# Fatigue Damage Estimation from Random Vibration Testing: Application to a notched specimen

**CSMA 2022**
15ème Colloque National en Calcul des Structures
16-20 mai 2022, Presqu'île de Giens (Var)

# Fatigue Damage Estimation from Random Vibration Testing: Application to a notched specimen

L. Campello[1,2], R. Serra[1], R. Sesana[2], C. Delprete[2]

[1] *INSA Centre Val de Loire, Laboratoire de Mécanique Gabriel Lamé, France, {luca.campello@insa-cvl.fr; roger.serra@insa-cvl.fr*
[2]*Politecnico di Torino, DIMEAS, , Italy, luca.campello@studenti.polito.it; raffaella.sesana@polito.it; cristiana.delprete@polito.it*

**Résumé** — Vibrations are random in a wide range of applications and they are the main cause of mechanical failure. To prevent such failure, it is necessary to evaluate the fatigue life using test or analysis techniques. For computing the severity of the damage many methods are available in literature, but the estimation damage is just an approximation. The objective of this study is to propose a numerical model, together with experimental validation, in order to estimate fatigue damage caused by random vibrations in metallic materials undergoing uniaxial fatigue testing.

**Mots clefs** — Random vibrations test, Fatigue Damage, Virtual test; frequency-domain counting method.

## 1. Introduction

Many mechanical components are subjected to vibrations generating in the material a stress history, related to vibration amplitude and frequency. Vibration can lead to fatigue failure, as a result of accumulated damage [12]. An adequate design of components and systems allows to estimate the damage of the components subject to vibrations and to predict its life [12,18]. Most of the vibrations are non-deterministic vibrations, so called, *random vibrations* [7]. They are non-deterministic excitations, a type of oscillation whose behaviour is non-predictable and non-repeatable, which make the design of the component harder [1]. Many methods of predicting the vibration response of mechanical and structural systems to random vibrations are presented in technical literature [14, 18]. The fatigue damage assessment can be approached both in time and in frequency domain [14,3]. In time domain, Rainflow Cycle Counting and linear damage rule are used; in the frequency domain several methods are available [14]. The approach in frequency domain carries a significant benefit in terms of calculation time and a more efficient assessment of random process [14]. In frequency domain the load is described by Power Spectral Density (PSD) function, which represents the distribution of power into frequency components composing the time-signal [1]. However, the damage estimation results are approximated, and, for a reliable estimation of damage, large and expensive testing campaigns are required. Damage and life simulation models are a helpful tool nowadays. The purpose of this work is to implement and validate a Finite Element Method (FEM)-model whose goal is to predict with a reliable accuracy the fatigue damage and the residual useful life of a AISI 304 specimen loaded with a random vibration.

## 2. Theoretical background

### 2.1. Random vibration

Zero mean, static Gaussian Random vibrations $X(t)$ are generally described in the frequency domain by means of Power Spectral Density (PSD) functions $S(\omega)$[7]. Generally, instead of using the PSD, it is customary to use the one-side spectral density $G_{xx}$. A random signal is characterized by a set of spectral moment [1].

$$m_i = \int_0^\infty \omega^i G_x x(\omega) d\omega \tag{1}$$

which represent important properties like variance and its derivative

$$m_0 = \sigma_x^2 m_2 = \sigma_{\dot{x}}^2 m_4 = \sigma_{\ddot{x}}^4 \tag{2}$$

Others important parameters are *expecting positive zero crossing rate* and *expecting peak*, defined respectively

$$E_0 = \sqrt{\frac{m_2}{m_0}} \quad (3) \qquad E_p = \sqrt{\frac{m_4}{m_2}} \quad (4)$$

The spectral density could be described also by an *irregularity factor* or *bandwidth parameters*

$$\alpha_m = \frac{m_i}{\sqrt{m_0 m_{2m}}} \quad \text{con m=1…2} \tag{5}$$

The most used are [3]

$$\alpha_1 = \frac{m_1}{\sqrt{m_0 m_2}} \quad (6) \qquad \alpha_2 = \frac{m_2}{\sqrt{m_0 m_4}} \quad (7)$$

If $\alpha_1$ tends to 0 the vibration is called a *wideband vibration* where the PSD has significant value in a wide frequency range, instead if $\alpha_1$ tends to 1, the process is called a *narrowband vibration* where PSD has significant value only in short band centered around a frequency value [12, 1].

The probability distribution of the peaks value of the random signal is well described by *Rice distribution* [1,18]. In the case of a narrow band vibration, the probability distribution is described by the *Rayleigh distribution* [1].

## 2.2. Structural Dynamic and Modal Analysis

The aim of the dynamic analysis of flexible structures is to characterise the response of the structures excited by dynamic loads. A flexible structure can be described as a multi-degree of freedom (MDOF) [5]:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \tag{8}$$

where [M] is the mass matrix, [C] is the damping matrix and [K] is the stiffness matrix of the structure, {f} is the external excitation force vector and {x} is the displacement vector of the degrees of freedom. The solution of the eigenvalue problem gives the natural frequencies $\omega_i$ and the modal shapes $\{\psi_i\}$, characterizing the dynamic behavior of the structure [11, 5]. Once the natural frequencies and the mode shapes are obtained, it is possible to write the Frequency Response Function (FRF) of the system, which is [5]:

$$H(\omega)_{jk} = \sum_{i=1}^{N} \frac{\{\psi_{jr}\}^T\{\psi_{kr}\}}{\omega_r + \omega^2 + 2i\zeta\omega_i\omega} \tag{9}$$

Where $2\xi\omega_i\eta_i$ is related to the damping. The FRF is a transfer function, expressed in the frequency-domain which relates the output of the system to the unit input at each frequency.

The PSD of the excitation $S_{XX}(\omega)$ can be related to the PSD of the response $S_{YY}(\omega)$ by means of the FRF [15] as follows:

$$S_{YY} = H(\omega)^2 S_{XX}(\omega) \tag{10}$$

The response characteristic can be expressed in term of displacement, stress or acceleration [15].

### 2.3. Spectral fatigue damage estimation

In uniaxial testing of a specimen, the relation between the stress level σ and the number of cycles before failure *N* are described by the Basquin equation [12]:

$$\sigma_i = CN^{-1/b} \tag{11}$$

where σ is the stress amplitude, *C* is the fatigue strength coefficient and *b* is the fatigue exponent, which describes the behavior of the Wöhler diagram [12]. In case of variable amplitude loading, the common way to compute the fatigue damage is under the hypothesis of the linear accumulation principle expressed by the Palmgreen and Miner's rule [12,13]:

$$D_i = \sum \frac{n_i}{N_i} \tag{12}$$

where $n_i$ is the number of cycles in the stress amplitude $\sigma_i$ and $N_i$ are the allowable number of cycles at the stress amplitude $\sigma_I$. When *D=1*, it is assumed that structural failure occurs [12,13].

Knowing the probability distribution function *p(σ)* of the random vibration and the Wöhler curve parameters, the damage could be estimated using the following expression [3]:

$$D_i = E_p C^{-1} \int_0^\infty \sigma^k p(\sigma) * \Delta\sigma \tag{13}$$

with *k=1/b*. Many methods for estimating the fatigue damage are available [4, 11]. The principal difference between them is the way they approximate the probability density function (*PDF*) *p(σ).*

Some of them, as Wirsching-Light method [20], Ortiz Chen method and Tovo-Benasciutti method [2], start from the PDF of a narrowband vibration and propose a correction factor for the fatigue life in order to obtain a broad band distribution fatigue life. Other methods, like Dirlik method [4], Zao-Baker method [21], Lalanne method [8] and Larsen-Lutes method [9], propose an alternative interpretation of the PDF [15, 19].

# 3. Experimental test

The experimental part is made of three steps: experimental modal analysis (EMA), steady state dynamic analysis (SSD) and vibration fatigue tests.

## 3.1. Specimen parameters

The specimen used for the experiments is made of AISI 304 (Figure 1). It is designed with two notches to speed to induce the failure in the notch [16]. Two specimens (named A-1 and A-2) were used for model calibration and validation. The properties of the specimens are reported in Table 1.

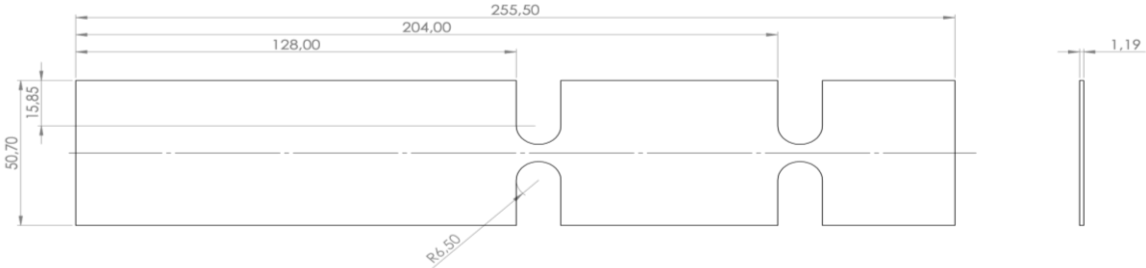


Figure 1 - Specimen geometry

Table 1 - Physical properties of the specimens

| Properties | Specimen A-1 | Specimen A-2 |
|---|---|---|
| Thickness [mm] | 1,19 | 1,19 |
| Mass [kg] | 0,1127 | 0,0113 |
| Density [kg/m$^3$] | 7982,50 | 8003,75 |
| Fatigue exponent b [17] | 0,2 | 0,2 |

## 3.2. Specimen dynamic response

In the EMA, the specimens were fixed as a cantilever beam as illustrated in the Figure 2. The A-1 specimen was excited by means of an impact hammer in 6 different points on longitudinal axis with the aim of obtaining the flexional mode shapes excited as shown in Figure 3.

The specimen response was evaluated by an accelerometer placed in the free end. Hammer and accelerometer signal were acquired by means of LMS Test lab software.

Figure 2 - Experimental test bench

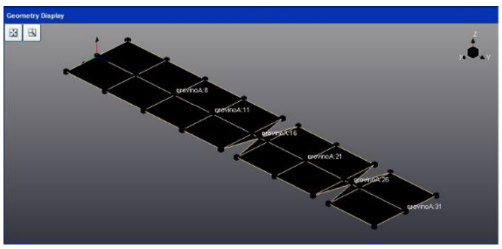

Figure 1 - Positions of the impacts

Successively, a steady state dynamic analysis has been performed, with the aim of determining the acceleration FRF. Tests were run by means of an electrodynamic shaker and with a closed loop control system [19], connected to a computer (Figure 4), the two specimens have been subjected to an acceleration of 1g over the frequency range [5-300 Hz]. The value of natural frequencies and modal damping extracted are reported in Table 3.

### 3.3. Random vibration fatigue tests

Random vibration fatigue tests were performed with the electrodynamic shaker and the closed loop control system as shown in Figure 4.

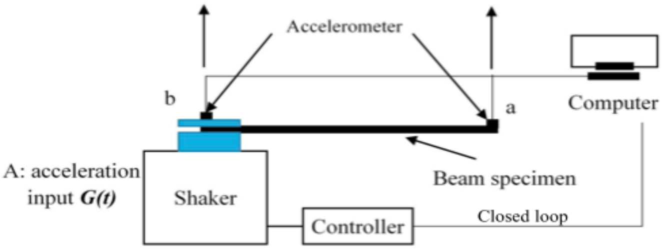


Figure 4 - General layout of the system, shaker, closed loop control system and accelerometer

The acceleration was described by the corresponding PSD. For analyzing the fatigue properties of the material, Locati method was used, by means of step loading starting below the fatigue limit and gradually increasing of a set step size until specimen failure [19]. According to the frequency obtained with SSD analysis, the load frequency range was 43-123 Hz, centered to the second frequency 83 Hz; the corresponding bandwidth was 80 Hz. The initial PSD load was set to 0,25 $g^2$/Hz and the step size was defined as 0,05 $g^2$/Hz (Table 2). The specimen A1 failed at 0,75 $g^2$/Hz after 10h42min and the specimen A2 failed at 0,8 $g^2$/Hz after 11h15min (Figure 5 & 6)

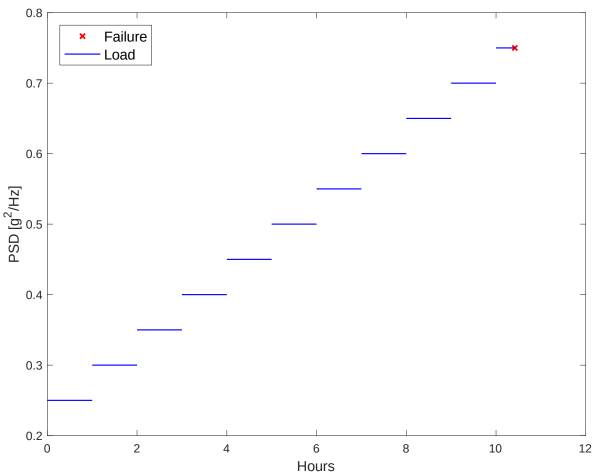


Figure 5 – Load history of specimen A1

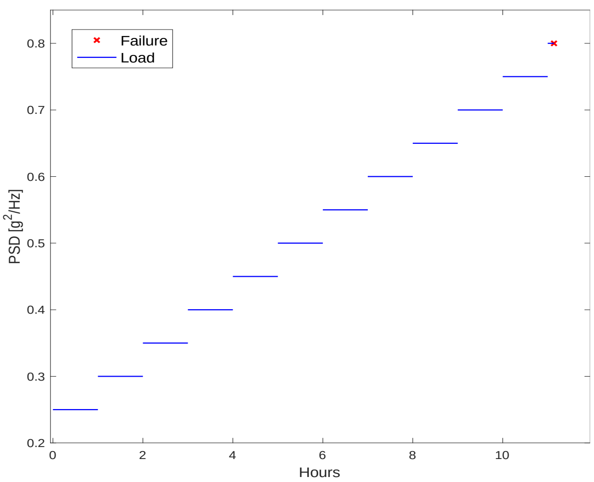


Figure 6 – Load history of specimen A2

| **Test** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PSD [$g^2$/Hz] | 0,25 | 0,3 | 0,35 | 0,4 | 0,45 | 0,5 | 0,55 | 0,6 | 0,65 | 0,7 | 0,75 | 0,8(only for A_2) |

Table 2 – PSD of the step input

## 4. Simulations

Simulation model representing with reliable accuracy the dynamic behavior of the specimens, are proposed by finite element analysis. Modal analysis and SSD are executed with ABAQUS with 4408 hexagonal elements which 2 elements in the thickness. The quadratic interpolation is chosen.
The value of density required is the average of those in Table 1, instead Young's Module (*E*), with the value of *E*=190MPa, is achieved from natural frequencies [10] extracted from EMA (Table 3) in order to obtain a more accurate value (Table 3).

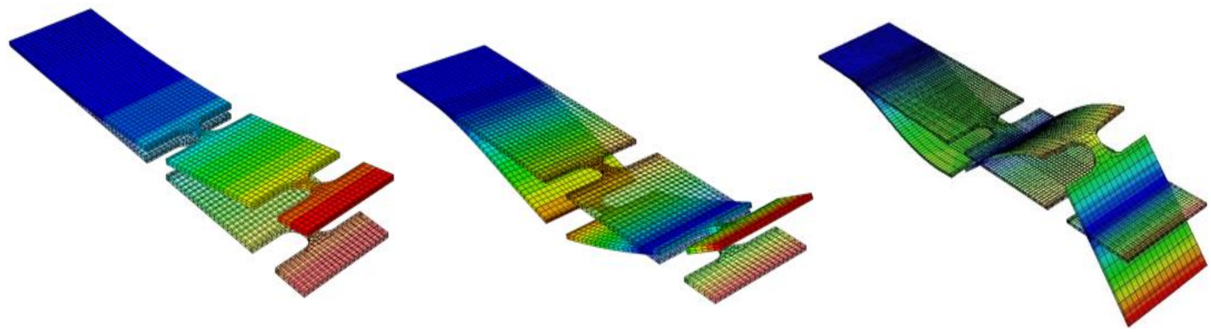

Figure 7 – Simulation of the 3 first flexional modes

For estimating the simulated-FRF of the model, the values of structural damping are needed. The firsts attempt value are those extracted from EMA, then they were updated in order to achieve close to the experimental FRF one (Figure 8). The natural frequencies of flexional mode shapes are given in Table 3 as well as the relative error between the simulation and the average of experimental data

| | **Mode** | **1** | **2** | **3** |
|---|---|---|---|---|
| Experimental Modal Analysis results | Frequency [Hz] | 14,02 | 76,30 | 238,10 |
| Experimental SSD analysis | Frequency (A1) [Hz] | 15,89 | 81,54 | 232 |
| | Frequency (A2) [Hz] | 16,08 | 83,29 | 237,16 |
| ABAQUS (after updating E) | Frequency [Hz] | 15,5 | 82,95 | 224,3 |
| Experimental /ABAQUS | Mean Relative error frequency (%) | 1 | 3,2 | 4,8 |
| Experimental SSD /ABAQUS FRF | Mean Relative error amplitude (%) | 2 | 3,7 | 12 |

Table 3 - Frequencies values and errors

Table 4 - Damping Values

| | **Mode** | **1** | **2** | **3** |
|---|---|---|---|---|
| Experimental Modal Analysis results | $\eta$ **[%]** | 7,8 | 2,6 | 1,4 |
| ABAQUS FRF (after updating $\eta$) | $\eta$ **[%]** | 6,8 | 3,1 | 3 |

Finally, the stress-FRF over the frequency range on the notch is shown in Figure 9.

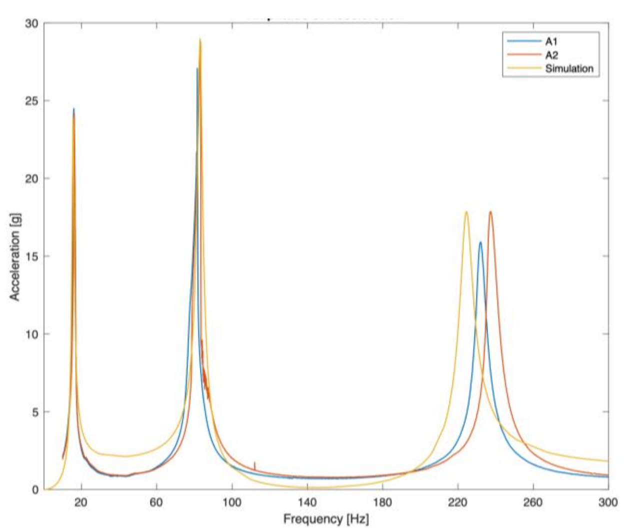


Figure 8 – Experimental and simulated FRF accelerations.

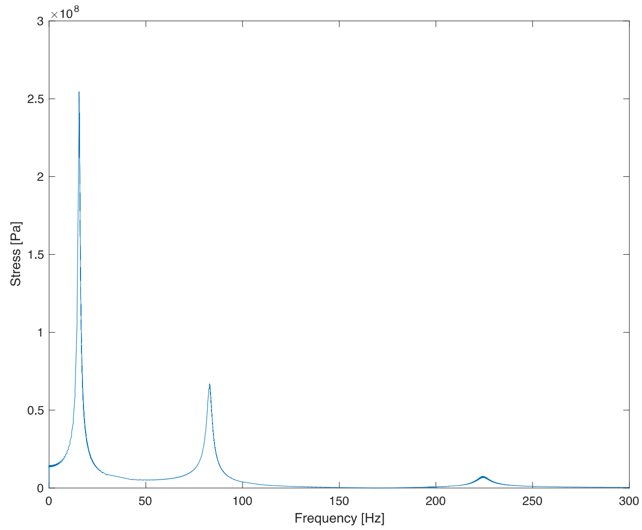


Figure 9 – Simulated max stress-FRF

## 5. Validation

From ABAQUS FRF model and according with the Locati method, the spectral fatigue damage value corresponding to every PSD-loading history is estimated by means of the Palmgreen-Miner model (eq.12) on the notch (max stress). In table 5 and 6, the results from the simulation for each of the two specimens are shown according with their experimental values.

Table 5 - Specimen A-1 : Spectral fatigue Damage estimation

| **PSD[g²/Hz]** | 0,25 | 0,3 | 0,35 | 0,4 | 0,45 | 0,5 | 0,55 | 0,6 | 0,65 | 0,7 | 0,75 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **gRMS [g]** | 4,5 | 4,9 | 5,3 | 5,7 | 6 | 6,3 | 6,6 | 6,9 | 7,2 | 7,5 | 7,8 | |
| **Time [s]** | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 2520 | $\Sigma D_i$ |
| **Damage** | 0,008 | 0,016 | 0,029 | 0,046 | 0,069 | 0,095 | 0,127 | 0,164 | 0,210 | 0,253 | 0,127 | 1,1 |

Table 6 - Specimen A-2 : Spectral fatigue Damage estimation

| **PSD [g²/Hz]** | 0,25 | 0,3 | 0,35 | 0,4 | 0,45 | 0,5 | 0,55 | 0,6 | 0,65 | 0,7 | 0,75 | 0,8 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **gRMS [g]** | 4,5 | 4,9 | 5,3 | 5,7 | 6 | 6,3 | 6,6 | 6,9 | 7,2 | 7,5 | 7,8 | 8,1 | |
| **Time [s]** | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 876 | $\Sigma D_i$ |
| **Damage** | 0,008 | 0,016 | 0,03 | 0,05 | 0,07 | 0,1 | 0,13 | 0,16 | 0,21 | 0,25 | 0,31 | 0,09 | 1,3 |

## 6. Conclusion

To summarize, in this paper, it has been presented a validated FEM model to simulate virtual random fatigue tests and to analyse the fatigue behaviour of the component. At the beginning, a dynamic analysis of the specimen was required. The results of simulated modal analysis and the simulated FRF were compared to results obtained from EMA and SSD. As illustrated in table 3, the error between the average of the experimental resonant frequencies and the simulated ones are less than 5%. The damping values used for the simulation are the result of an iteration starting from the experimental value. The fatigue test was conducting according the Locati method and damage value of each PSD load level, was extracted. The simulation fatigue cycle load, reproduced the Locati method, offers result fairly close to experimental ones, under the hypothesis of linear damage accumulation. So, it could be concluded that the model is a good representation of the fatigue behaviour of the metallic material

An extension of this research may be the comparison of the Max stress with strain gauges. In addition, it may be an interested the study of multiaxial random loading, considering that he presented model takes into consideration the uniaxial case

## References


[1] Bendat J.S., Piersol A.G., *Random Data: Analysis and Measurement Procedures,* Wiley & Sons, inc., New York, 1986

[2] Benasciutti D. and Tovo R. *Spectral methods for lifetime prediction under wide-band stationary random processes*. Int. J. Fatigue, 27(8):867–877, 2005.

[3] Benasciutti D. and Tovo R., *Comparison of spectral methods for fatigue analysis of broad-band Gaussian random processes,* Probabilistic Engineering Mechanics 21 (2006) 287–299

[4] Dirlik T. *Application of Computers in Fatigue Analysis*. PhD thesis, The University of Warwick, 1985.

[5] Genta Giancarlo, *Vibration Dynamics and Control*, Springer, 2009

[6] Irvine T., *Spectral moment notes,* Vibrationdata, 2015

[7] Kumar Santosh M, *Analyzing Random Vibration Fatigue*, ANSYS Advantage, Volume II, Issue 3, 2008

[8] Lalanne, C., *Mechanical Vibration and Shock*, Vol. 3, 4 and 5, Hermes Penton Science, London, 2009.

[9] Larsen C., Lutes L., *Predicting the Fatigue Life of Offshore Structures by the Single-Moment Spectral Method*, Probabilistic Engineering Mechanics, 6(2), 1991.

[10] McConnell Kenneth, *Vibration Testing: theory and practice*, Wiley & Sons, inc., New York, 1997

[11] Maia N.M.M., Silva J.M.M., *Theoretical and experimental Modal Analysis,* Wiley & Sons, inc., New York, 1997

[12] Milella Pietro Paolo, *Fatigue and corrosion in metals,* Springer, 2013

[13] Miner, M. A., *Cumulative Damage in Fatigue*, Journal of Applied Mechanics, Vol. 67, A159-A164, 1945.

[14] Mršnik Matjaž, Slavič Janko and Boltežar Miha, *Multiaxial Vibration Fatigue – A Theoretical and Experimental Comparison*. Mechanical Systems and Signal Processing, Vol. 76–77, p. 409–423, 2016.

[15] Mršnik Matjaž, Slavič Janko and Boltežar Miha, *Frequency-domain methods for a vibration-fatigue-life estimation - Application to real data*. International Journal of Fatigue, Vol. 47, p. 8-17, 2013.

[16] Nagulapalli Vinod Kumar, Gupta Abhijit, and Fan Shaofeng, *Estimation of Fatigue Life of Aluminum Beams subjected to Random Vibration*, Proceedings of International Modal Analysis Conference XXIV, paper# 268, pp. 1-6, 2007.

[17] Premium Material Database, nCode, 2009

[18] Quigley, J., Lee, Y., and Wang, L., *Review and Assessment of Frequency-Based Fatigue Damage Models,* SAE Int. J. Mater. Manf. 2016

[19] Wang Yuzhu, Serra Roger et Argoul Pierre, *Adapted Locati method used for accelerated fatigue test under random vibrations,* Procedia Structural Integrity 19 (2019) 674–681

[20] Wirsching P.H., Light M.C., *Fatigue under wide band random stresses*, J. Struct. Div., ASCE, 106(7), p. 1593-1607, 1980.

[21] Zhao W. and Baker M. J. *On the probability density function of rainflow stress range for stationary gaussian processes*. Int. J. Fatigue, 14(2):121– 135, March 1992.