability, allowing to handle hundreds of input parameters (250 parameters are considered in this study)
- Application to a large benchmark IEEE power transmission system where the electrical parameters of the transmission lines are varied in a wide range around their nominal value
- Feasible tool for the reliability assessment of a power network with many parameters

plementation, such as the sparse PCE, can be considered as the reference solution for UQ [7], [8]. A large research literature is available along with toolboxes implementing the most advanced techniques [9]. Recently, Machine Learning has also led to a proven viable and robust alternative offering comparable or even better results in many applications [10]–[12].

In this study, the uncertainty of branch elements in a power system is analyzed in order to possibly take into account the changes in the physical connection medium. The electrical parameters of transmission lines such as the equivalent resistance and inductance are thus considered as the varying parameters in our system. A Machine Learning method based on the so-called Least-Squares Support Vector machine (LS-SVM) regression is used along with principal component analysis (PCA) to build a compact and efficient surrogate model of all the nodal voltages of the power network with hundreds of uncertain parameters. According to [13], the combination of LS-SVM regression and PCA allows providing a compressed model of a multi-output system with several output variables. Therefore, such modeling approach can be seen as a promising candidate for the considered application. A systematic discussion of model performance is carried out with a comparison, in terms of convergence, accuracy and efficiency, of the proposed modeling scheme with the sparse PCE expansion. Results are validated via MC simulations with the computational model, which are assumed as the reference responses.

II. Macromodeling via Machine Learning

The discussion starts by considering a set of training data $\{(\mathbf{x}_l, \mathbf{y}_l)\}_{l=1}^{L}$ calculated via a generic multi-output computational model $\mathbf{y} = \mathcal{M}(\mathbf{x})$, which provides a non-linear map between the input parameters $\mathbf{x} = [x_1, \ldots, x_d]^T \in \mathbb{R}^d$ and the output vector $\mathbf{y} = [y_1, \ldots, y_M]^T \in \mathbb{R}^M$, collecting the nodal voltages of a power transmission or distribution network.

Our goal is to find a surrogate model $\tilde{\mathcal{M}}$ such that:

$$y_{l,m} \approx \tilde{\mathcal{M}}(\mathbf{x}_l), \tag{1}$$

for $m = 1 \ldots, M$ and $l = 1, \ldots, L$, where $y_{l,m}$ denotes the mth output variable for the lth configuration of the training samples.

This work investigates the combination of PCA with two different advanced approaches for the model construction, the sparse PCE and the LS-SVM regression, being the latter discussed in detailes hereater in this section. PCE is instead only used as a reference tool for comparison (see [8], [9], [13]).

A. Least-Square Support Vector Machine (LS-SVM) Regression

Given a subset $D_m = \{(\mathbf{x}_l, y_{l,m})\}_{l=1}^{L}$ of the training set in which the scalar values $y_{l,m}$ correspond to m-th system output, we are looking for a surrogate model constructed via the LS-SVM regression, which in its dual space formulation reads:

$$\tilde{\mathcal{M}}_{LS\text{-}SVM,m}(\mathbf{x}) = \sum_{l=1}^{L} \beta_{l,m} K(\mathbf{x}_l, \mathbf{x}) + b_m \tag{2}$$

where $\beta_{l,m} \in \mathbb{R}$ are scalar coefficients for the mth output, $K(\cdot,\cdot) : \mathbb{R}^d \to \mathbb{R}$ is the kernel function and $b_m \in \mathbb{R}$ is the bias term.

For each output variable, the LS-SVM regression estimates the optimum set of pramaters $\beta_{l,m}$ and b_m by minimizing the sum of the squared errors between the model predictions and the training samples. The above constraint, along with the one on the model flatness, allows estimating the regression coefficients via the solution of a least-square problem. In contrast to PCE [13], the dual space formulation of the LS-SVM regression in (2) provides a non-parametric model in which the number of regression unknowns to be estimated for each output variable is independent of the number of input parameters (i.e., d) and it equals to the number of training samples (i.e., L).

B. Principal Component Analysis (PCA)

Unfortunately, the direct application of the above "black-box" regression techniques to the modeling of systems with many outputs (e.g., in the order of thousands) is rather cumbersome, as it would require building a separate model for each of the output variables of interest. PCA can be seen as a promising technique able to mitigate the above issue. The key idea is to consider the possible correlation among the output variables in order to compress them and reduce the number of surrogate models to be trained.

The complete dataset of training responses $\{\mathbf{y}_l\}_{l=1}^{L}$ is recast in terms of the matrix $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_L] \in \mathbb{R}^{M \times L}$. Then the matrix $\mathbf{Y}$ is used within the PCA algorithm to find out the smallest set of "principal components", approximating the training responses $\{\mathbf{y}_l\}_{i=1}^{L}$ as [14]:

$$\mathbf{y}_l \approx \boldsymbol{\mu}_{\boldsymbol{\zeta}} + \sum_{n=1}^{\bar{n}} Z_{l,n} \mathbf{u}_n, \tag{3}$$

for $l = 1, \ldots, L$ where:

$$Z_{l,n} = \mathbf{u}_n^{\mathsf{T}} (\mathbf{y}_l - \boldsymbol{\mu}), \tag{4}$$

the vector $\boldsymbol{\mu}$ is the column-wise mean, and the principal components $\{\mathbf{u}_n\}_{n=1}^{\tilde{n}}$ are the left singular vectors calculated via singular value decomposition (SVD) applied to matrix $\mathbf{Y}$. The number of components to be considered in the approximation (3) is identified by setting a threshold on the relative magnitude of the singular values. In this paper, the number of principal components $\tilde{n}$ is selected in such a way that the magnitude falls below 10% of the largest singular value.

The PCA coefficients $\{Z_{l,n}\}_{l,n=1}^{L,\tilde{n}}$ defined in (4) can be interpreted as a collection of L samples of $\tilde{n}$ new output system variables $Z_n(\mathbf{x})$, with $Z_{l,n} = Z_n(\mathbf{x}_l)$, describing the information pertaining to the entire set of system outputs $\mathbf{y}$. Since typically $\tilde{n} \lll M$, the PCA truncation leads to a

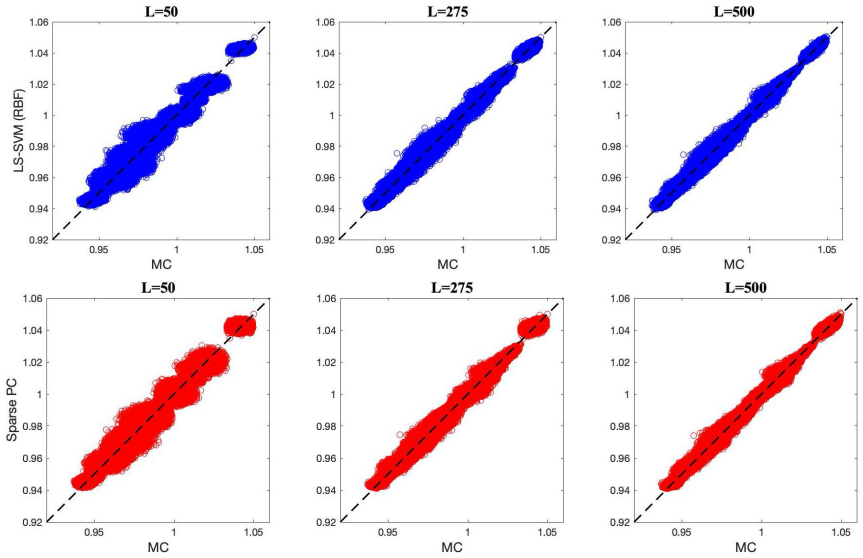


Fig. 1. LS-SVM scatter plots (top panel) and sparse PCE scatter plots (bottom panel) with increasing number of training samples.

substantial compression of the number of variables to be modeled. Each of these reduced variables can be approximated using any regression technique. Once a model is available for the compressed variables Z_n, new samples for the original output variables can be recovered via (3).

III. Numerical Results

The test case considered in this study is the IEEE-118 bus system. It represents a portion of the American Electric Power System, with some modifications [15]. It includes 19 generators, 35 synchronous condensers, 9 transformers, 91 loads and 177 transmission lines. The network has 118 voltage nodes (i.e., $M = 118$ output variables). The nominal minimum and maximum voltage (at nominal parameter values) of the network is 0.943 per unit (p.u.) and 1.050 p.u., respectively. The uncertain parameters considered are the equivalent resistance and inductance of the branch transmission lines. A total of 125 random branches are selected. The uncertain parameters are modeled as uniform random variables with a variability of 50% around their nominal values specified in [15], leading to an overall number of $d = 2 \times 125 = 250$ uncertain parameters. The above configuration scenario has been implemented within a MC simulation in which the solver Matpower [16] is used as load flow computational model.

Our goal is to construct a compact and accurate surrogate model able to predict the voltage magnitude of the 118 nodes of the network computed via a power flow analysis as a function of the considered 250 uncertain parameters. This means that we are looking for a model with $M = 118$ output variables and $d = 250$ uncertain input parameters. The proposed surrogate model based on the combination of LS-SVM and PCA is applied. The training samples are generated by means of a Latin Hypercube sampling (LHS) scheme [17]. An increasing number of training samples (i.e., $L = 50$, 275 and 500) is considered in the following analysis. First of all, the PCA is applied to the training responses. From the PCA results, only $\tilde{n} = 16$ coefficients out of the initial 118 are considered in order to achieve the required tolerance, leading to a compression rate of $7\times$. Then, the LS-SVM regression with a Radial Basis Functions (RBF) kernel is adopted to train the 16-coefficient surrogate model by using the LS-SVMlab toolbox [12]. A corresponding model based on sparse PCE with maximum order 2 has been trained via the UQlab tool [9].

Figure 1 (top panel) shows the scatter plots of the correlation between the nodal voltage values (i.e. the output vector $\mathbf{y}$) calculated from a MC simulation with 10000 samples and the corresponding predictions provided by the proposed surrogate model based on the LS-SVM regression trained with increasing number of training samples. Similarly, the bottom panel of the same figure shows the same scatter plots generated by a surrogate model built with a second-order PCE. The above comparison highlights the convergence of the considered methods with increasing training samples and the comparable and very good accuracy of both LS-SVM and PCE surrogates. However, it is important to point out that the order of the PCE can be hardly increased in high dimensional cases, as the one considered in this paper, due to the memory issues in the calculation of higher-order coefficients [2].

To provide a further comparison between LS-SVM and PCE, Fig. 2 shows the PDFs of the nodal voltage profile obtained from the MC samples (red bars), with the predictions of the proposed surrogate model LS-SVM (blue bars) and the predictions of PCE (green bars), both trained with 500 training samples. Again, the results confirm the capability of the two surrogates in providing a good estimation of the statistical behavior of the network nodal voltages.

Finally, Tab. I provides a summary comparison of methods by means of some quantitative numbers on model performance with increasing number of training samples for both LS-SVM and PCE. The table reports the root mean squares error (RMSE) computed from the MC and model responses, the CPU time required by (i) the generation of training samples (see the information about "cost" inserted nearby indication

TABLE I
MODEL PERFORMANCE IN TERMS OF BOTH ACCURACY AND EFFICIENCY FOR AN INCREASING NUMBER OF TRAINING SAMPLES L.

$\mathbf{d = 250}$	$L = 50$ (cost=4.45 s)			$L = 275$ (cost=12.38 s)			$L = 500$ (cost=31.17)		
Method	RMS Error	t_{model}	t_{cost}	RMS Error	t_{model}	t_{cost}	RMS Error	t_{model}	t_{cost}
MC	–	–	490.7 s	–	–	490.7 s	–	–	490.7 s
LS-SVM (RBF)	0.00178	7.48 s	0.49 s	0.00124	14.4 s	1.6 s	0.0001087	29.1 s	2.24 s
Sparse PC	0.0019	21.1 s	5.88 s	0.00129	41.4 s	7.3 s	0.0001089	76.6 s	7.9 s

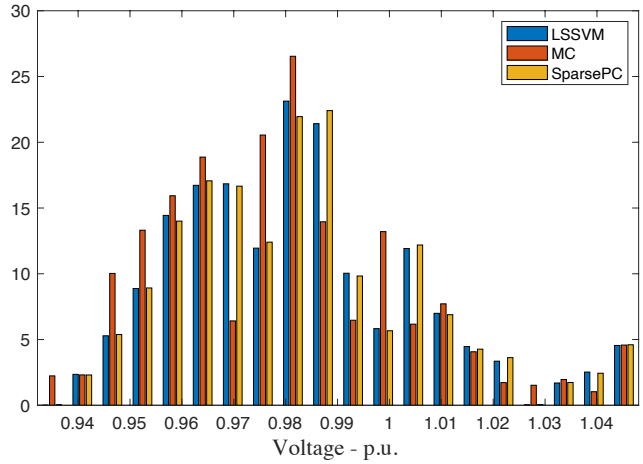


Fig. 2. PDFs produced by MC, LS-SVM and Sparse PCE for 500 training samples

of training samples L, in the header of the three sections of the table); (ii) the model generation (columns labelled as t_{model}) and computation of responses (columns labelled as t_{cost}) needed for the plots in Fig. 1. All the simulations have been performed with MATLAB on a Notebook with a Dual-Core Intel Core i5 CPU running at 3.1 GHz and 16 GB of RAM. From the numbers given in Tab. I, it is relevant to notice that the performance of both models, the LS-SVM and the sparse PCE, is always very good both in terms of RMSE and efficiency with respect to the plain MC simulation. However, when comparing LS-SVM and PCE, the amount of time required by LS-SVM is much less when using fewer training samples.

IV. CONCLUSION

Machine Learning is used in this paper to generate a compact parametric model of a power network in which the electrical parameters of the transmission lines are varied. A large variability of $\pm 50\%$ around the nominal values of the equivalent resistances and inductances is considered. Two approaches are considered in this study, namely the LS-SVM regression and the sparse PCE, and their results are compared with the ones provided by a MC simulation. A systematic assessment in terms of their accuracy, efficiency and convergence as a function of the number of training samples is carried out. LS-SVM has been proven to be an alternative viable solution for both uncertainty quantification and parametric assessment of a power network with large size and large number of parameters.

ACKNOWLEDGMENT

This research has been supported by Politecnico di Torino under the “Joint research projects with top universities”.

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