

Effects of cell quality in grid boundary layer on the simulated flow around a square cylinder

*Original*

Effects of cell quality in grid boundary layer on the simulated flow around a square cylinder / Bruno, Luca; Oberto, Davide. - In: COMPUTERS & FLUIDS. - ISSN 0045-7930. - ELETTRONICO. - 238:(2022), p. 105351. [10.1016/j.compfluid.2022.105351]

*Availability:*

This version is available at: 11583/2958239 since: 2022-03-23T11:13:06Z

*Publisher:*

Elsevier

*Published*

DOI:10.1016/j.compfluid.2022.105351

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

Elsevier postprint/Author's Accepted Manuscript

© 2022. This manuscript version is made available under the CC-BY-NC-ND 4.0 license  
<http://creativecommons.org/licenses/by-nc-nd/4.0/>. The final authenticated version is available online at:  
<http://dx.doi.org/10.1016/j.compfluid.2022.105351>

(Article begins on next page)

# Effects of cell quality in Grid Boundary Layer on the simulated flow around a square cylinder

Luca Bruno<sup>a,\*</sup>, Davide Oberto<sup>b</sup>

<sup>a</sup>*Politecnico di Torino, Department of Architecture and Design, Viale Mattioli 39, I-10125, Torino, Italy*

<sup>b</sup>*Politecnico di Torino, Department of Mathematical Sciences, Corso Duca degli Abruzzi 24, I-10129, Torino, Italy*

---

## Abstract

The flow around a square cylinder is widely studied as a paradigmatic case in bluff body aerodynamics. The effects of several physical parameters of the setup, and the errors induced by turbulence models, numerical schemes and grid density have been emphasized in a huge number of studies during the past two decades. Surprisingly, the effects of the grid quality on such a class of flow has been overlooked. The lack of a shared approach and suggested best practices for high-quality grid generation among scholars and practitioners follows. The present study aims at filling this gap. The cell skewness and non-orthogonality are adopted as metrics of the grid quality. The errors induced by poor quality cells and the possible corrective measures are discussed in a Finite Volume Method framework. The effects of the cell quality on the simulated flow are systematically evaluated by a parametrical study including four different types of grid boundary layer. The obtained results are compared among them and discussed in terms of instantaneous and time averaged flow fields, stress distribution at wall, and aerodynamic coefficients. Both the overall modelling error and the skewness-induced one are evaluated with reference to a huge number of data collected from previous studies. The local error induced by few, moderately skewed, near-wall cells upwind the cylinder propagates windward because

---

\*Corresponding author. Tel: + 39 011 090 4870. Fax: + 39 011 090 4999.  
Email address: [luca.bruno@polito.it](mailto:luca.bruno@polito.it) (Luca Bruno)

of the convection-dominated problem, and globally affects the boundary layer separation and the vortex shedding in the wake. Skewness around the trailing edge only affects the flow to a lower extent. The skewness error on bulk aerodynamic coefficients may largely prevails on the overall modelling error, in spite of the very simple turbulence model deliberately adopted in the study. Hybrid grid boundary layer made of structured cells along the cylinder sides and unstructured ones around its edges provides results analogous to the ones obtained with a fully orthogonal grid, in spite of some clusters of few skewed cells far from the wall. Hybrid grid boundary layer is recommended as a fine balance between accuracy and flexibility in grid generation, when full orthogonal grid boundary layer is not feasible around real-world engineering applications having complicate geometries with multiple obtuse or acute edges.

*Keywords:* Finite Volume Method, grid boundary layer, grid quality, skewness, square cylinder

---

## 1. Introduction

The high-Reynolds number, turbulent flow around rectangular cylinders is considered as a paradigmatic one around bluff bodies, of interest both for fundamental research and applications. In spite of the simple and nominally two-dimensional geometry, the flow is intricate, characterized by boundary layer separation, its possible reattachment depending on the ratio of the alongwind to crosswind dimensions, Kelvin-Helmholtz instability of the detached shear layer, Von-Karman-like vortex shedding along the wake. Such underlying flow regimes provide useful information on the aerodynamics of a wide range of bluff bodies of interest in civil engineering (e.g. long-span bridge decks or high-rise buildings) as well as in other engineering areas. The importance attached to such flows by the scientific community is testified by some international benchmarks, such as the ones on the flow around 1:1 square cylinder (Rodi, 2004; Rodi et al., 1997), and around 5:1 rectangular cylinder (Bruno and Salvetti, 2017; Bruno et al., 2014).

16 The studies within the benchmarks above have testified the high sensitivity  
 17 of the flow to both physical parameters of the setup, and to errors induced by  
 18 the components of the computational model. Among the former, special at-  
 19 tention was paid for instance to the cylinder edge roundness (Tamura et al.,  
 20 1998; Riberio, 2011; Cao and Tamura, 2017; Rocchio et al., 2020), Reynolds  
 21 number (Scruton et al., 1971; Mannini et al., 2010; Schewe, 2013), and to other  
 22 freestream conditions (e.g. Vickery, 1966; Lee, 1975; Mariotti et al., 2016; Cao  
 23 and Tamura, 2018). Among the latter, the effects of turbulence models on the  
 24 simulated flow have been emphasized in a huge number of studies (e.g. Franke  
 25 and Rodi, 1991; Kato and Launder, 1993; Rodi, 1997; Lee, 1997; Bosch and  
 26 Rodi, 1998; Sohankar et al., 2000; Ke, 2019). In their wake, the conscious and  
 27 adapted application of different turbulence models is currently widespread in  
 28 engineering practice. Even if to a minor extent, also the effects of the numerical  
 29 schemes on the simulated flow were scrutinized by a number of studies, with  
 30 special emphasis on the discretization of the convective terms (Shyy et al., 1992;  
 31 Lee, 1997; Tamura et al., 1998; Cao and Tamura, 2016; Zhu et al., 2020).

32 More recently, the exponential growth of HPC facilities is allowing the sys-  
 33 tematic evaluation of the effects of the *grid density*, both in the vertical along-  
 34 wind  $x - y$  plane (Cao and Tamura, 2016) and along the spanwise  $z$ -direction  
 35 (Bruno et al., 2012; Cao and Tamura, 2016; Cao et al., 2020; Zhang and Xu,  
 36 2020). The attention of other authors focused on Adaptive Mesh Refinement  
 37 in Finite Element Method (see e.g. Hoffman, 2005; Berrone and Marro, 2009)  
 38 and Finite Volume Method (FVM) (see e.g. Antepara et al., 2015). The errors  
 39 induced by poor *grid quality* in FVM are well known in fundamental research on  
 40 computational methods since the pioneering PhD Theses of Muzaferija (1994)  
 41 and Jasak (1996), up to the recent studies by e.g. Ahipo and Traoré (2009);  
 42 Traoré et al. (2009); Juretić and Gosman (2010); de Oliveira Samel Moraes  
 43 et al. (2013); Denner and van Wachem (2014, 2015). Conversely, grid quality  
 44 effects on the simulated flow around bluff bodies are surprisingly overlooked in  
 45 scientific and technical literature. In particular, the current and very pervasive  
 46 practice in the simulation of flow around cylinders implies that fully orthogonal



47 structured grids are generated by extrusion with constant grid spacing in the  
 48 spanwise direction, but non orthogonal and skewed grids are inevitably gener-  
 49 ated in the vertical alongwind plane orthogonal to the cylinder axis. To our best  
 50 knowledge, only a few number of studies partially cover such an issue. Nakane  
 51 (2013) applied FVM with orthogonal and non-orthogonal grids to the low-Re  
 52 ( $Re=550, 3000$ ) transient flow around a circular cylinder, with focus on the early  
 53 stage of the symmetrical steady reversed flow in the near wake. Haque et al.  
 54 (2016) paid attention to the effects of the cell height growing ratio across the  
 55 Grid Boundary Layer (GBL) on the high-Re flow around rectangular cylinders  
 56 and hexagonal bridge decks. The grid quality across the GBL is expected to  
 57 be of paramount importance, because of the high gradient of the velocity and  
 58 turbulent variables in the boundary layer. Shortage in studies on the effects  
 59 of grid quality in GBL results in the lack of a shared approach and suggested  
 60 best practices among scholars and practitioners. For instance, Figure 1 provides  
 61 some examples of different GBLs adopted in recent computational studies on  
 the aerodynamics of 1:1 and 5:1 rectangular cylinders. Even if a structured

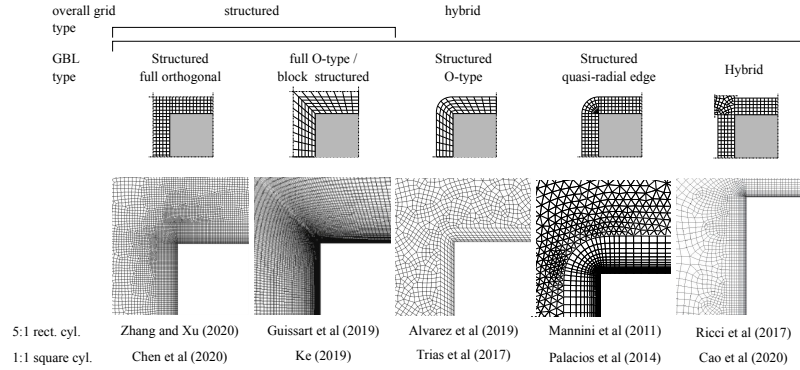


Figure 1: Types of GBL adopted in recent studies within benchmarks on the aerodynamics of 1:1 square and 5:1 rectangular cylinders

62  
 63 orthogonal GBL with quadrangular cells is generated in all studies along the  
 64 cylinder sides, they significantly differ in the meshing strategy around the sharp

65 (or slightly rounded) corners, in the shape of the cells around them, and in the  
 66 grid quality in turn. A non necessary exhaustive GBL categorization is ten-  
 67 tatively proposed in the figure, in the absence of a common nomenclature in  
 68 literature. All meshing strategies at corners are structured, except for the 'Hy-  
 69 brid GBL', where orthogonal boundary-fitted grid along the straight sides of the  
 70 cylinder alternates with unstructured grid in the quadrants departing from the  
 71 sharp corners. The latter inevitably includes almost regular, quasi orthogonal  
 72 and skewness-free cells in the first layers around the cylinder corners because  
 73 of the strong two-side constraint of the discretization of the adjacent edges of  
 74 the structured parts. Low quality cells take place moving away from the cylin-  
 75 der corners along and around the bisector of the quadrant. A single, largely  
 76 prevailing GBL meshing approach among the above categories does not exist in  
 77 research and engineering practice. For instance, Figure 2 provides statistics on  
 78 GBLs adopted in 23 studies published in the last decade on the aerodynamics  
 of the 5:1 rectangular cylinder.

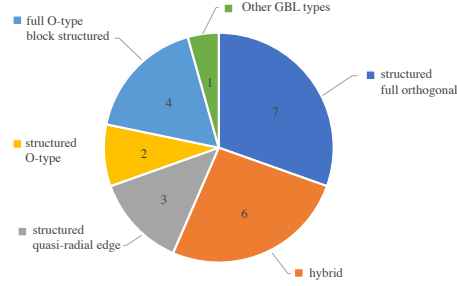


Figure 2: Statistics on types of GBL adopted in literature for the Benchmark on the Aerodynamics of a Rectangular 5:1 Cylinder (BARC)

79  
 80 An even larger variability of GBL meshing strategies occurs for other cylinder-  
 81 like, real-world engineering applications having more complicate geometries with  
 82 multiple obtuse or acute edges, such as bridge deck aerodynamics. For instance,  
 83 Tang et al. (2019) adopt a Hybrid GBL, Nieto et al. (2015) a block-structured  
 84 grid, Mannini et al. (2016) a quasi radial GBL, being full orthogonal GBL un-  
 85 feasible. The grid-induced errors are also expected to vary correspondingly. The

86 effects of low-quality grids could be even larger in fluid-structure interaction ap-  
 87 plications, such as bridge or cable aeroelasticity, where highly distorted cells can  
 88 arise from large amplitude pitching motion of the cylinder (see e.g. [Lee et al.,](#)  
 89 [2016](#), Fig.2).

90 This study aims to shed some light on the errors and effects of different  
 91 GBLs on the simulated flow around 1:1 square cylinder, and to compare such  
 92 grid-induced errors with the modelling errors. Besides this introduction, the  
 93 paper is organized into four more sections. In Section 2 the overall adopted  
 94 computational model is shortly recalled, while spatial grid quality metrics and  
 95 effects are detailed. The setup of the application is described in Section 3.  
 96 The obtained results are commented in Section 4. Finally, in Section 5 final  
 97 conclusions are drawn, and some research perspectives are briefly sketched.

## 98 2. Computational model

99 In the following, the adopted computational model is described in terms of  
 100 turbulence model, boundary conditions, numerical approach, and spatial grid  
 101 quality. Basic, well established and widely used turbulence model and numerical  
 102 approach are deliberately adopted in a 2D domain. They are expected to cor-  
 103 rectly reproduce the flow of interest in qualitative terms ([Lee, 1997](#)), although  
 104 with quantitative errors. Departures from top quality grid and induced errors  
 105 are detailed, the study being intended to quantitatively compare the overall er-  
 106 ror model with the one induced by low-quality GBLs in the vertical alongwind  
 107 plane orthogonal to the cylinder axis.

### 108 2.1. Turbulence model

109 The incompressible, turbulent, separated, unsteady flow around the 2D sec-  
 110 tion is modeled by the classical Unsteady Reynolds Averaged Navier-Stokes  
 111 (URANS) equations, which in indicial form read:

$$U_{i,i} = 0, \quad (1)$$

$$U_{i,t} + U_j U_{i,j} = -\frac{1}{\rho} P_{,i} + (\nu (U_{i,j} + U_{j,i}) - R_{ij})_{,j}, \quad (2)$$

where  $U_i$  is the averaged velocity component in the  $i$ -th direction,  $P$  is the averaged pressure,  $\rho$  the air density, and  $\nu$  its kinematic viscosity,  $R_{ij} = \overline{u_i u_j}$  the Reynolds stress, and  $u$  the velocity turbulent fluctuations. The  $k - \varepsilon$  RNG (Yakhot et al., 1992) turbulence model is used to close the URANS equations. The Reynolds stress is expressed by the well-know Boussinesq assumption as  $R_{ij} = -2\nu_t S_{ij}$ , where  $S_{ij}$  is the mean field strain rate, and the turbulent viscosity  $\nu_t$  is expressed as  $\nu_t = C_\mu \frac{k^2}{\varepsilon}$  in terms of the turbulent kinetic energy  $k$  and its dissipation rate  $\varepsilon$ . Their transport equations are

$$k_{,t} + U_j k_{,j} = -R_{ij} U_{i,j} + \left( \frac{\nu_t}{\sigma_k} k_{,i} \right)_{,i} - \varepsilon, \quad (3)$$

$$\varepsilon_{,t} + U_j \varepsilon_{,j} = -C_{\varepsilon_1} \frac{\varepsilon}{k} R_{ij} U_{i,j} + \left( \frac{\nu_t}{\sigma_\varepsilon} \varepsilon_{,i} \right)_{,i} - C_{\varepsilon_2}^* \frac{\varepsilon^2}{k}, \quad (4)$$

where  $C_{\varepsilon_2}^* = C_{\varepsilon_2} + C_\mu \eta^3 \frac{(1-\eta/\eta_0)}{1+\beta\eta^3}$ , and  $\eta = \frac{k}{\varepsilon} \sqrt{2S_{ij}S_{ij}}$  is the ratio between the turbulence time scale and the time scale of the mean field strain. The model constants are  $C_\mu = 0.085$ ,  $C_{\varepsilon_1} = 1.42$ ,  $C_{\varepsilon_2} = 1.68$ ,  $\eta_0 = 4.38$ ,  $\beta = 0.012$ ,  $\sigma_k = \sigma_\varepsilon = 1.393$ . The cell-centered nodal values of the flow variables adjacent to the wall are obtained by the standard wall function approach (Lauder and Spalding, 1974). Dirichlet boundary condition (b.c.) on the velocity field and on the turbulent variables are imposed at the inlet. Neumann b.c. on the normal component of the stress tensor, as well as the same Dirichlet b.c. on  $k$  and  $\varepsilon$ , are imposed at the outlet. Periodic b.c. are imposed on both the upper-lower boundaries. No-slip b.c. are imposed at the cylinder wall.

## 2.2. Numerical approach

A classic overall 2nd order approach in the framework of the Finite Volume Method is adopted to discretize the governing equations above. The cell-centre values of the variables are interpolated at face locations using the classic second-order Central Difference Scheme (CDS) for the diffusive terms. The convection terms are discretized by means of the QUICK scheme (Leonard, 1979). The fully implicit second-order Euler scheme is adopted for time discretization. The SIMPLE algorithm (Issa, 1986) is used for pressure-velocity coupling. The code Ansys Fluent© v.18 is used.

### 2.3. Grid quality effects

Non-orthogonality and skewness are common issues on non-Cartesian, arbitrary grids. A structured radial edge and an O-type GBLs are considered in Figure 3 as examples. Both non-orthogonality and skewness are qualities

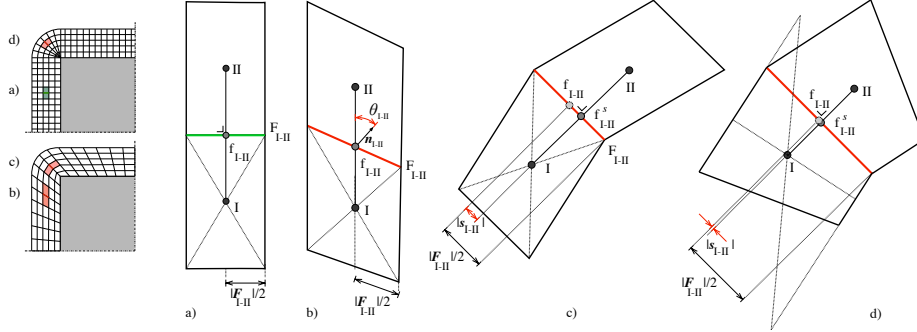


Figure 3: Face orthogonality  $\theta = 0$  (a), non-orthogonality  $\theta \neq 0$  (b), and skewness  $s$  (c,d) of cells around a sharp edge in different types of GBL

that refer to each face  $F_{I-II}$  shared between adjacent cells with centers I and II. Some pairs of cells are highlighted in Figure 3 for the sake of clarity. *Non-orthogonality* refers to the angle  $\theta_{I-II}$  between the normal vector  $\mathbf{n}_{I-II}$  and the vector connecting the two cell centers. The face is orthogonal if  $\theta_{I-II} = 0$  (Fig. 3-a, -c, -d), non-orthogonal otherwise (Fig. 3-b). *Skewness* refers to the vector  $\mathbf{s}_{I-II}$  from the face mid-point  $f_{I-II}$  to the interpolation point  $f_{I-II}^s$ , the latter being defined as the intersection between the face and the vector connecting the two cell centers. The face is not skewed if  $|\mathbf{s}_{I-II}| = 0$  (Fig. 3-a,b), skewed otherwise (Fig. 3-c,d). The examples in Figure 3 clearly underline that rectangular cells are perfectly orthogonal and not skewed, and that non-orthogonality does not necessarily implies skewness, and vice-versa. The above metrics can be made scalar and dimensionless as  $\theta_{I-II}^* = 2\theta_{I-II}/\pi$  and  $s_{I-II}^* = 2|\mathbf{s}_{I-II}|/|F_{I-II}|$ , where  $|F_{I-II}|$  is the length of the face  $F_{I-II}$  (Fig. 3). Both dimensionless metrics take values larger than unit for degenerate cells and interpolation points outside the face, respectively.

Both non-orthogonality and skewness are *potential sources of errors* in the

numerical fluxes evaluated by FVM. *Non-orthogonality-induced error* on the diffusive terms can be pointed out by splitting the face surface vector  $\mathbf{F}_{I-II}$  in the component  $\Delta \parallel \mathbf{d}_{I-II}$  and in a non-orthogonal one  $\mathbf{k}$  (Fig. 4-a, after Jasak, 1996). Whatever the chosen direction of  $\mathbf{k}$  is, the error induced on the diffusive flux of a generic variable  $\phi$  reads in vectorial form

$$E_{no}^d = \nabla \cdot (\nu_{no} \nabla \phi) \quad (5)$$

where  $\nu_{no} = \mathbf{U}_{f_{I-II}} \cdot \mathbf{k}$  is the so-called non-orthogonal viscosity, and  $\mathbf{U}_{f_{I-II}}$  is the velocity vector at the face mid-point.

*Skewness-induced error* can affect the convective terms, and results in a spurious non-physical diffusion flux evaluated by Jasak (1996) as

$$E_{sk}^c = \nabla \cdot (\nu_{sk} \nabla \phi) \quad (6)$$

where  $\nu_{sk} = \mathbf{U}_{f_{I-II}} \cdot \mathbf{s}$  is the so-called skewness viscosity. In other terms, grid skewness reduces the accuracy of interpolation to first order, and introduces a diffusion-like error in the discretized convective flux, analogously to the well-known leading truncation error of the first-order upwind scheme.

In short, both non-orthogonality and skewness introduce non-physical diffusion, if not properly dealt. In an engineering perspective such diffusive effects are not negligible for convection-dominated, high Reynolds number flow, where physical diffusive effects play a minor role. In other terms, both grid-induced viscosities can cumulate the kinematic viscosity and the modeled turbulent one, resulting in a simulated Re number lower than the one that characterizes the actual flow.

Two approaches can be adopted to counter this problem. *High-quality grid generation* is intended to eliminate the error source, i.e. to reduce the amplitude of the geometrical metrics  $\theta$  and  $s$ , and the induced errors in turn. High quality grid may be generated a priori, or obtained a posteriori by grid adaptation. Whatever the adopted technique is, such an approach is the most decisive, even if generation/adaptation criteria are required. *Corrective terms in interpolation schemes* are intended to mitigate the effects of low-quality grids, i.e. to reduce the errors (eq.s 5 and 6) without eradicate their causes.

188 Non-orthogonal correction strategies move from the surface vector splitting  
 $\mathbf{F}_{I-II} = \Delta + \mathbf{k}$  initially proposed by Jasak (1996) (Fig. 4-a).

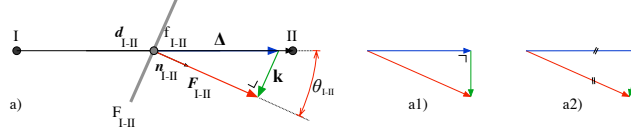


Figure 4: Approache for correction of non-orthogonality (after Jasak, 1996)

189  
 190 The diffusive flux for the generic variable  $\phi$  can be approximated as

$$\mathbf{F}_{I-II} \cdot (\nabla \phi)_{f_{I-II}} = \Delta \cdot (\nabla \phi)_{f_{I-II}} + \mathbf{k} \cdot (\nabla \phi)_{f_{I-II}} \approx |\Delta| \frac{\phi_{II} - \phi_I}{|d_{I-II}|} + \mathbf{k} \cdot (\widetilde{\nabla \phi})_{f_{I-II}}, \quad (7)$$

191 where the classic CDS is used for the orthogonal term. The non-orthogonal  
 192 correction second term depends on both the definition of  $\mathbf{k}$ , and the numer-  
 193 ical approximation of  $(\nabla \phi)_{f_{I-II}}$ . Different explicit non-orthogonal correction  
 194 schemes have been proposed in literature, e.g. Jasak (1996); Ahipo and Traoré  
 195 (2009); Traoré et al. (2009). Three possible choices for  $\mathbf{k}$  are depicted in Fig-  
 196 ures 4(a), (a1), (a2), where the former minimizes the correction. The iterative  
 197 numerical approximation  $(\widetilde{\nabla \phi})_{f_{I-II}}$  is generally inspired to the precursor ideas  
 198 in Khosla and Rubin (1974) and Muzaferija (1994). Generally speaking, the  
 199 non-orthogonality correction let to preserve the spatial second order accuracy.  
 200 However, when its contribution is larger than the orthogonal one for highly  
 201 non-orthogonal faces, the boundedness of the solution is no longer guaranteed  
 202 (Jasak, 1996). In order to ensure boundedness, the non-orthogonal correction  
 203 must be bounded or even discarded and the scheme becomes no longer second  
 204 order accurate (de Oliveira Samel Moraes et al., 2013).

205 Also skewness correction strategy is intended to correct standard interpolation  
 206 schemes for the quadrature values by additional term(s) accounting for  $\mathbf{s}$ . Start-  
 207 ing from the pioneering work of (Jasak, 1996) where the skewness problem has  
 208 been rigorously presented, some proposals for skewness correction have been  
 209 recently proposed in literature, e.g. de Oliveira Samel Moraes et al. (2013);

210 [Denner and van Wachem \(2014, 2015\)](#).

211 While non-orthogonal correction is widely implemented in industrial codes  
212 such as OpenFoam or Fluent, the same can not be said about the skewness cor-  
213 rection ([de Oliveira Samel Moraes et al., 2013](#)). In the light of this, the present  
214 study aims at evaluating different strategies of GBL generation to reduce the  
215 skewness geometrical metric  $|\mathbf{s}|$ , while GBL non-orthogonality is fixed according  
216 to the correction given in eq. (7).

### 217 3. Setup of the study

218 The adopted flow setup closely follows the one prescribed by the Ercoftac  
219 benchmark Ufr 2-02, detailed by [Rodi \(2004\)](#) in the QNET-CFD Knowledge  
220 Base Wiki ([Rodi, 2012](#)). The benchmark adopts as reference experimental tests  
221 the ones detailed in [Lyn and Rodi \(1994\)](#) and [Lyn et al. \(1995\)](#). The incoming  
222 flow is characterized by Reynolds number  $Re = UD/\nu = 2.2e + 4$ , where  $U$  is  
223 the free stream velocity and  $D$  the cylinder chord, incidence  $\alpha = 0$ , turbulence  
224 intensity  $It = 2\%$ , turbulent length scale  $Lt = 0.5D$ . The cylinder cross section  
225 has sharp edges and smooth surfaces. The dimensionless time step is set  $\Delta t^* =$   
226  $0.02tU/D$ . The simulation is extended over  $T \approx 300$  dimensionless time units  
227 in order to have a long enough statistical sample to secure converged statistics,  
228 after having excluded the initial transient. The dimensions of the 2D domain  
229 are given in Figure 5(a). The origin of the Cartesian coordinate is located at  
230 the center of the cylinder upwind face. The spatial grid is overall hybrid, being  
231 composed by both quadrangular and triangular cells, and by structured and  
232 unstructured partitions. A close-up view of the grid around the cylinder and its  
233 close wake is shown in Figure 5(b). In addition to usual strategies, subdomain  
234 partitioning and corresponding grid blocks are intended to ensure a dense, top-  
235 quality grid made of square cells ( $\Delta_x = \Delta_y = 0.042D$ ) in the wake region, and  
236 to keep the grid outside the GBL unchanged for all cases.

237 Four types of GBLs are generated in a  $D$ -tick square annular partition  
238 around the cylinder, as detailed in Figure 5(c). All GBLs share some common



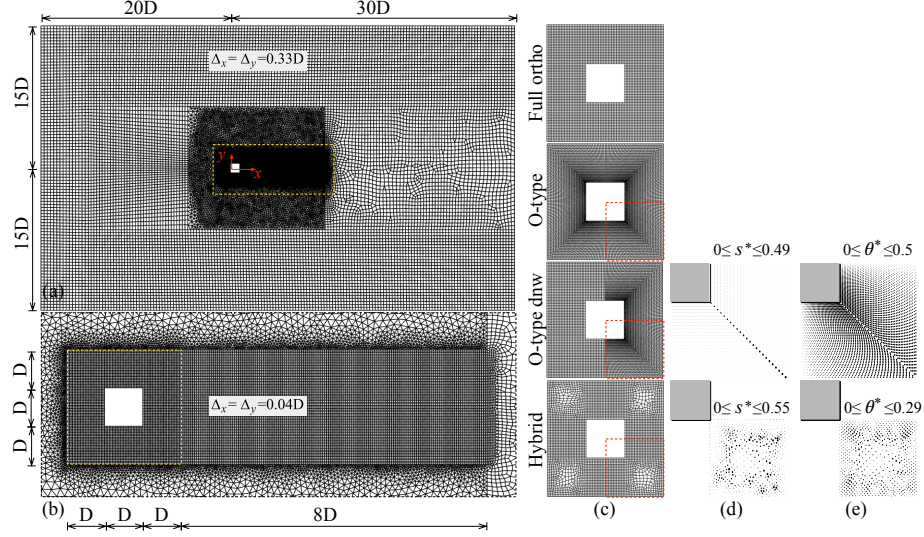


Figure 5: Computational domain (a), GBL and wake subdomain (b), GBL types (c) and their dimensionless skewness (d) and non-orthogonality (e)

239 features, in order to allow a proper comparative parametric study: cylinder side  
 240 uniformly discretized; height of the first cell at wall equal to  $\Delta n/D = 1/24$ ;  
 241 growing ratio equal to unit (nil grid gradient) and number of cells equal to 24  
 242 across the structured parts of the GBL.  $\Delta n$  is set to comply the requirements  
 243 of wall-function-based near-wall treatment about the height of the control vol-  
 244 umes at wall: the wall unit  $n^+ = nu_*/\nu$  results in the range  $25 \leq n^+ \leq 55$ . The  
 245 generated GBLs share the discretization of the quadrant edges, but differ in the  
 246 type of grid. The 'Full ortho' GBL has the same top quality grid a in the wake.  
 247 The 'O-type' one is characterized by a block-structured non-orthogonal grid.  
 248 The 'O-type dnw' GBL is a mix of the previous ones, where O-type grid is lim-  
 249 ited to the downwind half of the annular partition. In contrast to the previous  
 250 GBLs, a 'Hybrid' GBL is not uniquely defined by the discretization of the outer  
 251 edges of the quadrants, but it necessarily depends also on the specific algorithm  
 252 adopted for grid generation. In the present study, a paving, advancing-front-  
 253 based algorithm is employed (Blacker and Stephenson, 1991). The resulting,

254 specific realization of the Hybrid GBL shows the general distinctive features of  
 255 this class of GBLs as detailed in the introduction. However, in this specific case  
 256 the density and quality of the cells do not monotonically decrease moving away  
 257 from the wall as usually (see e.g. Fig. 1), because of nil grid gradient across  
 258 the structured parts of the GBL. Skewness and non-orthogonality are nil every-  
 259 where in the GBL for the 'Full ortho' GBL and the upwind part of the 'O-type  
 260 dnw' one. The spatial patterns of dimensionless skewness and non-orthogonal  
 261 metrics are given by scatterplots in Figures 5(d) and (e), respectively, for the  
 262 other cases and limited to the quadrant enclosed by the red dashed line. The  
 263  $s^*$  pattern in O-type GBLs groups skewness  $s^* \neq 0$  along the diagonal only,  
 264 while  $\theta^* \neq 0$  is distributed in the rest of the field. Significant values of both  
 265  $s^*$  and  $\theta^*$  are more scattered in the generated Hybrid GBL, with the highest  
 266 values in correspondence of the transition between dense and coarse grid. In  
 267 particular, the present Hybrid GBL shows nearly nil skewness in the first 3 cell  
 268 rows around the cylinder corners (corresponding to a width of  $1/8D$ ), because  
 269 of the constraints imposed by the adjacent edges of the structured parts on the  
 270 adopted mesh generation algorithm. It follows that the Hybrid GBL is locally  
 271 quite similar to the Full ortho GBL in such a region.  
 272 A quantitative statistical description of the patterns above is given in Table  
 273 1. Overall, two main comments follow. First, the maximum values of both  
 274 metrics are comparable among GBLs, and moderate ( $\max(\phi) \lesssim 0.5$ ). In other  
 275 terms, cells in GBLs are moderately skewed (the intersection point is no longer  
 276 distant than  $|\mathbf{F}_{\text{I-II}}|/4$  from the face mid-point) and moderately non-orthogonal  
 277 ( $\theta \leq 45^\circ$ ). Second,  $s^*$  and  $\theta^*$  values significantly greater than zero ( $\phi \geq 0.1$ )  
 278 rarely occur, except for  $\theta^*$  in O-type GBL. In other terms, GBL are very locally  
 279 skewed only. It follows that average and median values are very low, in turn.  
 280 Bearing in mind that non-orthogonality errors will be numerically corrected, it  
 281 might be inferred that all GBLs have comparable overall good quality, while  
 282 point-wise moderately-skewed cells are source of errors only.

Table 1: Statistics on dimensionless skewness and non-orthogonality for different GBL types

$\phi$	GBL	$\max(\phi)$	$\%_f  \phi  > 0.1$	$\text{avg}(\phi)$	$\text{median}(\phi)$	discrete distributions
$\theta^*$	Hybrid	0.2857	7.2%	0.0223	0	
	O-type	0.4955	83.9%	0.2778	0.2952	
$s^*$	Hybrid	0.5532	8.5%	0.0241	1e-6	
	O-type	0.4884	0.7%	0.0061	0.0047	

#### 4. Results

The present results are compared and discussed among them, in order to evaluate the errors induced by skewed cells in different regions of the GBL (Hybrid, O-type and O-type dnw grids) with respect to the full orthogonal grid (Full ortho). Furthermore, the present results are compared with a huge, even if not necessarily exhaustive, number of published studies on the same flow around square cylinder at  $\text{Re}=22,000$ , in the wake of the Ercoftac QNET-CFD Knowledge Base Wiki benchmark (Rodi, 2004, 2012). The goal of such a comparison is twofold: first, assessing the overall modelling error of the GBL-error-free simulation (Full ortho) with respect to the most accurate results in literature, and to compare it with the skewness-induced errors; second, placing the present results in the perspective of the whole scientific production on the topic, with special interest to the variability of the results obtained with different approaches. Both experimental tests spanning about 50 years (Pocha, 1971; Wilkinson, 1974; Lee, 1975; Otsuki et al., 1978; Bearman and Obasaju, 1982; Durao et al., 1988; Sakamoto et al., 1989; Lyn and Rodi, 1994; Lyn et al., 1995; Nishimura, 2001; Noda and Nakayama, 2003; Liu et al., 2015; Moore et al., 2019, in chronological order), and computational studies are considered. Among the latter, different approaches to turbulence modelling are considered: from the

302 pioneering so-called No-Model Simulations (NMS, [Tamura and Ito, 1996](#)) and  
 303 LES ([Murakami and Mochida, 1995](#); [Yu and Kareem, 1996](#)), to URANS ([Franke](#)  
 304 [and Rodi, 1991](#); [Kato and Launder, 1993](#); [Deng et al., 1994](#); [Wang and Vanka,](#)  
 305 [1997](#); [Bosch and Rodi, 1998](#); [Shimada and Ishihara, 2002](#); [Younis and Przulj,](#)  
 306 [2005](#)) and hybrid models ([Ke, 2019](#)), up to LES ([Sohankar et al., 2000](#); [Oka and](#)  
 307 [Ishihara, 2009](#); [Cao and Tamura, 2016](#); [Chen et al., 2020](#)), recent NMS ([Cao](#)  
 308 [et al., 2020](#)), and DNS ([Trias et al., 2015](#)).

309 For the sake of clarity, in the following the obtained results are schematically  
 310 arranged in three subsections. The simulated flow field around the cylinder is  
 311 discussed first, the distributions of the stresses at wall are commented in the  
 312 second section, while the bulk force coefficients are compared in the last one.

#### 313 *4.1. Flow field*

314 In order to provide an overall sound phenomenological picture of the sim-  
 315 ulated flow topology, the instantaneous flow patterns around the cylinder and  
 316 in its wake are plotted in Figure 6. All the flow patterns refer to the same  
 317 vortex-shedding phase, corresponding to the local maxima of the lift force. For  
 318 the sake of brevity, the isocontours of the vorticity magnitude  $|\omega_z|$  from two  
 319 grids are included only, corresponding to the highest (Full ortho, Fig. 6-d) and  
 320 lowest (O-type, Fig. 6-e) grid quality. The present results are compared be-  
 321 tween them and with analogous flow visualizations from past studies: DNS by  
 322 [Trias et al. \(2015\)](#) (background color according to the magnitude of the pres-  
 323 sure gradient); NMS by [Cao et al. \(2020\)](#) (densest  $z$ -wise grid, background color  
 324 according to the streamwise velocity component); 2D URANS simulation by  
 325 [Shimada and Ishihara \(2002\)](#) (isocontours of the vorticity magnitude). Visual-  
 326 izations are sorted by expected decreasing overall accuracy of the computational  
 327 model. In spite of their different state variables and contour values, qualitative  
 328 and quantitative remarks can be drawn. The von Kármán-like vortex street  
 329 clearly emerges in the wake in every simulation. However, the simulated vortex  
 330 pattern exhibits significant differences. A common trend can be recognized in  
 331 visualizations from a) to d): the lower the expected overall model accuracy, the

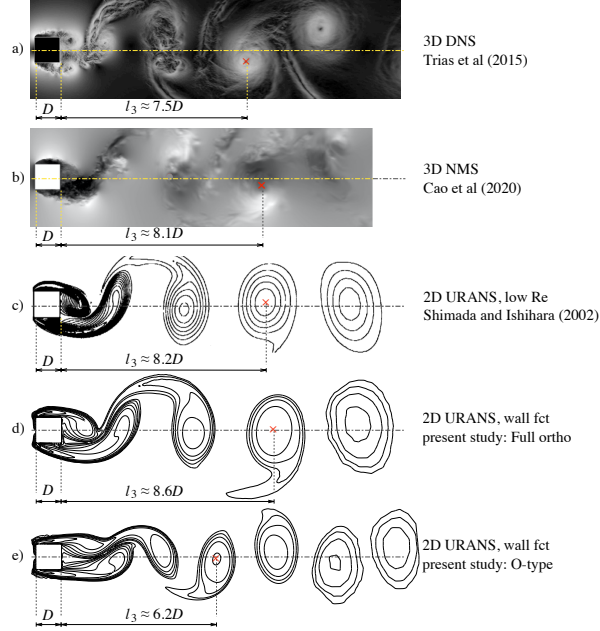


Figure 6: Instantaneous flow fields along the wake at the same vortex-shedding phase ( $t = \argmax(C_L)$ ) simulated by different computational models

332 slightly longer the longitudinal spacing between successive vortices. Conversely,  
 333 this trend is no longer monotonic moving from the Full ortho (d) to the skewed  
 334 O-type GBL (e): the vortex spacing drastically shortens. The distance  $l_3$  be-  
 335 tween the trailing edge and the centre of the 3rd vortex in the wake is estimated  
 336 by hand-picking to quantitatively support such a qualitative trend. Correspond-  
 337 ingly, the wake width simulated by skewed O-type GBL (e) is clearly narrower  
 338 than in Full ortho GBL (d). The wake features above appear correlated to the  
 339 curvature of the shear layer along the side surface just downwind the separation  
 340 point at the leading edge, and to the vortex shedding close to the trailing edge,  
 341 in turn. Simulations (a)-(d) are qualitatively consistent in predicting a highly  
 342 curved shear layer, and shedding point close to the rear face. Conversely, skewed  
 343 O-type GBL returns a lower curvature of the shear layer, and a shedding point  
 344 further away in the wake.

Figure 7 shows the time averaged vorticity  $t - avg(\omega_z)$  field around the cylinder base surface. The experimental PIV-based visualization by Moore et al.

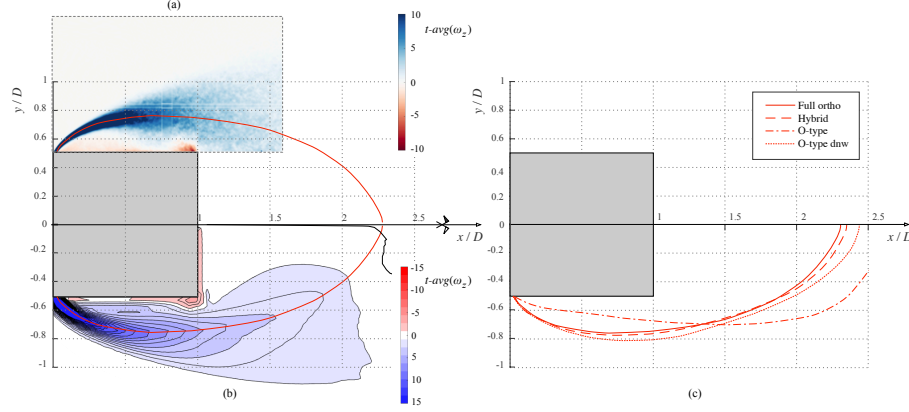


Figure 7: Mean flow field around the cylinder: time averaged vorticity  $t - avg(\omega_z)$  measured by Moore et al. (2019) (a), and simulated by the full orthogonal GBL (b); shape of the shear layer by loci of  $max(t - avg(\omega_z))$  simulated by different GBLs (c)

(2019) is given in Figure 7(a), while the corresponding field simulated by the Full ortho GBL is plotted in Figure 7(b). Both fields refer to the lower side surface, the experimental field being mirrored along the  $x$  axis to facilitate comparison. The red curve in both subfigures is the locus of maxima  $max(t - avg(\omega_z))$  simulated by Full ortho GBL, i.e. it visualizes the shape of the simulated shear layer. The Full ortho simulation is in excellent qualitative agreement with the experiment, in spite of the relatively simple overall computational model. The red curve closely follows the axis of the experimental blue plume, i.e. the separated shear layer. Finally, the Full ortho GBL simulates the negative time-averaged vorticity induced by the secondary clockwise vortex at the trailing edge, in qualitative agreement with PIV measurements. Figure 7(c) collects the loci of  $max(t - avg(\omega_z))$  for the four different GBLs. It can be observed that: i. the shapes of the shear layer obtained by the Full ortho and the Hybrid GBLs are very close in spite of the scattered skewness shown by the latter; ii. the O-type dnw curve slightly differs from the previous two, especially in the

362 wake; iii. the O-type curve completely departs from the others. On the one  
 363 hand, such findings quantitatively confirm in time average what discussed in  
 364 qualitative terms about the instantaneous patterns (Fig. 6-d, -e). On the other  
 365 hand, they suggest that very local skewness-induced errors along the diagonals  
 366 of the square annular partition globally affect the whole separated flow. The  
 367 closer the error source to the separation point at the cylinder edge, the more  
 368 significant its global effects (see e.g. the O-type GBL in Fig. 5-d). Conversely,  
 369 a local high quality of the cell closely around the separation point preserves the  
 370 accuracy of the results in spite of high skewness elsewhere (see e.g. the Hybrid  
 371 GBL in Fig. 5-d). Such errors are introduced along the diagonals of the square  
 372 annular partition, and in particular close to the separation point at the cylinder  
 373 edge in the O-type GBL.

374 In order to check such a conjecture, Figure 8 relates the skewness viscos-  
 ity to other simulated flow variables in the lower near wall region. The time

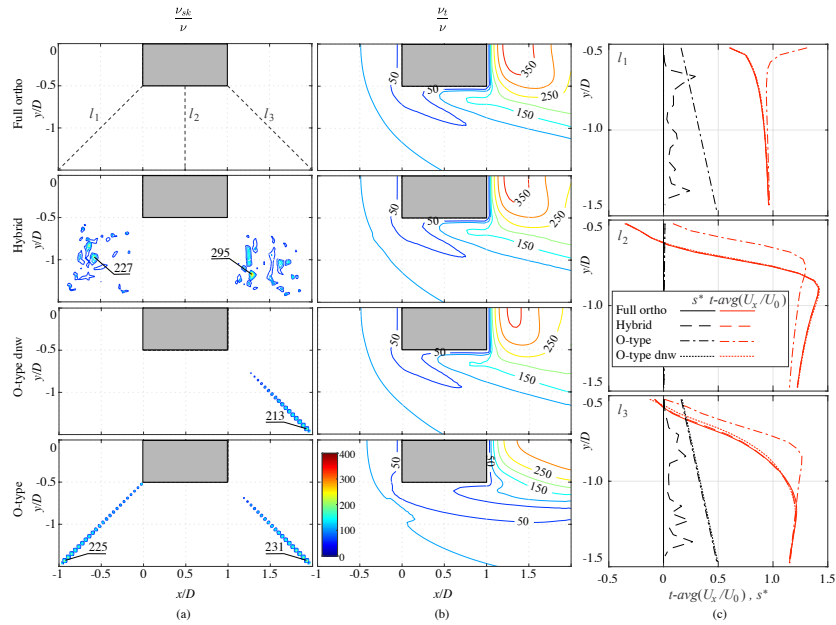


Figure 8: Time-averaged skewness viscosity ratio (a), turbulent viscosity ratio (b),  $x$ -velocity (red lines) and skewness (black lines) profiles (c)

376 averaged skewness viscosity ( $\nu_{sk}$ , Eq. 6, Fig. 8-a) are compared to the time av-  
 377 eraged turbulence eddy viscosity of the URANS model ( $\nu_t$ , Eq.s 3-4, Fig. 8-b).  
 378 Both quantities are normalized with respect to the kinematic viscosity  $\nu$ . Two  
 379 main remarks follow. First,  $\nu_{sk}$  is not distributed all over the domain but only  
 380 where skewed faces are located (Fig. 5-d). Second,  $\nu_{sk}$  and  $\nu_t$  take on values  
 381 within the same order of magnitude, hundreds of time larger than the kinematic  
 382 viscosity: in other term, false skewness viscosity potentially affects the solution  
 383 to the same extent of the turbulence model. In particular,  $\nu_{sk}$  has a dramatic  
 384 relative importance upwind the leading edge, where transition to turbulence not  
 385 yet occurs, and  $\nu_t$  is very low. Three different 1D domains  $l_1, l_2, l_3$  are defined,  
 386 as sketched in Figure 8(a), first row. Figure 8(c) shows the profiles of the di-  
 387 mensionless skewness and time-averaged horizontal velocity  $U_x$  along such lines.  
 388 Three main remarks follow. First, skewed faces lie along  $l_1$  in both Hybrid and  
 389 O-type GBL. However, only the O-type  $U_x$  profile departs from the others and  
 390 shows a non-physical flow acceleration very close to the wall. We can infer that  
 391 skewness viscosity plays a dramatic detrimental role mainly close to the wall,  
 392 approximately in the first 2 / 4 cell rows (see Fig. 5-d). Second, an analogous  
 393 scatter between O-type GBL and the other simulations is observed along  $l_2$ ,  
 394 even if the cells are locally unskewed and orthogonal in all GBLs. It means that  
 395 the *local skewness error* introduced by O-type GBL along  $l_1$  is *transported by*  
 396 *convection* along the side surface, analogously to the modeled state variables.  
 397 From a physical point of view, this implies *global effects*: the reversed flow pre-  
 398 dicted by Full ortho, Hybrid and O-type dnw GBLs is not adequately grasped  
 399 by the O-type GBL. Finally, the very small scatter between the  $U_x$  profiles by  
 400 Full ortho and O-type dnw GBLs along  $l_3$  testifies that the skewness viscosity,  
 401 its error and its effects not depend only on  $s^*$ , but on the velocity field, too (Eq.  
 402 6).

403 In order to quantitatively compare the present results with the ones in  
 404 literature, the distributions of the dimensionless streamwise velocity  $U_x/U_0$  av-  
 405 eraged in time ( $t-avg$  in the following) and in the spanwise direction ( $z-avg$  in  
 406 the following, relevant to 3D models) are plotted in Figure 9(a) along the wake



centerline  $y = 0$ . In particular, available experimental measurements (Durao  
et al., 1988; Lyn and Rodi, 1994), high-fidelity DNS (Trias et al., 2015) and  
LES (Cao and Tamura, 2016), and other LES and URANS results are consid-  
ered. In general, the considered high-fidelity simulations provide consistent

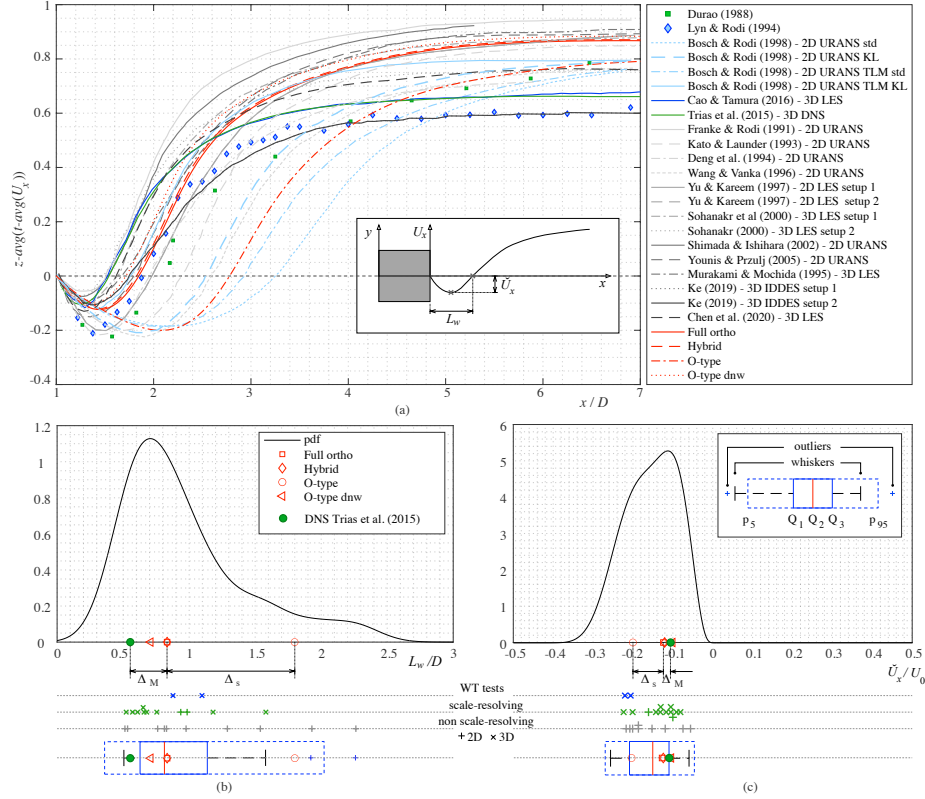


Figure 9: Distribution of time-averaged  $x$ -velocity along  $y = 0$  (a), statistics of the wake recirculation length  $L_w$  (b) and velocity defect  $\tilde{U}_x$  (c)

results. However, as pointed out by the same Trias et al. (2015), their computa-  
tional results slightly but systematically differ from the reference experimental  
data of Lyn and Rodi (1994), with remarkable differences in the near wake re-  
circulation region, where the simulated reversed velocity is nearly half of the  
measured one. The Lyn and Rodi (1994) measurements significantly depart in  
turn from the previous measures by Durao et al. (1988), and even larger variabil-

ity takes place among other computational results, even if the same approach to  
 turbulence modelling is adopted. Such overall variability of the velocity profiles  
 results in scattered values of the recirculation length, of the maximum reversed  
 speed, and of the velocity plateau along the fare wake, if any. In particular,  
 the streamwise velocity profiles predicted by low-skewness grids (Full ortho and  
 Hybrid) are in satisfactory agreement with experimental data in the near wake,  
 while the velocity recovery is overestimated in the far wake  $x/D > 2.5$ . The  
 dense and orthogonal grid in the wake (Fig. 5), and the 3rd order scheme used  
 for the convective terms suggest that most of the underlying error can be tenta-  
 tively ascribed to the 2D domain, and/or to adopted turbulence model, namely  
 to the empirical modelling of  $C_{\varepsilon_2}^*$ . The velocity distribution issued from O-type  
 grid dramatically suffers the effects of the skewness viscosity (eq. 6), resulting  
 in the striking overestimation of the recirculation length. Conversely, skewness  
 around the trailing edge only (O-type dnw) has the opposite effects of the skew-  
 ness around both leading and trailing edges (O-type), i.e. it shortens the recircu-  
 lation length and reduces the velocity defect with respect to the Full ortho grid.  
 Two local metrics are retained in order to synthetically describe the near wake  
 recirculation region: the wake recirculation length  $L_w$ , and the velocity defect  
 $\check{U}_x$ , graphically defined in Figure 9(a) for the sake of clarity and conciseness. In  
 the light of the high variability highlighted above, statistics are obtained on each  
 of them, by referring to the ensemble of realizations collected in literature and  
 plotted in Figure 9(a). Statistics are visualized in Figures 9(b) and (c) for  $L_w/D$   
 and  $\check{U}_x/U_0$ , respectively. All realizations are reported and categorised in Wind  
 Tunnel (WT) measurements, non scale-resolving (RANS/URANS), and scale-  
 resolving (DNS, NMS, LES, DES, IDDES) simulations. The *a-priori* expected  
 highest-fidelity model (DNS, Trias et al., 2015) is pointed out. Non paramet-  
 rical Probability Density Functions (PDF) with positive and negative support  
 are fitted on the whole ensemble for  $L_w$  and  $\check{U}_x$ , respectively, and accompanied  
 by the corresponding box plots. Such a postprocessing allows a twofold reading  
 of the obtained results. In a deterministic reading, the present overall mod-  
 elling error  $\Delta_M$  is evaluated by the scatter between the highest-fidelity model

(DNS, [Trias et al., 2015](#)) and the present highest-quality GBL (Full ortho),  
 while the highest skewness-induced error  $\Delta_s$  is given by the scatter between the  
 highest (Full ortho) and lowest (O-type) GBL quality. In a purely statistical  
 reading, each realization can be associated with the corresponding percentile,  
 i.e. the number where a certain percentage of scores fall below it. Here, we are  
 aware that the ensemble cardinality ( $\#=22$ ) is significant but not necessarily  
 high enough to ensure full statistical convergence. Hence, we prefer a coarse  
 statistical mapping of the present results by referring to outliers, and to the  
 intervals between percentiles  $p_5, p_{25} = Q_1, p_{50} = Q_2, p_{75} = Q_3, p_{95}$ . Selected  
 remarks follow from both perspectives: i. for both metrics the skewness-induced  
 error is by far larger than the overall modelling error, despite the latter results  
 form top- (3D DNS) and low- (2D URANS) ranked models; ii.  $L_w/D$  shows a  
 variability (coefficient of variation  $c.o.v.(L_w) = \sigma(L_w)/|\mu(L_w)| = 0.236$ ) lower  
 than  $\check{U}_x/U_0$  ( $c.o.v.(\check{U}_x) = 0.364$ ); iii. the values provided by the Full ortho and  
 Hybrid grids are within the  $[Q_1Q_3]$  range for both metrics, while the  $L_w/D$   
 value from O-type grid is clearly an outlier, and the corresponding  $\check{U}_x/U_0$  is  
 close to  $Q_1$ .

#### 4.2. Stress distributions at wall

The distributions of the dimensionless wall shear stress  $\tau_w$  averaged in time  
 and in the spanwise direction are plotted in Figure 10 versus the curvilinear  
 axis  $s$  along the upper half perimeter. Positive values correspond to  $\tau_w$  with the  
 same direction of the curvilinear axis. The distributions obtained by different  
 GBLs are compared with the ones obtained by [Cao and Tamura \(2016\)](#) with  
 different grid densities. A closed view along the side surface is given, in anal-  
 ogy to what done by [Cao and Tamura \(2016\)](#) as well. The distributions by  
 this study follows from a wall-function modelling approach and, consequently,  
 the size of the cells along the wall is much larger than the ones in [Cao and  
 Tamura \(2016\)](#) in both  $s$  and  $n$  directions. The  $s$ -wise low grid density implies  
 a scarce resolution of the wall shear stress profile, testified by the coarse place-  
 ment of the peaks immediately upstream and downstream the leading corner B.

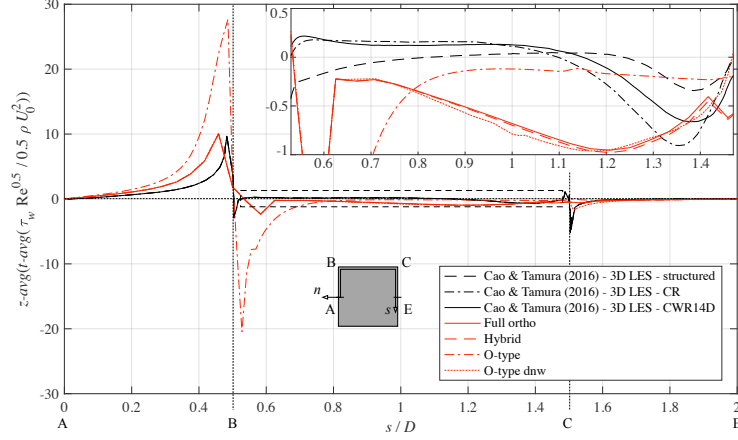


Figure 10: Distribution of time-averaged dimensionless wall shear stress along the upper half perimeter

However, the maxima of  $|\tau_w \text{Re}^{0.5} / 0.5 \rho U_0^2| \approx 10$  and  $2.5$  predicted by Full ortho, Hybrid and O-type dnw GBLs well matches the LES results in Cao and Tamura (2016). Both  $s$  and  $n$ -wise low grid density does not allows the simulation of the small-size counterclockwise secondary vortices downstream B ( $0.5 < s/D \lesssim 0.7$ ) and upstream the trailing corner C ( $1.47 \lesssim s/D < 1.5$ ) visualized in Cao and Tamura (2016)-Fig. 10, and Cao et al. (2020)-Fig. 10. This yields to reversed flow all along the side surface, and corresponding negative  $\tau_w$ . The "3D LES-structured" simulation by Cao and Tamura (2016) is affected by an analogous issue. The O-type distribution drastically differs from the others because of both local and global effects of the skewness viscosity. The local peaks around B are, in absolute value, from 3 to 8 times higher than the ones issued by the other GBLs. After a shear recovery, the distribution attains a plateau value close to zero along the downwind half of the side surface, i.e. the flow is unrealistically quasi-stagnant. Conversely, the effects of the skewness viscosity on  $\tau_w$  induced by O-type dnw GBL mainly take place around the trailing corner only, because of the convection-dominated flow.

The distributions of the time-averaged and time-standard deviation ( $t - std$

495 in the following) of the pressure coefficient  $C_p$ , also averaged spanwise and be-  
 496 tween the upper and lower half perimeters (*side-avg* in the following) are  
 497 plotted in Figure 11 (a) and (b), respectively. They are compared to available  
 498 experimental measurements, high-fidelity DNS (Trias et al., 2015) and LES  
 499 (Cao and Tamura, 2016), and several other LES and URANS simulations. In  
 500 spite of the same nominal setup, both experimental and computational results  
 501 are significantly scattered along the whole base surface, especially in regard to  
 502  $t - std(C_p)$ . Interestingly, distribution from *a-priori* high-fidelity simulations  
 503 are extremal occurrences at the lower and upper bounds of the ensemble for  
 504  $t - avg(C_p)$  and  $t - std(C_p)$ , respectively. Conversely, Full ortho GBL distribu-  
 505 tions lie around the upper and lower bounds of the ensemble for  $t - avg(C_p)$  and  
 506  $t - std(C_p)$ , respectively. In short, the adopted computational model slightly  
 507 underestimates the mean suction and the pressure fluctuations, and notably  
 508 fails in predicting the local maximum of  $t - std(C_p)$  in the downwind half of the  
 509 side surface. Hybrid GBL provides the same mean distribution, and negligible  
 510 differences take place for the pressure standard deviation. O-type dnw GBL  
 511 does not significantly affect the mean distribution, while the local error made at  
 512 the trailing edge C makes the  $t - std(C_p)$  distribution closer to the average of  
 513 the experimental measurements. We conjecture that such a misleading agree-  
 514 ment is induced by a deeper footprint of the vortex shedding on the side surface,  
 515 induced by small changes in the flow structure in the near wake. Highly skewed  
 516 O-type GBL dramatically and qualitatively affects both the distributions of the  
 517  $C_p$  mean and standard deviation. Their trend is typical of a streamlined body  
 518 rather than of a bluff body: deep mean suction occurs at the leading edge B;  
 519 mean pressure plateau is missing downstream it, and mean pressure recovery  
 520 takes place along the side surface; pressure standard deviation is one order of  
 521 magnitude lower than the average of the results in literature.

522 Analogously to what done for the streamwise velocity in the wake, four *s*-  
 523 wise averaged metrics are defined to synthetically describe the time-averaged  
 524 and time-fluctuating pressure field along the side ( $l - avg$ ) and rear ( $b - avg$ )  
 525 surfaces. The bounds of the averaging lengths  $l_1$ ,  $l_2$ ,  $b$  are intended to discard

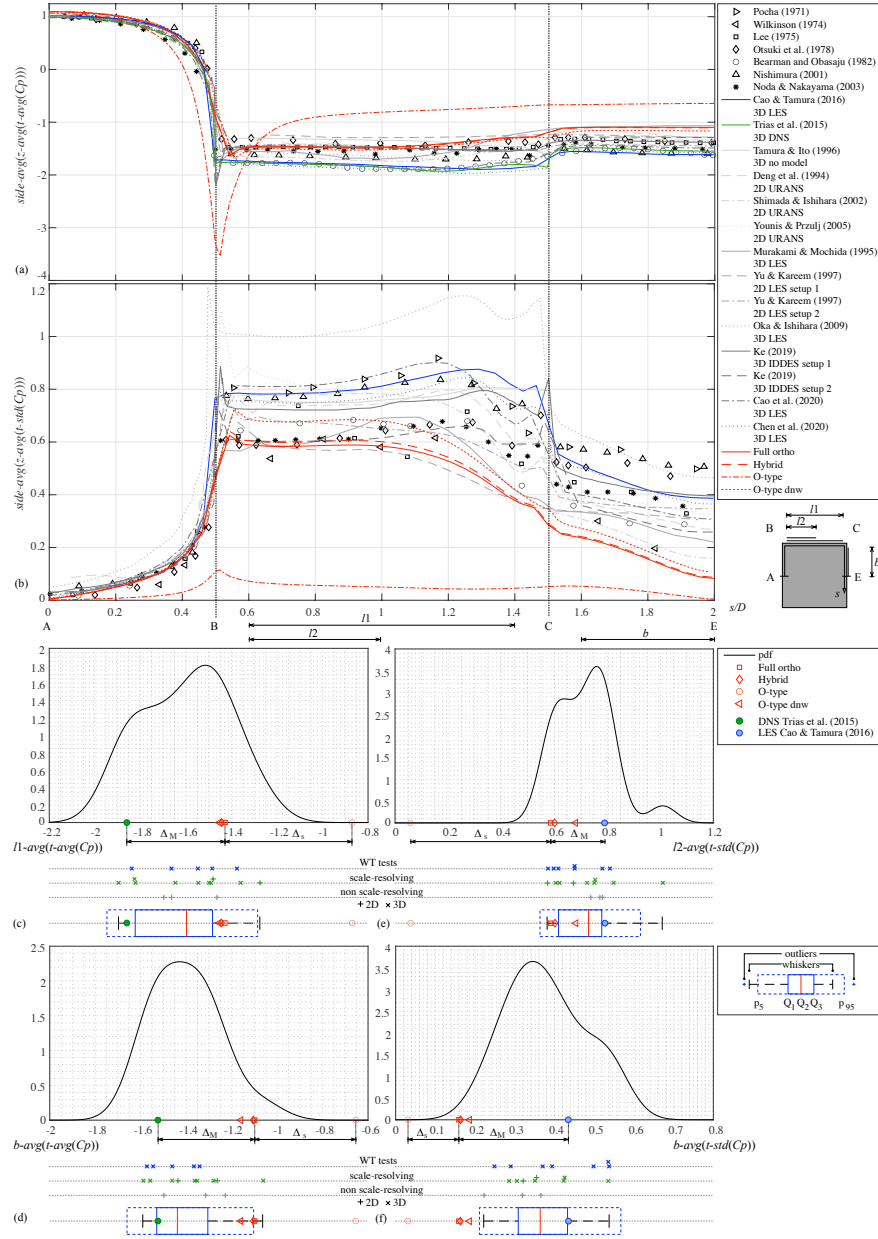


Figure 11:  $C_p$  distribution: time-averaged value (a), time-standard deviation (b), statistics of the side (c,e) and base (d,f) pressure

526 the neighbourhood of the sharp edges, and related local high pressure gradients.  
 527 The cardinality of the ensemble of the realizations ( $\#=21$ ) is close to the one  
 528 used for the streamwise velocity in the wake. The 3D LES by [Cao and Tamura](#)  
 529 ([2016](#)) is chosen as *a-priori* highest-fidelity model when DNS results from [Trias](#)  
 530 [et al. \(2015\)](#) are not publicly available. Figures [11\(c\)](#) and [\(d\)](#) show the distribu-  
 531 tions of the side and rear surface  $t - avg(Cp)$  plateaux. The following remarks  
 532 can be drawn: i. the coefficients of variation of the literature results are similar  
 533 ( $c.o.v.(l1 - avg(t - avg(Cp))) = 0.12$ ,  $c.o.v.(b - avg(t - avg(Cp))) = 0.1$ ) and  
 534 not negligible; ii. in a statistical setting, the Full ortho, Hybrid and O-type  
 535 dnw realizations are close to  $Q_3$  of  $l1 - avg(t - avg(Cp))$  and to and  $p_{95}$  of  
 536  $b - avg(t - avg(Cp))$ , while the O-type ones are outliers in both cases; iii. in a  
 537 deterministic setting, the skewness errors  $\Delta_s$  are comparable to the model ones  
 538  $\Delta_M$ . Figures [11\(e\)](#) and [\(f\)](#) show the distributions of  $t - std(Cp)$  over  $l2$  and  $b$ ,  
 539 respectively. The following remarks can be drawn: i. the c.o.v. of the distribu-  
 540 tions of the  $Cp$  standard deviation in literature are overall higher than the ones  
 541 of the time-averaged pressure. In particular, the c.o.v. of the pressure standard  
 542 deviation along the rear face ( $c.o.v.(b - avg(t - std(Cp))) = 0.25$ , [Fig. 11-f](#)) is  
 543 higher than the one on the side surface ( $c.o.v.(l2 - avg(t - std(Cp))) = 0.15$ , [Fig.](#)  
 544 [11-e](#)), and impressive in absolute terms. The slight bi-modality of the PDF in  
 545 [Figure 11\(e\)](#) does not seem to depend on the approach/model used; ii. over the  
 546  $l2$  domain, while the Full ortho and Hybrid realizations are located near  $Q_1$  and  
 547 the O-type dnw one in between  $Q_1$  and  $Q_2$ , the O-type realization is an outlier;  
 548 iii. over the domain  $b$ , all the realizations of the present work are outliers even if  
 549 Full ortho, Hybrid and O-type dnw are close to  $p_5$ ; iv. the deterministic model  
 550 error  $\Delta_M$  is prevailing over the skewness one  $\Delta_s$  along  $b$  only, while the opposite  
 551 is the case along  $l2$ .

#### 552 4.3. Bulk forces

553 Bulk force coefficients are scarcely useful in shedding light on phenomenolog-  
 554 ical local features of the simulated flows and skewness effects on them. Neverthe-  
 555 less, they are of great importance for design purposes in engineering applications.

556 The main bulk force coefficients obtained in the present study are compared and  
 557 discussed with reference to statistics on the same metrics obtained in different  
 558 wind tunnel and computational studies in Figure 12:  $t - avg(C_D)$  is the time-  
 559 and spanwise-averaged drag coefficient per unit length;  $t - std(C_L)$  is the stan-  
 560 dard deviation of the time variation of the lift coefficient;  $St = f_s D / U_0$  is the  
 561 Strouhal number, where the shedding frequency  $f_s$  is evaluated from the time  
 562 fluctuations of the lift coefficient. Statistics are obtained from ensembles having  
 563 a cardinality equal to  $\# = 39$  for  $t - avg(C_D)$  and  $St$ , and  $\# = 29$  for  $t - std(C_L)$ .  
 564 Realizations are collected from Liu et al. (2015), Trias et al. (2015), Cao and  
 Tamura (2016), and references therein. PDFs of  $t - avg(C_D)$  and  $t - std(C_L)$

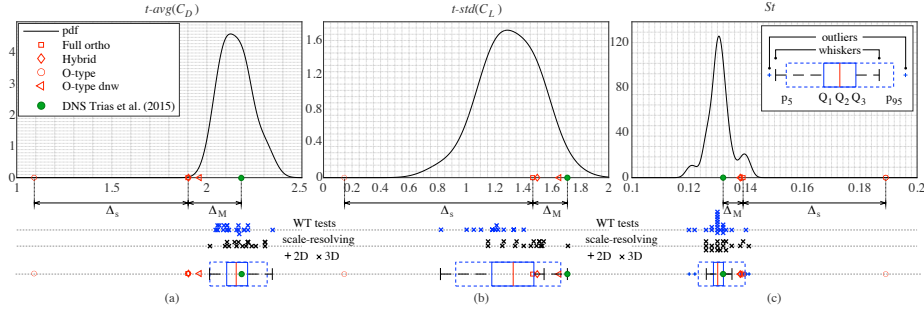


Figure 12: Bulk parameters: statistics of time-averaged drag coefficient (a), standard deviation  
 of the lift coefficient (b), Strouhal number (c)

565  
 566 are moderately skewed and unimodal. The three-mode PDF of the  $St$  number  
 567 seems not due to the selected approach: for instance, WT realizations contribute  
 568 to all modes. The coefficient of variation is small for the time-averaged drag  
 569 ( $c.o.v.(t - avg(C_D)) = 0.04$ ) and Strouhal number ( $c.o.v.(St) = 0.03$ ), while  
 570 it is significant for lift fluctuations ( $c.o.v.(t - std(C_L)) = 0.15$ ). The *a priori*  
 571 high fidelity model closely fits the ensemble average value of  $t - avg(C_D)$  and  
 572  $St$ . Conversely, DNS significantly overestimates the  $t - std(C_L)$  in respect to  
 573 the whole ensemble, and to the experimental measurements in particular. We  
 574 conjecture this can be ascribed to the unsatisfactory simulation of the spanwise  
 575 pressure correlation, probably due to too short spanwise domain size, and/or to



576 poor grid density in the same direction (Bruno et al., 2012). Both modelling and  
 577 skewness errors imply the reduction of  $t - avg(C_D)$  and  $t - std(C_L)$ , while the  
 578 Strouhal number increases under the effects of both  $\Delta_M$  and  $\Delta_S$ . The skewness  
 579 error is much higher than the modelling one for all bulk metrics: 2.9 times for  
 580  $t - avg(C_D)$ , 5.4 times for  $t - std(C_L)$ , 7.2 times for  $St$ . Let us consider the  
 581 drag coefficient as an example (Fig. 12-a). The ensemble median is  $Q_2 \approx 2.15$ ,  
 582 and DNS returns a reference value  $t - avg(C_D)_{rep} \approx 2.18$  (71-th percentile in in  
 583 probabilistic terms), while Full ortho and Hybrid GBL predict  $t - avg(C_D) \approx 1.9$   
 584 (corresponding to the 0.01-th percentile, and resulting in  $\Delta_M \approx 0.28$ ), and O-  
 585 type GBL  $t - avg(C_D) \approx 1.1$  (corresponding to a full outlier, and resulting in  
 586  $\Delta_s \approx 1.08$ ). It follows that both modelling and skewness errors lead to the  
 587 unsafe estimation of the drag force. However, the design could be effectively  
 588 secured versus  $\Delta_M$  by adopting a partial safety factor  $\gamma_Q$  for variable actions,  
 589 also accounting for model errors/uncertainties (e.g.  $\gamma_Q = 1.5$  in EN 1990, 2002;  
 590 EN 1991-1-4, 2005, for wind action in structural engineering). The same usual  
 591 value of the partial safety factor is not effective face to the large skewness error.  
 592 Analogous comments are relevant to  $t - std(C_L)$  and  $St$ , if the deviation of the  
 593 Strouhal number has unfavourable effects on the structural response.

## 594 **5. Conclusions**

595 The flow around a square cylinder at  $Re=22,000$  is simulated under four  
 596 types of Grid Boundary Layers, the other components of the computational  
 597 model being unchanged. GBLs differ in cell quality, while they share common  
 598 other features, such as the geometry of their boundaries, their size, the grid den-  
 599 sity at wall, the grid in the outer part of the computational domain. Thanks to  
 600 this, the effects of the cell skewness are separately discussed with respect to the  
 601 ideal full orthogonal GBL. The attention paid in the recent past to the adopted  
 602 benchmark by the scientific community allowed a huge number of previous re-  
 603 sults to be collected and statistically analyzed. Hence, the obtained results are  
 604 compared not only among them, but also with such a huge database.

605 Some conclusions can be summarized:

- 606 • The local error induced by few, moderately skewed, near-wall cells upwind  
607 the cylinder propagates windward because of the convection-dominated  
608 problem. The error globally and significantly affects the boundary layer  
609 separation and the vortex shedding in the wake. The same skewed cells  
610 around the trailing edge only affect the flow to a lower extent;
- 611 • The skewness error on bulk aerodynamic coefficients may largely prevails  
612 on the overall modelling error, despite a very simple turbulence model and  
613 a second-order accurate numerical approach are deliberately adopted in  
614 the study;
- 615 • O-type, block structured, skewed GBL results in the qualitative simulation  
616 of the instantaneous vortex street in the wake, and in the concurrent  
617 dramatic underestimation of the lift fluctuating component by a factor  
618 5. Such results call to mind the well renowned quote by [Ferziger \(1993\)](#):  
619 *"The greatest disaster one can encounter in computation is not instability*  
620 *or lack of convergence, but results that are simultaneously good enough to*  
621 *be believable, but bad enough to cause trouble"*;
- 622 • Hybrid GBL with unstructured patterns around the cylinder edges can  
623 provide results analogous to the ones obtained with a full orthogonal grid,  
624 provided that from 2 to 4 cell rows nearly orthogonal and skewness-free are  
625 generated near the wall (corresponding to a width from  $1/12D$  to  $1/6D$ ),  
626 while clusters of skewed cells are kept further from the wall;
- 627 • Hybrid GBL is judged to be a fine balance between accuracy and flexibility  
628 in grid generation, with the above due care and attention to detail. In  
629 particular, hybrid GBLs are nearly unescapable for real-world engineering  
630 applications with intricate geometries, e.g. with multiple obtuse or acute  
631 edges.

632 Analogous studies on grid quality effects are recommended for other paradig-  
633 matic high-Re flows in bluff body aerodynamics, such as the one around elon-

634 gated rectangular cylinders with separated and reattached boundary layer, and  
635 around real world structures, such as bridge decks and tall buildings. The rela-  
636 tive significance of skewness viscosity with respect to other modelling approaches  
637 to turbulence such as wall resolved LES would be worth to be investigated as  
638 well.

## 639 Acknowledgments

640 The Authors acknowledge the collective effort paid by all the colleagues  
641 involved in the benchmark on the flow around square cylinder: the present  
642 study is grounded on their results. The Authors warmly thank D. D'Ambrosio,  
643 A. Bianco and A. López Sanfeliciano for the preliminary discussions about the  
644 topic of this study.

## 645 References

- 646 Ahipo, Y., Traoré, P., 2009. A robust iterative scheme for finite volume dis-  
647 cretization of diffusive flux on highly skewed meshes. *Journal of Computa-*  
648 *tional and Applied Mathematics* 231, 478–491. doi:[10.1016/j.cam.2009.03.](https://doi.org/10.1016/j.cam.2009.03.015)  
649 [015](https://doi.org/10.1016/j.cam.2009.03.015).
- 650 Antepara, O., Lehmkuhl, O., Borrell, R., Chiva, J., Oliva, A., 2015. Paral-  
651 lel adaptive mesh refinement for large-eddy simulations of turbulent flows.  
652 *Computers and Fluids* 110, 48–61. doi:[10.1016/j.compfluid.2014.09.050](https://doi.org/10.1016/j.compfluid.2014.09.050).
- 653 Bearman, P.W., Obasaju, E.D., 1982. An experimental study of pressure fluc-  
654 tuations on fixed and oscillating square-section cylinders. *Journal of Fluid*  
655 *Mechanics* 119, 297–321. doi:[10.1017/s0022112082001360](https://doi.org/10.1017/s0022112082001360).
- 656 Berrone, S., Marro, M., 2009. Space-time adaptive simulations for unsteady  
657 navier-stokes problems. *Computers and Fluids* 38, 1132–1144. doi:[10.1016/](https://doi.org/10.1016/j.compfluid.2008.11.004)  
658 [j.compfluid.2008.11.004](https://doi.org/10.1016/j.compfluid.2008.11.004).

- 659 Blacker, T., Stephenson, M., 1991. Paving: A new approach to automated  
660 quadrilateral mesh generation. *International Journal for Numerical Methods*  
661 *in Engineering* 32, 811–847. doi:[10.1002/nme.1620320410](https://doi.org/10.1002/nme.1620320410).
- 662 Bosch, G., Rodi, W., 1998. Simulation of vortex shedding past a square cylinder  
663 with different turbulence models. *International Journal for Numerical Meth-*  
664 *ods in Fluids* 28, 601–616. doi:[10.1002/\(sici\)1097-0363\(19980930\)28:](https://doi.org/10.1002/(sici)1097-0363(19980930)28:4<601::aid-fld732>3.0.co;2-f)  
665 [4<601::aid-fld732>3.0.co;2-f](https://doi.org/10.1002/(sici)1097-0363(19980930)28:4<601::aid-fld732>3.0.co;2-f).
- 666 Bruno, L., Coste, N., Fransos, D., 2012. Simulated flow around a rectangular 5:1  
667 cylinder: Spanwise discretisation effects and emerging flow features. *Journal*  
668 *of Wind Engineering and Industrial Aerodynamics* 104-106. doi:[10.1016/j.](https://doi.org/10.1016/j.jweia.2012.03.018)  
669 [jweia.2012.03.018](https://doi.org/10.1016/j.jweia.2012.03.018).
- 670 Bruno, L., Salvetti, M., Ricciardelli, F., 2014. Benchmark on the aerodynamics  
671 of a rectangular 5:1 cylinder: An overview after the first four years of activity.  
672 *Journal of Wind Engineering and Industrial Aerodynamics* 126. doi:[10.1016/](https://doi.org/10.1016/j.jweia.2014.01.005)  
673 [j.jweia.2014.01.005](https://doi.org/10.1016/j.jweia.2014.01.005).
- 674 Bruno, L., Salvetti, M.V., 2017. Ufr 2-15, benchmark on the aerodynamics of a  
675 rectangular 5:1 cylinder (bare). URL: [https://www.kbwiki.ercoftac.org/](https://www.kbwiki.ercoftac.org/w/index.php/UFR_2-15_Description)  
676 [w/index.php/UFR\\_2-15\\_Description](https://www.kbwiki.ercoftac.org/w/index.php/UFR_2-15_Description).
- 677 Cao, Y., Tamura, T., 2016. Large-eddy simulations of flow past a square cylinder  
678 using structured and unstructured grids. *Computers and Fluids* 137, 36–54.  
679 doi:[10.1016/j.compfluid.2016.07.013](https://doi.org/10.1016/j.compfluid.2016.07.013).
- 680 Cao, Y., Tamura, T., 2017. Supercritical flows past a square cylinder with  
681 rounded corners. *Physics of Fluids* 29, 085110. doi:[10.1063/1.4998739](https://doi.org/10.1063/1.4998739).
- 682 Cao, Y., Tamura, T., 2018. Aerodynamic characteristics of a rounded-corner  
683 square cylinder in shear flow at subcritical and supercritical reynolds numbers.  
684 *Journal of Fluids and Structures* 82, 473–491. doi:[https://doi.org/10.](https://doi.org/10.1016/j.jfluidstructs.2018.07.012)  
685 [1016/j.jfluidstructs.2018.07.012](https://doi.org/10.1016/j.jfluidstructs.2018.07.012).

686 Cao, Y., Tamura, T., Kawai, H., 2020. Spanwise resolution requirements for the  
687 simulation of high-reynolds-number flows past a square cylinder. *Computers*  
688 & *Fluids* 196, 104320. doi:[10.1016/j.compfluid.2019.104320](https://doi.org/10.1016/j.compfluid.2019.104320).

689 Chen, Y., Djidjeli, K., Xie, Z.T., 2020. Large eddy simulation of flow past  
690 stationary and oscillating square cylinders. *Journal of Fluids and Structures*  
691 97, 103107. doi:[10.1016/j.jfluidstructs.2020.103107](https://doi.org/10.1016/j.jfluidstructs.2020.103107).

692 Deng, G., Piquet, J., Queutey, P., Visonneau, M., 1994. 2-d computations of  
693 unsteady flow past a square cylinder with the baldwin-lomax model. *Journal*  
694 of *Fluids and Structures* 8, 663–680. doi:[10.1016/s0889-9746\(94\)90140-6](https://doi.org/10.1016/s0889-9746(94)90140-6).

695 Denner, F., van Wachem, B., 2014. Compressive vof method with skewness  
696 correction to capture sharp interfaces on arbitrary meshes. *Journal of Com-*  
697 *putational Physics* 279, 127–144. doi:[10.1016/j.jcp.2014.09.002](https://doi.org/10.1016/j.jcp.2014.09.002).

698 Denner, F., van Wachem, B., 2015. Tvd differencing on three-dimensional un-  
699 structured meshes with monotonicity-preserving correction of mesh skewness.  
700 *Journal of Computational Physics* 298, 466–479. doi:[10.1016/j.jcp.2015.](https://doi.org/10.1016/j.jcp.2015.06.008)  
701 [06.008](https://doi.org/10.1016/j.jcp.2015.06.008).

702 Durao, D.F.G., Heitor, M.V., Pereira, J.C.F., 1988. Measurements of turbulent  
703 and periodic flows around a square cross-section cylinder. *Experiments in*  
704 *Fluids* 6, 298–304. doi:[10.1007/bf00538820](https://doi.org/10.1007/bf00538820).

705 EN 1990, 2002. Eurocode - basis of structural design.

706 EN 1991-1-4, 2005. Eurocode 1: Actions on structures - part 1-4: General  
707 actions - wind actions.

708 Ferziger, J.H., 1993. Estimation and reduction of numerical error, in: *Proceed-*  
709 *ings of the Symposium on Quantification of Uncertainty in Computational*  
710 *Fluid Dynamics*, pp. 1–8.

711 Franke, R., Rodi, W., 1991. Calculation of vortex shedding past a square  
712 cylinder with various turbulence models, in: *Eighth Symposium on turbu-*

713     lent Shear Flows, Paper 20-1, Technical University of Munich, Germany.  
714     doi:[10.1007/978-3-642-77674-8\\_14](https://doi.org/10.1007/978-3-642-77674-8_14).

715     Haque, M., Katsuchi, H., Yamada, H., Nishio, M., 2016. Strategy to de-  
716     velop efficient grid system for flow analysis around two-dimensional bluff  
717     bodies. KSCE Journal of Civil Engineering 20, 1913–1924. doi:[10.1007/  
718     s12205-015-0696-2](https://doi.org/10.1007/s12205-015-0696-2).

719     Hoffman, J., 2005. Computation of mean drag for bluff body problems using  
720     adaptive dns/les. SIAM Journal on Scientific Computing 27, 184–207. doi:[10.  
721     1137/040614463](https://doi.org/10.1137/040614463).

722     Issa, R.I., 1986. Solution of implicitly discretized fluid-flow equations by op-  
723     erator splitting. Journal of Computational Physics 62, 40–65. doi:[10.1016/  
724     0021-9991\(86\)90099-9](https://doi.org/10.1016/0021-9991(86)90099-9).

725     Jasak, H., 1996. Error Analysis and Estimation for the Finite Volume Method  
726     with Applications to Fluid Flows. Ph.D. thesis.

727     Juretić, F., Gosman, A.D., 2010. Error analysis of the finite-volume method  
728     with respect to mesh type. Numerical Heat Transfer, Part B: Fundamentals  
729     57, 414–439. doi:[10.1080/10407791003685155](https://doi.org/10.1080/10407791003685155).

730     Kato, M., Launder, B.E., 1993. The modeling of turbulent flow around station-  
731     ary and vibrating square cylinders, in: Ninth Symposium on Turbulent Shear  
732     Flows, Paper 10-4, Kyoto, Japan.

733     Ke, J., 2019. RANS and hybrid LES/RANS simulations of flow over a square  
734     cylinder. Advances in Aerodynamics 1. doi:[10.1186/s42774-019-0012-9](https://doi.org/10.1186/s42774-019-0012-9).

735     Khosla, P., Rubin, S., 1974. A diagonally dominant second-order accu-  
736     rate implicit scheme. Computers and Fluids 2, 207–209. doi:[10.1016/  
737     0045-7930\(74\)90014-0](https://doi.org/10.1016/0045-7930(74)90014-0).

738     Launder, B., Spalding, D., 1974. The numerical computation of turbulent  
739     flows. Computer Methods in Applied Mechanics and Engineering 3, 269–289.  
740     doi:[10.1016/b978-0-08-030937-8.50016-7](https://doi.org/10.1016/b978-0-08-030937-8.50016-7).

- 741 Lee, B.E., 1975. The effect of turbulence on the surface pressure field of  
742 a square prism. *Journal of Fluid Mechanics* 69, 263–282. doi:[10.1017/  
743 S0022112075001437](https://doi.org/10.1017/S0022112075001437).
- 744 Lee, N., Lee, H., Baek, C., Lee, S., 2016. Aeroelastic analysis of bridge deck  
745 flutter with modified implicit coupling method. *Journal of Wind Engineering  
746 and Industrial Aerodynamics* 155, 11–22. doi:[10.1016/j.jweia.2016.04.  
747 010](https://doi.org/10.1016/j.jweia.2016.04.010).
- 748 Lee, S., 1997. Unsteady aerodynamic force prediction on a square cylinder  
749 using  $k - \varepsilon$  turbulence models. *Journal of Wind Engineering and Industrial  
750 Aerodynamics* 67-68, 79–90. doi:[10.1016/S0167-6105\(97\)00064-0](https://doi.org/10.1016/S0167-6105(97)00064-0).
- 751 Leonard, B., 1979. A stable and accurate convective modelling procedure based  
752 on quadratic upstream interpolation. *Computer Methods in Applied Mechan-  
753 ics and Engineering* 19, 59–98. doi:[10.1016/0045-7825\(79\)90034-3](https://doi.org/10.1016/0045-7825(79)90034-3).
- 754 Liu, M., Xiao, L., Yang, L., 2015. Experimental investigation of flow charac-  
755 teristics around four square-cylinder arrays at subcritical reynolds numbers.  
756 *International Journal of Naval Architecture and Ocean Engineering* 7, 906–  
757 919. doi:[10.1515/ijnaoe-2015-0063](https://doi.org/10.1515/ijnaoe-2015-0063).
- 758 Lyn, D.A., Einav, S., Rodi, W., Park, J.H., 1995. A laser-doppler ve-  
759 locimetry study of ensemble-averaged characteristics of the turbulent wake  
760 of a square cylinder. *Journal of Fluid Mechanics* 304, 285–319. doi:[doi:  
761 10.1017/S0022112095004435](https://doi.org/10.1017/S0022112095004435).
- 762 Lyn, D.A., Rodi, W., 1994. The flapping shear layer formed by flow separation  
763 from the forward corner of a square cylinder. *Journal of Fluid Mechanics* 267,  
764 353. doi:[10.1017/S0022112094001217](https://doi.org/10.1017/S0022112094001217).
- 765 Mannini, C., Sbragi, G., Schewe, G., 2016. Analysis of self-excited forces for a  
766 box-girder bridge deck through unsteady rans simulations. *Journal of Fluids  
767 and Structures* 63, 57–76. doi:[10.1016/j.jfluidstructs.2016.02.007](https://doi.org/10.1016/j.jfluidstructs.2016.02.007).

- 768 Mannini, C., Šoda, A., Schewe, G., 2010. Unsteady rans modelling of flow past  
769 a rectangular cylinder: Investigation of reynolds number effects. *Computers*  
770 *and Fluids* doi:[10.1016/j.compfluid.2010.05.014](https://doi.org/10.1016/j.compfluid.2010.05.014).
- 771 Mariotti, A., Salvetti, M.V., Omrani, P.S., Witteveen, J.A., 2016. Stochastic  
772 analysis of the impact of freestream conditions on the aerodynamics of a  
773 rectangular 5:1 cylinder. *Computers and Fluids* doi:[10.1016/j.compfluid.](https://doi.org/10.1016/j.compfluid.2016.06.008)  
774 [2016.06.008](https://doi.org/10.1016/j.compfluid.2016.06.008).
- 775 Moore, D.M., Letchford, C.W., Amitay, M., 2019. Energetic scales in a bluff  
776 body shear layer. *Journal of Fluid Mechanics* doi:[10.1017/jfm.2019.480](https://doi.org/10.1017/jfm.2019.480).
- 777 Murakami, W., Mochida, A., 1995. On turbulent vortex shedding past 2d square  
778 cylinder predicted by cfd. *Journal of Wind Engineering and Industrial Aero-*  
779 *dynamics* 54-55, 191–211. doi:[10.1016/0167-6105\(94\)00043-D](https://doi.org/10.1016/0167-6105(94)00043-D).
- 780 Muzaferija, S., 1994. Adaptative finite volume method for flow prediction using  
781 unstructured meshes and multigrid approach. Ph.D. thesis.
- 782 Nakane, I., 2013. Effectiveness of grid orthogonality on body fitted curvilinear  
783 coordinate system in cfd. *International Journal of Environmental Science and*  
784 *Development* 4. doi:[10.7763/ijesd.2013.v4.329](https://doi.org/10.7763/ijesd.2013.v4.329).
- 785 Nieto, F., Owen, J.S., Hargreaves, D.M., Hernandez, S., 2015. Bridge deck  
786 flutter derivatives: Efficient numerical evaluation exploiting their interde-  
787 pendence. *Journal of Wind Engineering and Industrial Aerodynamics* 136,  
788 138–150. doi:[10.1016/j.jweia.2014.11.006](https://doi.org/10.1016/j.jweia.2014.11.006).
- 789 Nishimura, H., 2001. Fundamental study of bluff body aerodynamics (in  
790 Japanese). Ph.D. thesis.
- 791 Noda, H., Nakayama, A., 2003. Free-stream turbulence effects on the instanta-  
792 neous pressure and forces on cylinders of rectangular cross section. *Experi-*  
793 *ments in Fluids* 34, 332–344. doi:[10.1007/s00348-002-0562-0](https://doi.org/10.1007/s00348-002-0562-0).



794 Oka, S., Ishihara, T., 2009. Numerical study of aerodynamic characteristics of a  
795 square prism in a uniform flow. *Journal of Wind Engineering and Industrial*  
796 *Aerodynamics* 97, 548–559. doi:[10.1016/j.jweia.2009.08.006](https://doi.org/10.1016/j.jweia.2009.08.006).

797 de Oliveira Samel Moraes, A., da Cunha Lage, P., Cunha, G., Silva, L., 2013.  
798 Analysis of the non-ortogonality correction of finite volume discretization on  
799 unstructured meshes,, in: 22nd International Congress of Mechanical Engi-  
800 neering (COBEM 2013).

801 Otsuki, Y., Fuji, K., Washizu, K., Ohya, A., 1978. Wind tunnel experiments  
802 on aerodynamic forces and pressure distributions of rectangular cylinders in  
803 a uniform flow, in: *Proceedings of the Fifth Symposium on Wind Effects on*  
804 *Structures*, pp. 169–176.

805 Pocha, J.J., 1971. On unsteady flow past cylinders of square cross-section. Ph.D.  
806 thesis.

807 Riberio, A.F.P., 2011. Unsteady rans modelling of flow past a rectangular  
808 5:1 cylinder: investigation of edge sharpness effects. *Proceedings of the*  
809 *Thirteenth International Conference on Wind Engineering*, Amsterdam, The  
810 Netherlands .

811 Rocchio, B., Mariotti, A., Salvetti, M., 2020. Flow around a 5:1 rectangular  
812 cylinder: Effects of upstream-edge rounding. *Journal of Wind Engineering &*  
813 *Industrial Aerodynamics* 204, 104237. doi:[10.1016/j.jweia.2020.104237](https://doi.org/10.1016/j.jweia.2020.104237).

814 Rodi, W., 1997. Comparison of les and rans calculations of the flow around  
815 bluff bodies. *Journal of Wind Engineering and Industrial Aerodynamics* 69-  
816 71, 55–75. doi:[10.1016/s0167-6105\(97\)00147-5](https://doi.org/10.1016/s0167-6105(97)00147-5).

817 Rodi, W., 2004. Ufr 2-02, flow past cylinder. URL: [https://www.kbwiki.](https://www.kbwiki.ercoftac.org/w/index.php?title=UFR_2-02_Description)  
818 [ercoftac.org/w/index.php?title=UFR\\_2-02\\_Description](https://www.kbwiki.ercoftac.org/w/index.php?title=UFR_2-02_Description).

819 Rodi, W., 2012. The ERCOFTAC knowledge base wiki - an aid for validating  
820 CFD models, in: *Process and Plant Safety*. Wiley-VCH Verlag GmbH & Co.  
821 KGaA, pp. 179–187. doi:[10.1002/9783527645725.ch11](https://doi.org/10.1002/9783527645725.ch11).

- 822 Rodi, W., Ferziger, J.H., Breuer, M., Pourquié, M., 1997. Status of large-eddy  
823 simulation: Results of a workshop. *Journal of Fluids Engineering* 119, 248–  
824 262. doi:[10.1115/1.2819128](https://doi.org/10.1115/1.2819128).
- 825 Sakamoto, H., Haniu, H., Kobayashi, Y., 1989. Fluctuating force acting on rect-  
826 angular cylinders in uniform flow on rectangular cylinders with fully separated  
827 flow. *Transactions of the Japan Society of Mechanical Engineers, Series B* 55,  
828 2310–2317.
- 829 Schewe, G., 2013. Reynolds-number-effects in flow around a rectangular cylinder  
830 with aspect ratio 1:5. *Journal of Fluids and Structures* 39, 15–26. doi:[10.1016/j.jfluidstructs.2013.02.013](https://doi.org/10.1016/j.jfluidstructs.2013.02.013).
- 832 Scruton, C., Rogers, E., Menzies, J., Scorer, R., 1971. Steady and unsteady wind  
833 loading of buildings and structures [and discussion]. *Philosophical Transac-*  
834 *tions of the Royal Society of London Series A, Mathematical and Physical*  
835 *Sciences* 269, 353–383. URL: <https://www.jstor.org/stable/73873>.
- 836 Shimada, K., Ishihara, T., 2002. Application of a modified  $k-\varepsilon$  model to the pre-  
837 diction of aerodynamic characteristics of rectangular cross-section cylinders.  
838 *Journal of Fluids and Structures* 16, 465–485. doi:[10.1006/jfls.2001.0433](https://doi.org/10.1006/jfls.2001.0433).
- 839 Shyy, W., Thakur, S., Wright, J., 1992. Second-order upwind and central differ-  
840 ence schemes for recirculating flow computation. *AIAA Journal* 4, 923–932.  
841 doi:[10.2514/3.11010](https://doi.org/10.2514/3.11010).
- 842 Sohankar, A., Davidson, L., Norberg, C., 2000. Large eddy simulation of flow  
843 past a square cylinder: Comparison of different subgrid scale models. *Journal*  
844 *of Fluids Engineering* 122, 39–47. doi:[10.1115/1.483224](https://doi.org/10.1115/1.483224).
- 845 Tamura, T., Ito, Y., 1996. Aerodynamic characteristics and flow structures  
846 around a rectangular cylinder with a section of various depth/breadth ratios.  
847 *Journal of Structural and Construction Engineering (Transactions of Archi-*  
848 *tectural Institute of Japan)* 486, 153–162. doi:[10.3130/aijs.61.153\\_2](https://doi.org/10.3130/aijs.61.153_2).

- 849 Tamura, T., Miyagi, T., Kitagishi, T., 1998. Numerical prediction of un-  
850 steady pressures on a square cylinder with various corner shapes. *Journal*  
851 *of Wind Engineering and Industrial Aerodynamics* 74-76, 531–542. doi:[10.](https://doi.org/10.1016/s0167-6105(98)00048-8)  
852 [1016/s0167-6105\(98\)00048-8](https://doi.org/10.1016/s0167-6105(98)00048-8).
- 853 Tang, H., Shum, K., Li, Y., 2019. Investigation of flutter performance of a twin-  
854 box bridge girder at large angles of attack. *Journal of Wind Engineering and*  
855 *Industrial Aerodynamics* 186, 192–203. doi:[10.1016/j.jweia.2019.01.010](https://doi.org/10.1016/j.jweia.2019.01.010).
- 856 Traoré, P., Ahipo, Y., Louste, C., 2009. A robust and efficient finite volume  
857 scheme for the discretization of diffusive flux on extremely skewed meshes  
858 in complex geometries. *Journal of Computational Physics* 228, 5148–5159.  
859 doi:[10.1016/j.jcp.2009.04.007](https://doi.org/10.1016/j.jcp.2009.04.007).
- 860 Trias, F.X., Gorobets, A., Oliva, A., 2015. Turbulent flow around a square  
861 cylinder at reynolds number 22000: a dns study. *Computer and Fluids* 123,  
862 87–98. doi:[10.1016/j.compfluid.2015.09.013](https://doi.org/10.1016/j.compfluid.2015.09.013).
- 863 Vickery, B., 1966. Fluctuating lift and drag on a long cylinder of square cross  
864 section in a smooth and turbulent flow. *Journal of Fluid Mechanics* 25, 481–  
865 494. doi:[10.1017/S002211206600020X](https://doi.org/10.1017/S002211206600020X).
- 866 Wang, G., Vanka, S.P., 1997. Les of flow over a square cylinder, in: Chollet,  
867 J.P., Voke, P.R., Kleiser, L. (Eds.), *Direct and Large-Eddy Simulation II*, pp.  
868 397–400. doi:[10.1007/978-94-011-5624-0\\_37](https://doi.org/10.1007/978-94-011-5624-0_37).
- 869 Wilkinson, R., 1974. On the vortex-induced loading on long bluff cylinders.  
870 Ph.D. thesis.
- 871 Yakhot, V., Orszag, S., Thangam, S., Gatski, T., Speziale, G., 1992. Develop-  
872 ment of turbulence models for shear flows by a double expansion technique.  
873 *Physics of Fluids A* 4, 1510–1520. doi:[10.1063/1.858424](https://doi.org/10.1063/1.858424).
- 874 Younis, B.A., Przulj, V.P., 2005. Computation of turbulent vortex shedding.  
875 *Computational Mechanics* 37, 408–425. doi:[10.1007/s00466-005-0713-2](https://doi.org/10.1007/s00466-005-0713-2).

- 876 Yu, D., Kareem, A., 1996. Two dimensional simulation of flow around rectan-  
877 gular prisms. *Journal of Wind Engineering and Industrial Aerodynamics* ,  
878 131–161doi:[10.1016/s0167-6105\(96\)00076-1](https://doi.org/10.1016/s0167-6105(96)00076-1).
- 879 Zhang, Z., Xu, F., 2020. Spanwise length and mesh resolution effects on simu-  
880 lated flow around a 5:1 rectangular cylinder. *Journal of Wind Engineering &*  
881 *Industrial Aerodynamics* 202, 104186. doi:[10.1016/j.jweia.2020.104186](https://doi.org/10.1016/j.jweia.2020.104186).
- 882 Zhu, G., Huang, S., Li, Q., 2020. Large-eddy simulation of the inflow turbulence  
883 transport and aerodynamics of a rectangular 5:1 cylinder with high-order  
884 numerical methods. *Journal of Wind Engineering & Industrial Aerodynamics*  
885 207, 104366. doi:[10.1016/j.jweia.2020.104366](https://doi.org/10.1016/j.jweia.2020.104366).