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(Article begins on next page)

Effects of cell quality in Grid Boundary Layer on the simulated flow around a square cylinder

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Abstract

The flow around a square cylinder is widely studied as a paradigmatic case in bluff body aerodynamics. The effects of several physical parameters of the setup, and the errors induced by turbulence models, numerical schemes and grid density have been emphasized in a huge number of studies during the past two decades. Surprisingly, the effects of the grid quality on such a class of flow has been overlooked. The lack of a shared approach and suggested best practices for high-quality grid generation among scholars and practitioners follows. The present study aims at filling this gap. The cell skewness and non-orthogonality are adopted as metrics of the grid quality. The errors induced by poor quality cells and the possible corrective measures are discussed in a Finite Volume Method framework. The effects of the cell quality on the simulated flow are systematically evaluated by a parametrical study including four different types of grid boundary layer. The obtained results are compared among them and discussed in terms of instantaneous and time averaged flow fields, stress distribution at wall, and aerodynamic coefficients. Both the overall modelling error and the skewness-induced one are evaluated with reference to a huge number of data collected from previous studies. The local error induced by few, moderately skewed, near-wall cells upwind the cylinder propagates windward because

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of the convection-dominated problem, and globally affects the boundary layer separation and the vortex shedding in the wake. Skewness around the trailing edge only affects the flow to a lower extent. The skewness error on bulk aerodynamic coefficients may largely prevails on the overall modelling error, in spite of the very simple turbulence model deliberately adopted in the study. Hybrid grid boundary layer made of structured cells along the cylinder sides and unstructured ones around its edges provides results analogous to the ones obtained with a fully orthogonal grid, in spite of some clusters of few skewed cells far from the wall. Hybrid grid boundary layer is recommended as a fine balance between accuracy and flexibility in grid generation, when full orthogonal grid boundary layer is not feasible around real-world engineering applications having complicate geometries with multiple obtuse or acute edges.

Keywords: Finite Volume Method, grid boundary layer, grid quality, skewness, square cylinder

1. Introduction

The high-Reynolds number, turbulent flow around rectangular cylinders is considered as a paradigmatic one around bluff bodies, of interest both for fundamental research and applications. In spite of the simple and nominally two-dimensional geometry, the flow is intricate, characterized by boundary layer separation, its possible reattachment depending on the ratio of the alongwind to crosswind dimensions, Kelvin-Helmholtz instability of the detached shear layer, Von-Karman-like vortex shedding along the wake. Such underlying flow regimes provide useful information on the aerodynamics of a wide range of bluff bodies of interest in civil engineering (e.g. long-span bridge decks or high-rise buildings) as well as in other engineering areas. The importance attached to such flows by the scientific community is testified by some international benchmarks, such as the ones on the flow around 1:1 square cylinder (Rodi, 2004; Rodi et al., 1997), and around 5:1 rectangular cylinder (Bruno and Salvetti, 2017; Bruno et al., 2014).

16 The studies within the benchmarks above have testified the high sensitivity
17 of the flow to both physical parameters of the setup, and to errors induced by
18 the components of the computational model. Among the former, special at-
19 tention was paid for instance to the cylinder edge roundness (Tamura et al.,
20 1998; Riberio, 2011; Cao and Tamura, 2017; Rocchio et al., 2020), Reynolds
21 number (Scruton et al., 1971; Mannini et al., 2010; Schewe, 2013), and to other
22 freestream conditions (e.g. Vickery, 1966; Lee, 1975; Mariotti et al., 2016; Cao
23 and Tamura, 2018). Among the latter, the effects of turbulence models on the
24 simulated flow have been emphasized in a huge number of studies (e.g. Franke
25 and Rodi, 1991; Kato and Launder, 1993; Rodi, 1997; Lee, 1997; Bosch and
26 Rodi, 1998; Sohankar et al., 2000; Ke, 2019). In their wake, the conscious and
27 adapted application of different turbulence models is currently widespread in
28 engineering practice. Even if to a minor extent, also the effects of the numerical
29 schemes on the simulated flow were scrutinized by a number of studies, with
30 special emphasis on the discretization of the convective terms (Shyy et al., 1992;
31 Lee, 1997; Tamura et al., 1998; Cao and Tamura, 2016; Zhu et al., 2020).

32 More recently, the exponential growth of HPC facilities is allowing the sys-
33 tematic evaluation of the effects of the *grid density*, both in the vertical along-
34 wind $x - y$ plane (Cao and Tamura, 2016) and along the spanwise z -direction
35 (Bruno et al., 2012; Cao and Tamura, 2016; Cao et al., 2020; Zhang and Xu,
36 2020). The attention of other authors focused on Adaptive Mesh Refinement
37 in Finite Element Method (see e.g. Hoffman, 2005; Berrone and Marro, 2009)
38 and Finite Volume Method (FVM) (see e.g. Antepará et al., 2015). The errors
39 induced by poor *grid quality* in FVM are well known in fundamental research on
40 computational methods since the pioneering PhD Theses of Muzafferija (1994)
41 and Jasak (1996), up to the recent studies by e.g. Ahipo and Traoré (2009);
42 Traoré et al. (2009); Juretić and Gosman (2010); de Oliveira Samel Moraes
43 et al. (2013); Denner and van Wachem (2014, 2015). Conversely, grid quality
44 effects on the simulated flow around bluff bodies are surprisingly overlooked in
45 scientific and technical literature. In particular, the current and very pervasive
46 practice in the simulation of flow around cylinders implies that fully orthogonal

47 structured grids are generated by extrusion with constant grid spacing in the
 48 spanwise direction, but non orthogonal and skewed grids are inevitably gener-
 49 ated in the vertical alongwind plane orthogonal to the cylinder axis. To our best
 50 knowledge, only a few number of studies partially cover such an issue. Nakane
 51 (2013) applied FVM with orthogonal and non-orthogonal grids to the low-Re
 52 ($Re=550, 3000$) transient flow around a circular cylinder, with focus on the early
 53 stage of the symmetrical steady reversed flow in the near wake. Haque et al.
 54 (2016) paid attention to the effects of the cell height growing ratio across the
 55 Grid Boundary Layer (GBL) on the high-Re flow around rectangular cylinders
 56 and hexagonal bridge decks. The grid quality across the GBL is expected to
 57 be of paramount importance, because of the high gradient of the velocity and
 58 turbulent variables in the boundary layer. Shortage in studies on the effects
 59 of grid quality in GBL results in the lack of a shared approach and suggested
 60 best practices among scholars and practitioners. For instance, Figure 1 provides
 61 some examples of different GBLs adopted in recent computational studies on
 the aerodynamics of 1:1 and 5:1 rectangular cylinders. Even if a structured

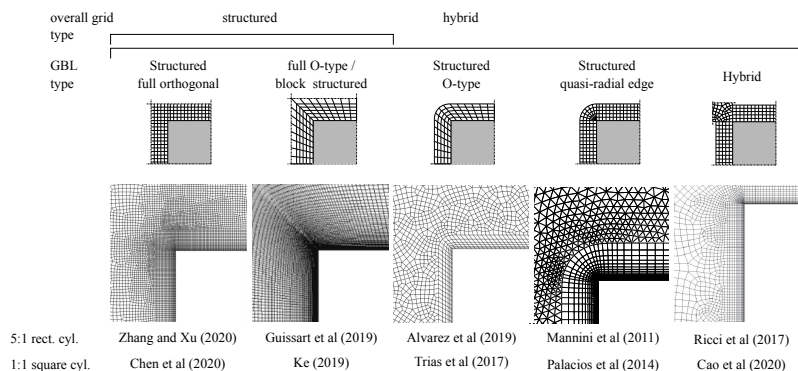


Figure 1: Types of GBL adopted in recent studies within benchmarks on the aerodynamics of 1:1 square and 5:1 rectangular cylinders

62
 63 orthogonal GBL with quadrangular cells is generated in all studies along the
 64 cylinder sides, they significantly differ in the meshing strategy around the sharp

65 (or slightly rounded) corners, in the shape of the cells around them, and in the
 66 grid quality in turn. A non necessary exhaustive GBL categorization is ten-
 67 tatively proposed in the figure, in the absence of a common nomenclature in
 68 literature. All meshing strategies at corners are structured, except for the 'Hy-
 69 brid GBL', where orthogonal boundary-fitted grid along the straight sides of the
 70 cylinder alternates with unstructured grid in the quadrants departing from the
 71 sharp corners. The latter inevitably includes almost regular, quasi orthogonal
 72 and skewness-free cells in the first layers around the cylinder corners because
 73 of the strong two-side constraint of the discretization of the adjacent edges of
 74 the structured parts. Low quality cells take place moving away from the cylin-
 75 der corners along and around the bisector of the quadrant. A single, largely
 76 prevailing GBL meshing approach among the above categories does not exist in
 77 research and engineering practice. For instance, Figure 2 provides statistics on
 78 GBLs adopted in 23 studies published in the last decade on the aerodynamics
 of the 5:1 rectangular cylinder.

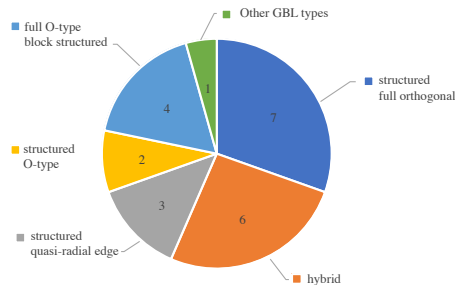


Figure 2: Statistics on types of GBL adopted in literature for the Benchmark on the Aerodynamics of a Rectangular 5:1 Cylinder (BARC)

79
 80 An even larger variability of GBL meshing strategies occurs for other cylinder-
 81 like, real-world engineering applications having more complicate geometries with
 82 multiple obtuse or acute edges, such as bridge deck aerodynamics. For instance,
 83 [Tang et al. \(2019\)](#) adopt a Hybrid GBL, [Nieto et al. \(2015\)](#) a block-structured
 84 grid, [Mannini et al. \(2016\)](#) a quasi radial GBL, being full orthogonal GBL un-
 85 feasible. The grid-induced errors are also expected to vary correspondingly. The

86 effects of low-quality grids could be even larger in fluid-structure interaction ap-
 87 plications, such as bridge or cable aeroelasticity, where highly distorted cells can
 88 arise from large amplitude pitching motion of the cylinder (see e.g. [Lee et al.,](#)
 89 [2016](#), Fig.2).

90 This study aims to shed some light on the errors and effects of different
 91 GBLs on the simulated flow around 1:1 square cylinder, and to compare such
 92 grid-induced errors with the modelling errors. Besides this introduction, the
 93 paper is organized into four more sections. In Section 2 the overall adopted
 94 computational model is shortly recalled, while spatial grid quality metrics and
 95 effects are detailed. The setup of the application is described in Section 3.
 96 The obtained results are commented in Section 4. Finally, in Section 5 final
 97 conclusions are drawn, and some research perspectives are briefly sketched.

98 2. Computational model

99 In the following, the adopted computational model is described in terms of
 100 turbulence model, boundary conditions, numerical approach, and spatial grid
 101 quality. Basic, well established and widely used turbulence model and numerical
 102 approach are deliberately adopted in a 2D domain. They are expected to cor-
 103 rectly reproduce the flow of interest in qualitative terms ([Lee, 1997](#)), although
 104 with quantitative errors. Departures from top quality grid and induced errors
 105 are detailed, the study being intended to quantitatively compare the overall er-
 106 ror model with the one induced by low-quality GBLs in the vertical alongwind
 107 plane orthogonal to the cylinder axis.

108 2.1. Turbulence model

109 The incompressible, turbulent, separated, unsteady flow around the 2D sec-
 110 tion is modeled by the classical Unsteady Reynolds Averaged Navier-Stokes
 111 (URANS) equations, which in indicial form read:

$$U_{i,i} = 0, \tag{1}$$

$$U_{i,t} + U_j U_{i,j} = -\frac{1}{\rho} P_{,i} + (\nu (U_{i,j} + U_{j,i}) - R_{ij})_{,j}, \tag{2}$$

113 where U_i is the averaged velocity component in the i -th direction, P is the
114 averaged pressure, ρ the air density, and ν its kinematic viscosity, $R_{ij} = \overline{u_i u_j}$
115 the Reynolds stress, and u the velocity turbulent fluctuations. The $k - \varepsilon$ RNG
116 (Yakhot et al., 1992) turbulence model is used to close the URANS equations.
117 The Reynolds stress is expressed by the well-know Boussinesq assumption as
118 $R_{ij} = -2\nu_t S_{ij}$, where S_{ij} is the mean field strain rate, and the turbulent vis-
119 cosity ν_t is expressed as $\nu_t = C_\mu \frac{k^2}{\varepsilon}$ in terms of the turbulent kinetic energy k
120 and its dissipation rate ε . Their transport equations are

$$k_{,t} + U_j k_{,j} = -R_{ij} U_{i,j} + \left(\frac{\nu_t}{\sigma_k} k_{,i} \right)_{,i} - \varepsilon, \quad (3)$$

$$\varepsilon_{,t} + U_j \varepsilon_{,j} = -C_{\varepsilon_1} \frac{\varepsilon}{k} R_{ij} U_{i,j} + \left(\frac{\nu_t}{\sigma_\varepsilon} \varepsilon_{,i} \right)_{,i} - C_{\varepsilon_2}^* \frac{\varepsilon^2}{k}, \quad (4)$$

121
122 where $C_{\varepsilon_2}^* = C_{\varepsilon_2} + C_\mu \eta^3 \frac{(1-\eta/\eta_0)}{1+\beta\eta^3}$, and $\eta = \frac{k}{\varepsilon} \sqrt{2S_{ij}S_{ij}}$ is the ratio between the
123 turbulence time scale and the time scale of the mean field strain. The model
124 constants are $C_\mu = 0.085$, $C_{\varepsilon_1} = 1.42$, $C_{\varepsilon_2} = 1.68$, $\eta_0 = 4.38$, $\beta = 0.012$,
125 $\sigma_k = \sigma_\varepsilon = 1.393$. The cell-centered nodal values of the flow variables adjacent
126 to the wall are obtained by the standard wall function approach (Lauder and
127 Spalding, 1974). Dirichlet boundary condition (b.c.) on the velocity field and
128 on the turbulent variables are imposed at the inlet. Neumann b.c. on the normal
129 component of the stress tensor, as well as the same Dirichlet b.c. on k and ε ,
130 are imposed at the outlet. Periodic b.c. are imposed on both the upper-lower
131 boundaries. No-slip b.c. are imposed at the cylinder wall.

132 2.2. Numerical approach

133 A classic overall 2nd order approach in the framework of the Finite Volume
134 Method is adopted to discretize the governing equations above. The cell-centre
135 values of the variables are interpolated at face locations using the classic second-
136 order Central Difference Scheme (CDS) for the diffusive terms. The convection
137 terms are discretized by means of the QUICK scheme (Leonard, 1979). The
138 fully implicit second-order Euler scheme is adopted for time discretization. The
139 SIMPLE algorithm (Issa, 1986) is used for pressure-velocity coupling. The code
140 Ansys Fluent© v.18 is used.

141 2.3. Grid quality effects

142 Non-orthogonality and skewness are common issues on non-Cartesian, ar-
 143 bitrary grids. A structured radial edge and an O-type GBLs are considered
 in Figure 3 as examples. Both non-orthogonality and skewness are qualities

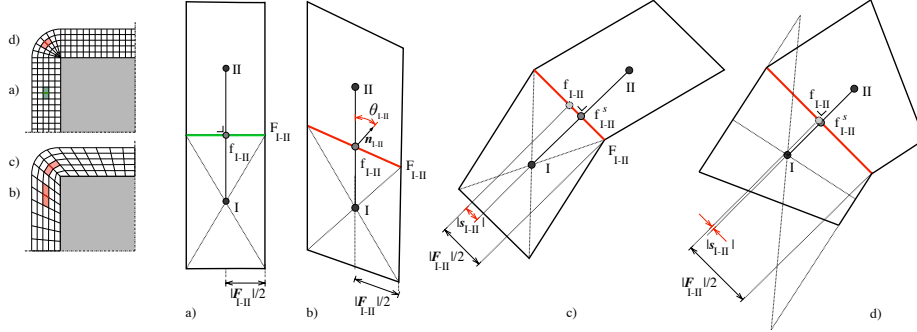


Figure 3: Face orthogonality $\theta = 0$ (a), non-orthogonality $\theta \neq 0$ (b), and skewness s (c,d) of cells around a sharp edge in different types of GBL

144

145 that refer to each face F_{I-II} shared between adjacent cells with centers I and II.
 146 Some pairs of cells are highlighted in Figure 3 for the sake of clarity. *Non-*
 147 *orthogonality* refers to the angle θ_{I-II} between the normal vector \mathbf{n}_{I-II} and the
 148 vector connecting the two cell centers. The face is orthogonal if $\theta_{I-II} = 0$ (Fig.
 149 3-a, -c, -d), non-orthogonal otherwise (Fig. 3-b). *Skewness* refers to the vector
 150 \mathbf{s}_{I-II} from the face mid-point f_{I-II} to the interpolation point f_{I-II}^s , the latter being
 151 defined as the intersection between the face and the vector connecting the two
 152 cell centers. The face is not skewed if $|\mathbf{s}_{I-II}| = 0$ (Fig. 3-a,b), skewed otherwise
 153 (Fig. 3-c,d). The examples in Figure 3 clearly underline that rectangular cells
 154 are perfectly orthogonal and not skewed, and that non-orthogonality does not
 155 necessarily implies skewness, and vice-versa. The above metrics can be made
 156 scalar and dimensionless as $\theta_{I-II}^* = 2\theta_{I-II}/\pi$ and $s_{I-II}^* = 2|\mathbf{s}_{I-II}|/|F_{I-II}|$, where
 157 $|F_{I-II}|$ is the length of the face F_{I-II} (Fig. 3). Both dimensionless metrics take
 158 values larger than unit for degenerate cells and interpolation points outside the
 159 face, respectively.

160 Both non-orthogonality and skewness are *potential sources of errors* in the

161 numerical fluxes evaluated by FVM. *Non-orthogonality-induced error* on the dif-
 162 fusive terms can be pointed out by splitting the face surface vector \mathbf{F}_{I-II} in the
 163 component $\Delta \parallel \mathbf{d}_{I-II}$ and in a non-orthogonal one \mathbf{k} (Fig. 4-a, after Jasak,
 164 1996). Whatever the chosen direction of \mathbf{k} is, the error induced on the diffusive
 165 flux of a generic variable ϕ reads in vectorial form

$$E_{no}^d = \nabla \cdot (\nu_{no} \nabla \phi) \quad (5)$$

166 where $\nu_{no} = \mathbf{U}_{f_{I-II}} \cdot \mathbf{k}$ is the so-called non-orthogonal viscosity, and $\mathbf{U}_{f_{I-II}}$ is the
 167 velocity vector at the face mid-point.

168 *Skewness-induced error* can affect the convective terms, and results in a spurious
 169 non-physical diffusion flux evaluated by Jasak (1996) as

$$E_{sk}^c = \nabla \cdot (\nu_{sk} \nabla \phi) \quad (6)$$

170 where $\nu_{sk} = \mathbf{U}_{f_{I-II}} \cdot \mathbf{s}$ is the so-called skewness viscosity. In other terms, grid
 171 skewness reduces the accuracy of interpolation to first order, and introduces a
 172 diffusion-like error in the discretized convective flux, analogously to the well-
 173 known leading truncation error of the first-order upwind scheme.

174 In short, both non-orthogonality and skewness introduce non-physical diffusion,
 175 if not properly dealt. In an engineering perspective such diffusive effects are not
 176 negligible for convection-dominated, high Reynolds number flow, where physical
 177 diffusive effects play a minor role. In other terms, both grid-induced viscosities
 178 can cumulate the kinematic viscosity and the modeled turbulent one, resulting
 179 in a simulated Re number lower than the one that characterizes the actual flow.

180 Two approaches can be adopted to counter this problem. *High-quality grid*
 181 *generation* is intended to eliminate the error source, i.e. to reduce the amplitude
 182 of the geometrical metrics θ and s , and the induced errors in turn. High quality
 183 grid may be generated a priori, or obtained a posteriori by grid adaptation.
 184 Whatever the adopted technique is, such an approach is the most decisive, even
 185 if generation/adaptation criteria are required. *Corrective terms in interpolation*
 186 *schemes* are intended to mitigate the effects of low-quality grids, i.e. to reduce
 187 the errors (eq.s 5 and 6) without eradicate their causes.

188 Non-orthogonal correction strategies move from the surface vector splitting
 189 $\mathbf{F}_{I-II} = \mathbf{\Delta} + \mathbf{k}$ initially proposed by [Jasak \(1996\)](#) (Fig. 4-a).

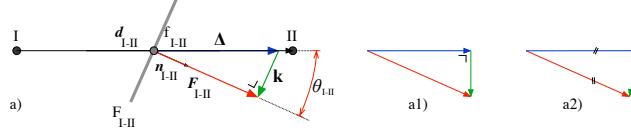


Figure 4: Approache for correction of non-orthogonality (after [Jasak, 1996](#))

189
 190 The diffusive flux for the generic variable ϕ can be approximated as

$$\mathbf{F}_{I-II} \cdot (\nabla \phi)_{f_{I-II}} = \mathbf{\Delta} \cdot (\nabla \phi)_{f_{I-II}} + \mathbf{k} \cdot (\nabla \phi)_{f_{I-II}} \approx |\mathbf{\Delta}| \frac{\phi_{II} - \phi_I}{|\mathbf{d}_{I-II}|} + \mathbf{k} \cdot (\widetilde{\nabla \phi})_{f_{I-II}}, \quad (7)$$

191 where the classic CDS is used for the orthogonal term. The non-orthogonal
 192 correction second term depends on both the definition of \mathbf{k} , and the numer-
 193 ical approximation of $(\nabla \phi)_{f_{I-II}}$. Different explicit non-orthogonal correction
 194 schemes have been proposed in literature, e.g. [Jasak \(1996\)](#); [Ahipo and Traoré](#)
 195 [\(2009\)](#); [Traoré et al. \(2009\)](#). Three possible choices for \mathbf{k} are depicted in Fig-
 196 ures 4(a), (a1), (a2), where the former minimizes the correction. The iterative
 197 numerical approximation $(\widetilde{\nabla \phi})_{f_{I-II}}$ is generally inspired to the precursor ideas
 198 in [Khosla and Rubin \(1974\)](#) and [Muzaferija \(1994\)](#). Generally speaking, the
 199 non-orthogonality correction let to preserve the spatial second order accuracy.
 200 However, when its contribution is larger than the orthogonal one for highly
 201 non-orthogonal faces, the boundedness of the solution is no longer guaranteed
 202 ([Jasak, 1996](#)). In order to ensure boundedness, the non-orthogonal correction
 203 must be bounded or even discarded and the scheme becomes no longer second
 204 order accurate ([de Oliveira Samel Moraes et al., 2013](#)).

205 Also skewness correction strategy is intended to correct standard interpolation
 206 schemes for the quadrature values by additional term(s) accounting for \mathbf{s} . Start-
 207 ing from the pioneering work of ([Jasak, 1996](#)) where the skewness problem has
 208 been rigorously presented, some proposals for skewness correction have been
 209 recently proposed in literature, e.g. [de Oliveira Samel Moraes et al. \(2013\)](#);

210 [Denner and van Wachem \(2014, 2015\)](#).

211 While non-orthogonal correction is widely implemented in industrial codes
212 such as OpenFoam or Fluent, the same can not be said about the skewness cor-
213 rection ([de Oliveira Samel Moraes et al., 2013](#)). In the light of this, the present
214 study aims at evaluating different strategies of GBL generation to reduce the
215 skewness geometrical metric $|\mathbf{s}|$, while GBL non-orthogonality is fixed according
216 to the correction given in eq. (7).

217 3. Setup of the study

218 The adopted flow setup closely follows the one prescribed by the Ercoftac
219 benchmark Ufr 2-02, detailed by [Rodi \(2004\)](#) in the QNET-CFD Knowledge
220 Base Wiki ([Rodi, 2012](#)). The benchmark adopts as reference experimental tests
221 the ones detailed in [Lyn and Rodi \(1994\)](#) and [Lyn et al. \(1995\)](#). The incoming
222 flow is characterized by Reynolds number $Re = UD/\nu = 2.2e + 4$, where U is
223 the free stream velocity and D the cylinder chord, incidence $\alpha = 0$, turbulence
224 intensity $It = 2\%$, turbulent length scale $Lt = 0.5D$. The cylinder cross section
225 has sharp edges and smooth surfaces. The dimensionless time step is set $\Delta t^* =$
226 $0.02tU/D$. The simulation is extended over $T \approx 300$ dimensionless time units
227 in order to have a long enough statistical sample to secure converged statistics,
228 after having excluded the initial transient. The dimensions of the 2D domain
229 are given in Figure 5(a). The origin of the Cartesian coordinate is located at
230 the center of the cylinder upwind face. The spatial grid is overall hybrid, being
231 composed by both quadrangular and triangular cells, and by structured and
232 unstructured partitions. A close-up view of the grid around the cylinder and its
233 close wake is shown in Figure 5(b). In addition to usual strategies, subdomain
234 partitioning and corresponding grid blocks are intended to ensure a dense, top-
235 quality grid made of square cells ($\Delta_x = \Delta_y = 0.042D$) in the wake region, and
236 to keep the grid outside the GBL unchanged for all cases.

237 Four types of GBLs are generated in a D -tick square annular partition
238 around the cylinder, as detailed in Figure 5(c). All GBLs share some common

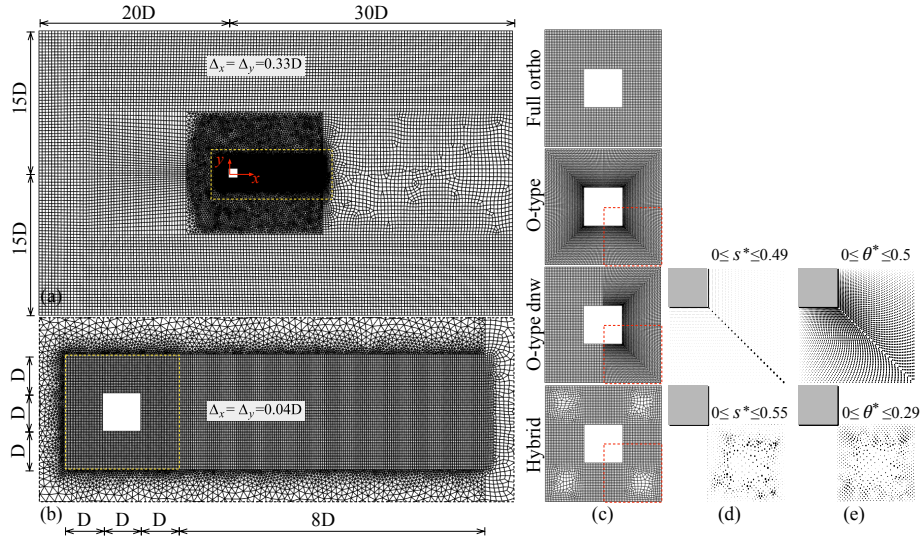


Figure 5: Computational domain (a), GBL and wake subdomain (b), GBL types (c) and their dimensionless skewness (d) and non-orthogonality (e)

239 features, in order to allow a proper comparative parametric study: cylinder side
 240 uniformly discretized; height of the first cell at wall equal to $\Delta n/D = 1/24$;
 241 growing ratio equal to unit (nil grid gradient) and number of cells equal to 24
 242 across the structured parts of the GBL. Δn is set to comply the requirements
 243 of wall-function-based near-wall treatment about the height of the control vol-
 244 umes at wall: the wall unit $n^+ = nu_*/\nu$ results in the range $25 \leq n^+ \leq 55$. The
 245 generated GBLs share the discretization of the quadrant edges, but differ in the
 246 type of grid. The 'Full ortho' GBL has the same top quality grid a in the wake.
 247 The 'O-type' one is characterized by a block-structured non-orthogonal grid.
 248 The 'O-type dnw' GBL is a mix of the previous ones, where O-type grid is lim-
 249 ited to the downwind half of the annular partition. In contrast to the previous
 250 GBLs, a 'Hybrid' GBL is not uniquely defined by the discretization of the outer
 251 edges of the quadrants, but it necessarily depends also on the specific algorithm
 252 adopted for grid generation. In the present study, a paving, advancing-front-
 253 based algorithm is employed (Blacker and Stephenson, 1991). The resulting,

254 specific realization of the Hybrid GBL shows the general distinctive features of
 255 this class of GBLs as detailed in the introduction. However, in this specific case
 256 the density and quality of the cells do not monotonically decrease moving away
 257 from the wall as usually (see e.g. Fig. 1), because of nil grid gradient across
 258 the structured parts of the GBL. Skewness and non-orthogonality are nil every-
 259 where in the GBL for the 'Full ortho' GBL and the upwind part of the 'O-type
 260 dnw' one. The spatial patterns of dimensionless skewness and non-orthogonal
 261 metrics are given by scatterplots in Figures 5(d) and (e), respectively, for the
 262 other cases and limited to the quadrant enclosed by the red dashed line. The
 263 s^* pattern in O-type GBLs groups skewness $s^* \neq 0$ along the diagonal only,
 264 while $\theta^* \neq 0$ is distributed in the rest of the field. Significant values of both
 265 s^* and θ^* are more scattered in the generated Hybrid GBL, with the highest
 266 values in correspondence of the transition between dense and coarse grid. In
 267 particular, the present Hybrid GBL shows nearly nil skewness in the first 3 cell
 268 rows around the cylinder corners (corresponding to a width of $1/8D$), because
 269 of the constraints imposed by the adjacent edges of the structured parts on the
 270 adopted mesh generation algorithm. It follows that the Hybrid GBL is locally
 271 quite similar to the Full ortho GBL in such a region.

272 A quantitative statistical description of the patterns above is given in Table
 273 1. Overall, two main comments follow. First, the maximum values of both
 274 metrics are comparable among GBLs, and moderate ($\max(\phi) \lesssim 0.5$). In other
 275 terms, cells in GBLs are moderately skewed (the intersection point is no longer
 276 distant than $|\mathbf{F}_{L-II}|/4$ from the face mid-point) and moderately non-orthogonal
 277 ($\theta \leq 45^\circ$). Second, s^* and θ^* values significantly greater than zero ($\phi \geq 0.1$)
 278 rarely occur, except for θ^* in O-type GBL. In other terms, GBL are very locally
 279 skewed only. It follows that average and median values are very low, in turn.
 280 Bearing in mind that non-orthogonality errors will be numerically corrected, it
 281 might be inferred that all GBLs have comparable overall good quality, while
 282 point-wise moderately-skewed cells are source of errors only.

Table 1: Statistics on dimensionless skewness and non-orthogonality for different GBL types

ϕ	GBL	$\max(\phi)$	$\%_f \phi > 0.1$	$\text{avg}(\phi)$	$\text{median}(\phi)$	discrete distributions
θ^*	Hybrid	0.2857	7.2%	0.0223	0	
	O-type	0.4955	83.9%	0.2778	0.2952	
s^*	Hybrid	0.5532	8.5%	0.0241	1e-6	
	O-type	0.4884	0.7%	0.0061	0.0047	

283 4. Results

284 The present results are compared and discussed among them, in order to
 285 evaluate the errors induced by skewed cells in different regions of the GBL
 286 (Hybrid, O-type and O-type dnw grids) with respect to the full orthogonal
 287 grid (Full ortho). Furthermore, the present results are compared with a huge,
 288 even if not necessarily exhaustive, number of published studies on the same
 289 flow around square cylinder at $\text{Re}=22,000$, in the wake of the Ercoftac QNET-
 290 CFD Knowledge Base Wiki benchmark (Rodi, 2004, 2012). The goal of such a
 291 comparison is twofold: first, assessing the overall modelling error of the GBL-
 292 error-free simulation (Full ortho) with respect to the most accurate results in
 293 literature, and to compare it with the skewness-induced errors; second, placing
 294 the present results in the perspective of the whole scientific production on the
 295 topic, with special interest to the variability of the results obtained with different
 296 approaches. Both experimental tests spanning about 50 years (Pocha, 1971;
 297 Wilkinson, 1974; Lee, 1975; Otsuki et al., 1978; Bearman and Obasaju, 1982;
 298 Durao et al., 1988; Sakamoto et al., 1989; Lyn and Rodi, 1994; Lyn et al., 1995;
 299 Nishimura, 2001; Noda and Nakayama, 2003; Liu et al., 2015; Moore et al., 2019,
 300 in chronological order), and computational studies are considered. Among the
 301 latter, different approaches to turbulence modelling are considered: from the

302 pioneering so-called No-Model Simulations (NMS, [Tamura and Ito, 1996](#)) and
303 LES ([Murakami and Mochida, 1995](#); [Yu and Kareem, 1996](#)), to URANS ([Franke](#)
304 [and Rodi, 1991](#); [Kato and Launder, 1993](#); [Deng et al., 1994](#); [Wang and Vanka,](#)
305 [1997](#); [Bosch and Rodi, 1998](#); [Shimada and Ishihara, 2002](#); [Younis and Przulj,](#)
306 [2005](#)) and hybrid models ([Ke, 2019](#)), up to LES ([Sohankar et al., 2000](#); [Oka and](#)
307 [Ishihara, 2009](#); [Cao and Tamura, 2016](#); [Chen et al., 2020](#)), recent NMS ([Cao](#)
308 [et al., 2020](#)), and DNS ([Trias et al., 2015](#)).

309 For the sake of clarity, in the following the obtained results are schematically
310 arranged in three subsections. The simulated flow field around the cylinder is
311 discussed first, the distributions of the stresses at wall are commented in the
312 second section, while the bulk force coefficients are compared in the last one.

313 *4.1. Flow field*

314 In order to provide an overall sound phenomenological picture of the sim-
315 ulated flow topology, the instantaneous flow patterns around the cylinder and
316 in its wake are plotted in [Figure 6](#). All the flow patterns refer to the same
317 vortex-shedding phase, corresponding to the local maxima of the lift force. For
318 the sake of brevity, the isocontours of the vorticity magnitude $|\omega_z|$ from two
319 grids are included only, corresponding to the highest (Full ortho, [Fig. 6-d](#)) and
320 lowest (O-type, [Fig. 6-e](#)) grid quality. The present results are compared be-
321 tween them and with analogous flow visualizations from past studies: DNS by
322 [Trias et al. \(2015\)](#) (background color according to the magnitude of the pres-
323 sure gradient); NMS by [Cao et al. \(2020\)](#) (densest z -wise grid, background color
324 according to the streamwise velocity component); 2D URANS simulation by
325 [Shimada and Ishihara \(2002\)](#) (isocontours of the vorticity magnitude). Visual-
326 izations are sorted by expected decreasing overall accuracy of the computational
327 model. In spite of their different state variables and contour values, qualitative
328 and quantitative remarks can be drawn. The von Kármán-like vortex street
329 clearly emerges in the wake in every simulation. However, the simulated vortex
330 pattern exhibits significant differences. A common trend can be recognized in
331 visualizations from a) to d): the lower the expected overall model accuracy, the

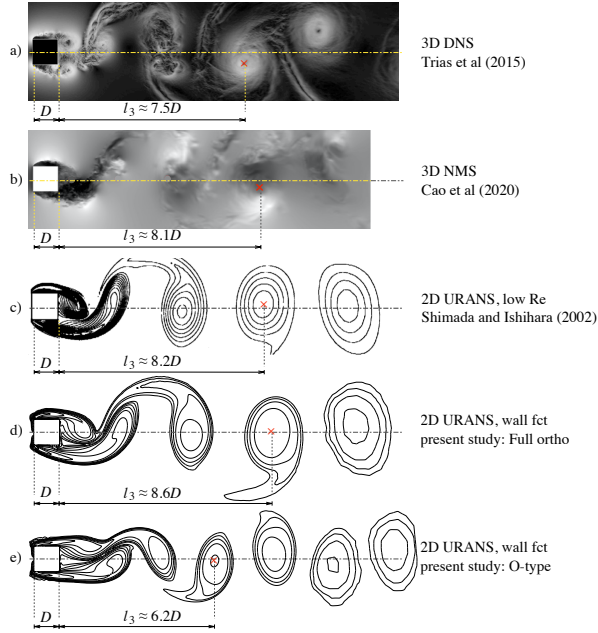


Figure 6: Instantaneous flow fields along the wake at the same vortex-shedding phase ($t = \text{argmax}(C_L)$) simulated by different computational models

332 slightly longer the longitudinal spacing between successive vortices. Conversely,
 333 this trend is no longer monotonic moving from the Full ortho (d) to the skewed
 334 O-type GBL (e): the vortex spacing drastically shortens. The distance l_3 be-
 335 tween the trailing edge and the centre of the 3rd vortex in the wake is estimated
 336 by hand-picking to quantitatively support such a qualitative trend. Correspond-
 337 ingly, the wake width simulated by skewed O-type GBL (e) is clearly narrower
 338 than in Full ortho GBL (d). The wake features above appear correlated to the
 339 curvature of the shear layer along the side surface just downwind the separation
 340 point at the leading edge, and to the vortex shedding close to the trailing edge,
 341 in turn. Simulations (a)-(d) are qualitatively consistent in predicting a highly
 342 curved shear layer, and shedding point close to the rear face. Conversely, skewed
 343 O-type GBL returns a lower curvature of the shear layer, and a shedding point
 344 further away in the wake.

345 Figure 7 shows the time averaged vorticity $t - avg(\omega_z)$ field around the
 cylinder base surface. The experimental PIV-based visualization by Moore et al.

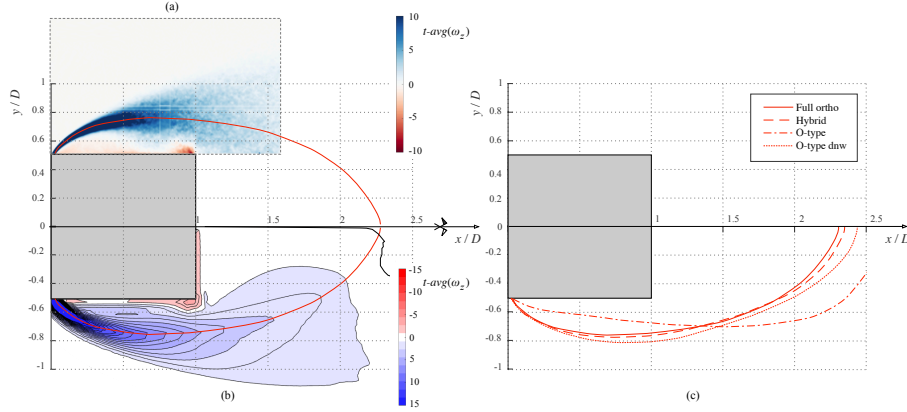


Figure 7: Mean flow field around the cylinder: time averaged vorticity $t - avg(\omega_z)$ measured by Moore et al. (2019) (a), and simulated by the full orthogonal GBL (b); shape of the shear layer by loci of $max(t - avg(\omega_z))$ simulated by different GBLs (c)

346
 347 (2019) is given in Figure 7(a), while the corresponding field simulated by the Full
 348 ortho GBL is plotted in Figure 7(b). Both fields refer to the lower side surface,
 349 the experimental field being mirrored along the x axis to facilitate comparison.
 350 The red curve in both subfigures is the locus of maxima $max(t - avg(\omega_z))$ sim-
 351 ulated by Full ortho GBL, i.e. it visualizes the shape of the simulated shear
 352 layer. The Full ortho simulation is in excellent qualitative agreement with
 353 the experiment, in spite of the relatively simple overall computational model.
 354 The red curve closely follows the axis of the experimental blue plume, i.e. the
 355 separated shear layer. Finally, the Full ortho GBL simulates the negative time-
 356 averaged vorticity induced by the secondary clockwise vortex at the trailing
 357 edge, in qualitative agreement with PIV measurements. Figure 7(c) collects
 358 the loci of $max(t - avg(\omega_z))$ for the four different GBLs. It can be observed
 359 that: i. the shapes of the shear layer obtained by the Full ortho and the Hybrid
 360 GBLs are very close in spite of the scattered skewness shown by the latter; ii.
 361 the O-type dnw curve slightly differs from the previous two, especially in the

362 wake; iii. the O-type curve completely departs from the others. On the one
 363 hand, such findings quantitatively confirm in time average what discussed in
 364 qualitative terms about the instantaneous patterns (Fig. 6-d, -e). On the other
 365 hand, they suggest that very local skewness-induced errors along the diagonals
 366 of the square annular partition globally affect the whole separated flow. The
 367 closer the error source to the separation point at the cylinder edge, the more
 368 significant its global effects (see e.g. the O-type GBL in Fig. 5-d). Conversely,
 369 a local high quality of the cell closely around the separation point preserves the
 370 accuracy of the results in spite of high skewness elsewhere (see e.g. the Hybrid
 371 GBL in Fig. 5-d). Such errors are introduced along the diagonals of the square
 372 annular partition, and in particular close to the separation point at the cylinder
 373 edge in the O-type GBL.

374 In order to check such a conjecture, Figure 8 relates the skewness viscos-
 ity to other simulated flow variables in the lower near wall region. The time

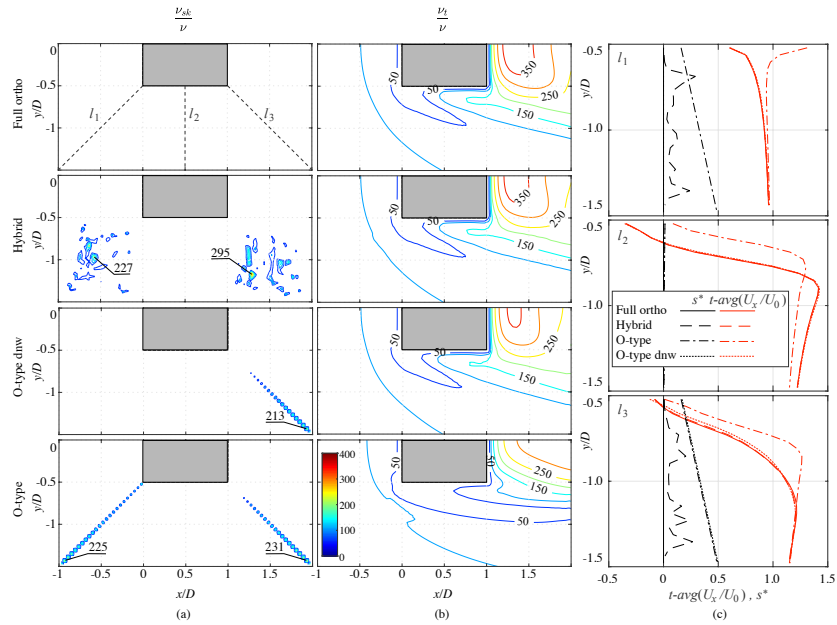


Figure 8: Time-averaged skewness viscosity ratio (a), turbulent viscosity ratio (b), x -velocity (red lines) and skewness (black lines) profiles (c)

376 averaged skewness viscosity (ν_{sk} , Eq. 6, Fig. 8-a) are compared to the time av-
 377 eraged turbulence eddy viscosity of the URANS model (ν_t , Eq.s 3-4, Fig. 8-b).
 378 Both quantities are normalized with respect to the kinematic viscosity ν . Two
 379 main remarks follow. First, ν_{sk} is not distributed all over the domain but only
 380 where skewed faces are located (Fig. 5-d). Second, ν_{sk} and ν_t take on values
 381 within the same order of magnitude, hundreds of time larger than the kinematic
 382 viscosity: in other term, false skewness viscosity potentially affects the solution
 383 to the same extent of the turbulence model. In particular, ν_{sk} has a dramatic
 384 relative importance upwind the leading edge, where transition to turbulence not
 385 yet occurs, and ν_t is very low. Three different 1D domains l_1, l_2, l_3 are defined,
 386 as sketched in Figure 8(a), first row. Figure 8(c) shows the profiles of the di-
 387 mensionless skewness and time-averaged horizontal velocity U_x along such lines.
 388 Three main remarks follow. First, skewed faces lie along l_1 in both Hybrid and
 389 O-type GBL. However, only the O-type U_x profile departs from the others and
 390 shows a non-physical flow acceleration very close to the wall. We can infer that
 391 skewness viscosity plays a dramatic detrimental role mainly close to the wall,
 392 approximately in the first 2 / 4 cell rows (see Fig. 5-d). Second, an analogous
 393 scatter between O-type GBL and the other simulations is observed along l_2 ,
 394 even if the cells are locally unskewed and orthogonal in all GBLs. It means that
 395 the *local skewness error* introduced by O-type GBL along l_1 is *transported by*
 396 *convection* along the side surface, analogously to the modeled state variables.
 397 From a physical point of view, this implies *global effects*: the reversed flow pre-
 398 dicted by Full ortho, Hybrid and O-type dnw GBLs is not adequately grasped
 399 by the O-type GBL. Finally, the very small scatter between the U_x profiles by
 400 Full ortho and O-type dnw GBLs along l_3 testifies that the skewness viscosity,
 401 its error and its effects not depend only on s^* , but on the velocity field, too (Eq.
 402 6).

403 In order to quantitatively compare the present results with the ones in
 404 literature, the distributions of the dimensionless streamwise velocity U_x/U_0 av-
 405 eraged in time ($t-avg$ in the following) and in the spanwise direction ($z-avg$ in
 406 the following, relevant to 3D models) are plotted in Figure 9(a) along the wake

407 centerline $y = 0$. In particular, available experimental measurements (Duroao
 408 et al., 1988; Lyn and Rodi, 1994), high-fidelity DNS (Trias et al., 2015) and
 409 LES (Cao and Tamura, 2016), and other LES and URANS results are consid-
 ered. In general, the considered high-fidelity simulations provide consistent

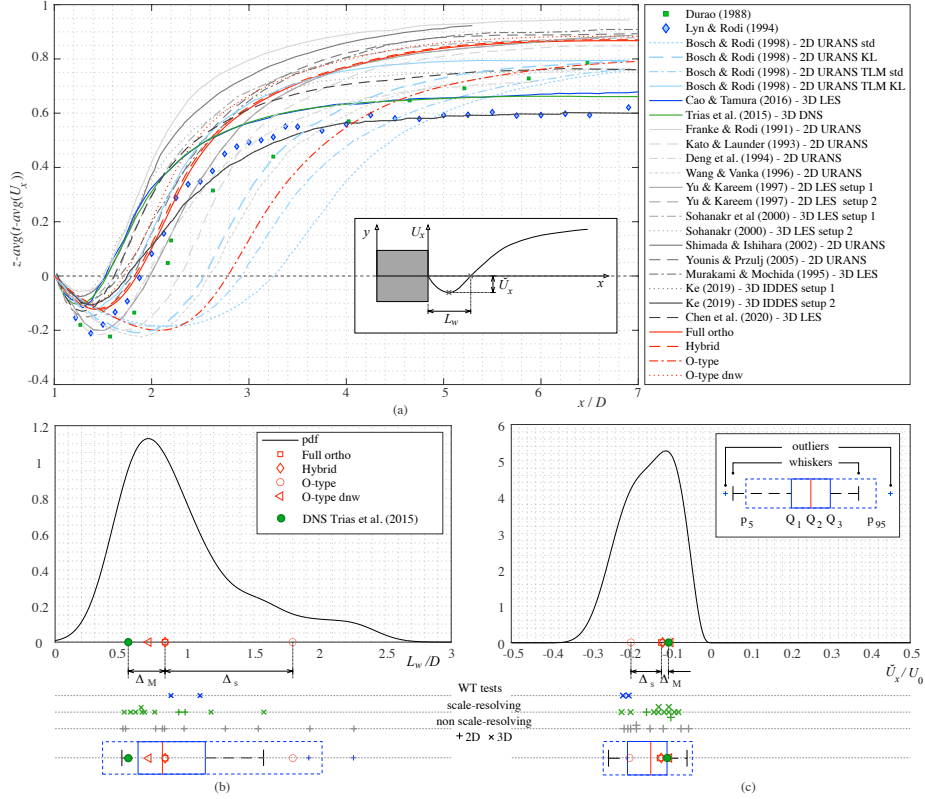


Figure 9: Distribution of time-averaged x -velocity along $y = 0$ (a), statistics of the wake recirculation length L_w (b) and velocity defect \tilde{U}_x (c)

410

411 results. However, as pointed out by the same Trias et al. (2015), their computa-
 412 tional results slightly but systematically differ from the reference experimental
 413 data of Lyn and Rodi (1994), with remarkable differences in the near wake re-
 414 circulation region, where the simulated reversed velocity is nearly half of the
 415 measured one. The Lyn and Rodi (1994) measurements significantly depart in
 416 turn from the previous measures by Duroao et al. (1988), and even larger variabil-

417 ity takes place among other computational results, even if the same approach to
 418 turbulence modelling is adopted. Such overall variability of the velocity profiles
 419 results in scattered values of the recirculation length, of the maximum reversed
 420 speed, and of the velocity plateau along the fare wake, if any. In particular,
 421 the streamwise velocity profiles predicted by low-skewness grids (Full ortho and
 422 Hybrid) are in satisfactory agreement with experimental data in the near wake,
 423 while the velocity recovery is overestimated in the far wake $x/D > 2.5$. The
 424 dense and orthogonal grid in the wake (Fig. 5), and the 3rd order scheme used
 425 for the convective terms suggest that most of the underlying error can be tenta-
 426 tively ascribed to the 2D domain, and/or to adopted turbulence model, namely
 427 to the empirical modelling of $C_{\varepsilon_2}^*$. The velocity distribution issued from O-type
 428 grid dramatically suffers the effects of the skewness viscosity (eq. 6), resulting
 429 in the striking overestimation of the recirculation length. Conversely, skewness
 430 around the trailing edge only (O-type dnw) has the opposite effects of the skew-
 431 ness around both leading and trailing edges (O-type), i.e. it shortens the recircu-
 432 lation length and reduces the velocity defect with respect to the Full ortho grid.
 433 Two local metrics are retained in order to synthetically describe the near wake
 434 recirculation region: the wake recirculation length L_w , and the velocity defect
 435 \check{U}_x , graphically defined in Figure 9(a) for the sake of clarity and conciseness. In
 436 the light of the high variability highlighted above, statistics are obtained on each
 437 of them, by referring to the ensemble of realizations collected in literature and
 438 plotted in Figure 9(a). Statistics are visualized in Figures 9(b) and (c) for L_w/D
 439 and \check{U}_x/U_0 , respectively. All realizations are reported and categorised in Wind
 440 Tunnel (WT) measurements, non scale-resolving (RANS/URANS), and scale-
 441 resolving (DNS, NMS, LES, DES, IDDES) simulations. The *a-priori* expected
 442 highest-fidelity model (DNS, Trias et al., 2015) is pointed out. Non paramet-
 443 rical Probability Density Functions (PDF) with positive and negative support
 444 are fitted on the whole ensemble for L_w and \check{U}_x , respectively, and accompanied
 445 by the corresponding box plots. Such a postprocessing allows a twofold reading
 446 of the obtained results. In a deterministic reading, the present overall mod-
 447 elling error Δ_M is evaluated by the scatter between the highest-fidelity model

448 (DNS, [Trias et al., 2015](#)) and the present highest-quality GBL (Full ortho),
 449 while the highest skewness-induced error Δ_s is given by the scatter between the
 450 highest (Full ortho) and lowest (O-type) GBL quality. In a purely statistical
 451 reading, each realization can be associated with the corresponding percentile,
 452 i.e. the number where a certain percentage of scores fall below it. Here, we are
 453 aware that the ensemble cardinality ($\#=22$) is significant but not necessarily
 454 high enough to ensure full statistical convergence. Hence, we prefer a coarse
 455 statistical mapping of the present results by referring to outliers, and to the
 456 intervals between percentiles $p_5, p_{25} = Q_1, p_{50} = Q_2, p_{75} = Q_3, p_{95}$. Selected
 457 remarks follow from both perspectives: i. for both metrics the skewness-induced
 458 error is by far larger than the overall modelling error, despite the latter results
 459 form top- (3D DNS) and low- (2D URANS) ranked models; ii. L_w/D shows a
 460 variability (coefficient of variation $c.o.v.(L_w) = \sigma(L_w)/|\mu(L_w)| = 0.236$) lower
 461 than \check{U}_x/U_0 ($c.o.v.(\check{U}_x) = 0.364$); iii. the values provided by the Full ortho and
 462 Hybrid grids are within the $[Q_1Q_3]$ range for both metrics, while the L_w/D
 463 value from O-type grid is clearly an outlier, and the corresponding \check{U}_x/U_0 is
 464 close to Q_1 .

465 4.2. Stress distributions at wall

466 The distributions of the dimensionless wall shear stress τ_w averaged in time
 467 and in the spanwise direction are plotted in [Figure 10](#) versus the curvilinear
 468 axis s along the upper half perimeter. Positive values correspond to τ_w with the
 469 same direction of the curvilinear axis. The distributions obtained by different
 470 GBLs are compared with the ones obtained by [Cao and Tamura \(2016\)](#) with
 471 different grid densities. A closed view along the side surface is given, in anal-
 472 ogy to what done by [Cao and Tamura \(2016\)](#) as well. The distributions by
 473 this study follows from a wall-function modelling approach and, consequently,
 474 the size of the cells along the wall is much larger than the ones in [Cao and](#)
 475 [Tamura \(2016\)](#) in both s and n directions. The s -wise low grid density implies
 476 a scarce resolution of the wall shear stress profile, testified by the coarse place-
 477 ment of the peaks immediately upstream and downstream the leading corner B.

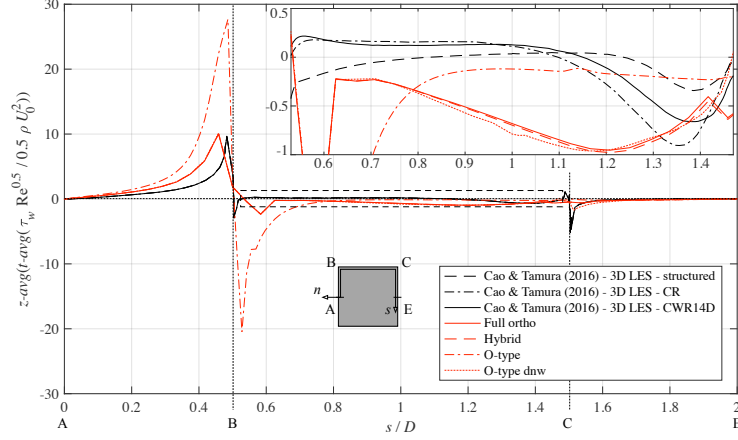


Figure 10: Distribution of time-averaged dimensionless wall shear stress along the upper half perimeter

478 However, the maxima of $|\tau_w \text{Re}^{0.5} / 0.5 \rho U_0^2| \approx 10$ and 2.5 predicted by Full ortho,
 479 Hybrid and O-type dnw GBLs well matches the LES results in [Cao and Tamura](#)
 480 [\(2016\)](#). Both s and n -wise low grid density does not allows the simulation of the
 481 small-size counterclockwise secondary vortices downstream B ($0.5 < s/D \lesssim 0.7$)
 482 and upstream the trailing corner C ($1.47 \lesssim s/D < 1.5$) visualized in [Cao and](#)
 483 [Tamura \(2016\)](#)-Fig. 10, and [Cao et al. \(2020\)](#)-Fig. 10. This yields to reversed
 484 flow all along the side surface, and corresponding negative τ_w . The "3D LES-
 485 structured" simulation by [Cao and Tamura \(2016\)](#) is affected by an analogous
 486 issue. The O-type distribution drastically differs from the others because of both
 487 local and global effects of the skewness viscosity. The local peaks around B are,
 488 in absolute value, from 3 to 8 times higher than the ones issued by the other
 489 GBLs. After a shear recovery, the distribution attains a plateau value close to
 490 zero along the downwind half of the side surface, i.e. the flow is unrealistically
 491 quasi-stagnant. Conversely, the effects of the skewness viscosity on τ_w induced
 492 by O-type dnw GBL mainly take place around the trailing corner only, because
 493 of the convection-dominated flow.

494 The distributions of the time-averaged and time-standard deviation ($t - std$

495 in the following) of the pressure coefficient C_p , also averaged spanwise and be-
 496 tween the upper and lower half perimeters (*side-avg* in the following) are
 497 plotted in Figure 11 (a) and (b), respectively. They are compared to available
 498 experimental measurements, high-fidelity DNS (Trias et al., 2015) and LES
 499 (Cao and Tamura, 2016), and several other LES and URANS simulations. In
 500 spite of the same nominal setup, both experimental and computational results
 501 are significantly scattered along the whole base surface, especially in regard to
 502 $t-std(C_p)$. Interestingly, distribution from *a-priori* high-fidelity simulations
 503 are extremal occurrences at the lower and upper bounds of the ensemble for
 504 $t-avg(C_p)$ and $t-std(C_p)$, respectively. Conversely, Full ortho GBL distribu-
 505 tions lie around the upper and lower bounds of the ensemble for $t-avg(C_p)$ and
 506 $t-std(C_p)$, respectively. In short, the adopted computational model slightly
 507 underestimates the mean suction and the pressure fluctuations, and notably
 508 fails in predicting the local maximum of $t-std(C_p)$ in the downwind half of the
 509 side surface. Hybrid GBL provides the same mean distribution, and negligible
 510 differences take place for the pressure standard deviation. O-type dnw GBL
 511 does not significantly affect the mean distribution, while the local error made at
 512 the trailing edge C makes the $t-std(C_p)$ distribution closer to the average of
 513 the experimental measurements. We conjecture that such a misleading agree-
 514 ment is induced by a deeper footprint of the vortex shedding on the side surface,
 515 induced by small changes in the flow structure in the near wake. Highly skewed
 516 O-type GBL dramatically and qualitatively affects both the distributions of the
 517 C_p mean and standard deviation. Their trend is typical of a streamlined body
 518 rather than of a bluff body: deep mean suction occurs at the leading edge B;
 519 mean pressure plateau is missing downstream it, and mean pressure recovery
 520 takes place along the side surface; pressure standard deviation is one order of
 521 magnitude lower than the average of the results in literature.

522 Analogously to what done for the streamwise velocity in the wake, four *s-*
 523 wise averaged metrics are defined to synthetically describe the time-averaged
 524 and time-fluctuating pressure field along the side (*l-avg*) and rear (*b-avg*)
 525 surfaces. The bounds of the averaging lengths l_1 , l_2 , b are intended to discard

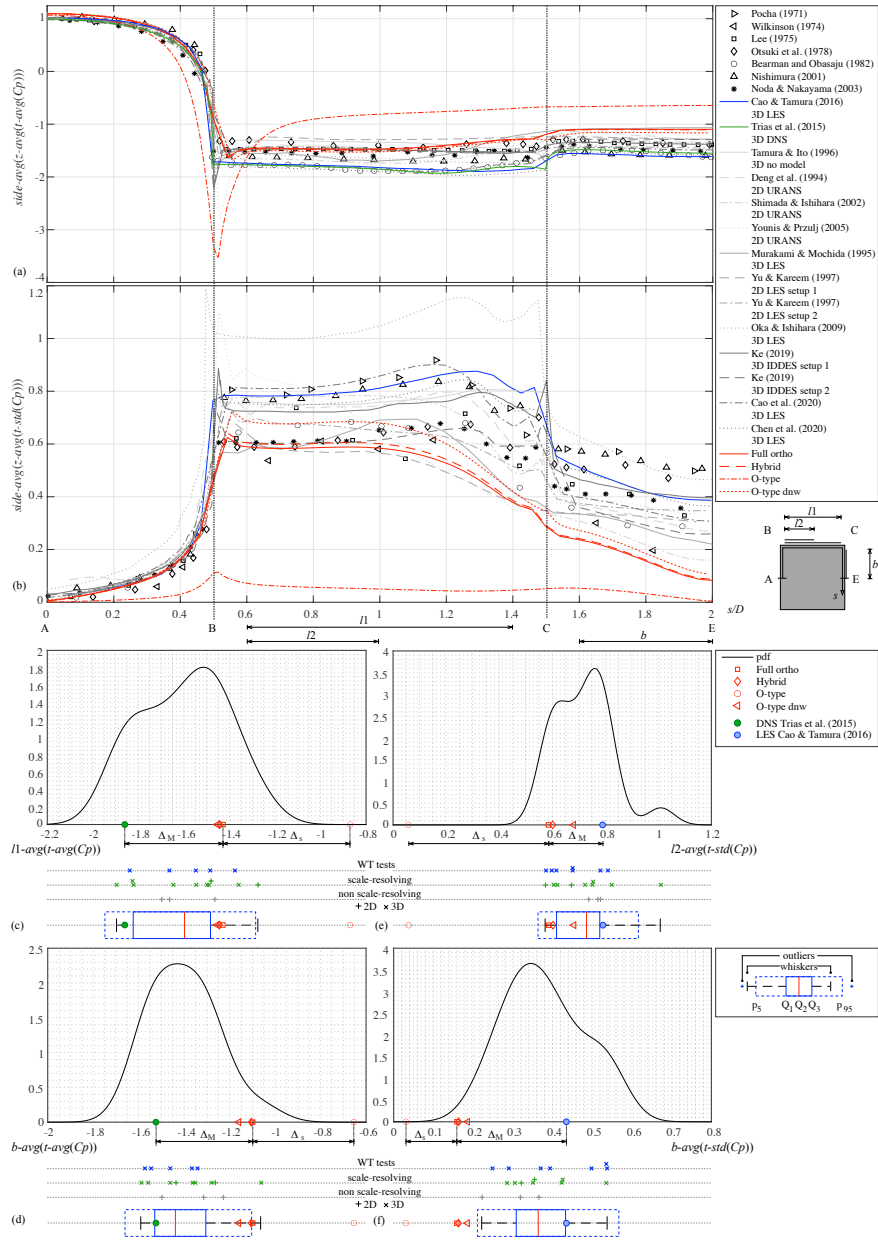


Figure 11: C_p distribution: time-averaged value (a), time-standard deviation (b), statistics of the side (c,e) and base (d,f) pressure

526 the neighbourhood of the sharp edges, and related local high pressure gradients.
 527 The cardinality of the ensemble of the realizations ($\#=21$) is close to the one
 528 used for the streamwise velocity in the wake. The 3D LES by [Cao and Tamura](#)
 529 [\(2016\)](#) is chosen as *a-priori* highest-fidelity model when DNS results from [Trias](#)
 530 [et al. \(2015\)](#) are not publicly available. Figures 11(c) and (d) show the distribu-
 531 tions of the side and rear surface $t - avg(Cp)$ plateaux. The following remarks
 532 can be drawn: i. the coefficients of variation of the literature results are similar
 533 ($c.o.v.(l1 - avg(t - avg(Cp))) = 0.12$, $c.o.v.(b - avg(t - avg(Cp))) = 0.1$) and
 534 not negligible; ii. in a statistical setting, the Full ortho, Hybrid and O-type
 535 dnw realizations are close to Q_3 of $l1 - avg(t - avg(Cp))$ and to and p_{95} of
 536 $b - avg(t - avg(Cp))$, while the O-type ones are outliers in both cases; iii. in a
 537 deterministic setting, the skewness errors Δ_s are comparable to the model ones
 538 Δ_M . Figures 11(e) and (f) show the distributions of $t - std(Cp)$ over $l2$ and b ,
 539 respectively. The following remarks can be drawn: i. the c.o.v. of the distribu-
 540 tions of the Cp standard deviation in literature are overall higher than the ones
 541 of the time-averaged pressure. In particular, the c.o.v. of the pressure standard
 542 deviation along the rear face ($c.o.v.(b - avg(t - std(Cp))) = 0.25$, Fig. 11-f) is
 543 higher than the one on the side surface ($c.o.v.(l2 - avg(t - std(Cp))) = 0.15$, Fig.
 544 11-e), and impressive in absolute terms. The slight bi-modality of the PDF in
 545 Figure 11(e) does not seem to depend on the approach/model used; ii. over the
 546 $l2$ domain, while the Full ortho and Hybrid realizations are located near Q_1 and
 547 the O-type dnw one in between Q_1 and Q_2 , the O-type realization is an outlier;
 548 iii. over the domain b , all the realizations of the present work are outliers even if
 549 Full ortho, Hybrid and O-type dnw are close to p_5 ; iv. the deterministic model
 550 error Δ_M is prevailing over the skewness one Δ_s along b only, while the opposite
 551 is the case along $l2$.

552 4.3. Bulk forces

553 Bulk force coefficients are scarcely useful in shedding light on phenomenolog-
 554 ical local features of the simulated flows and skewness effects on them. Neverthe-
 555 less, they are of great importance for design purposes in engineering applications.

556 The main bulk force coefficients obtained in the present study are compared and
 557 discussed with reference to statistics on the same metrics obtained in different
 558 wind tunnel and computational studies in Figure 12: $t - avg(C_D)$ is the time-
 559 and spanwise-averaged drag coefficient per unit length; $t - std(C_L)$ is the stan-
 560 dard deviation of the time variation of the lift coefficient; $St = f_s D / U_0$ is the
 561 Strouhal number, where the shedding frequency f_s is evaluated from the time
 562 fluctuations of the lift coefficient. Statistics are obtained from ensembles having
 563 a cardinality equal to $\# = 39$ for $t - avg(C_D)$ and St , and $\# = 29$ for $t - std(C_L)$.
 564 Realizations are collected from Liu et al. (2015), Trias et al. (2015), Cao and
 Tamura (2016), and references therein. PDFs of $t - avg(C_D)$ and $t - std(C_L)$

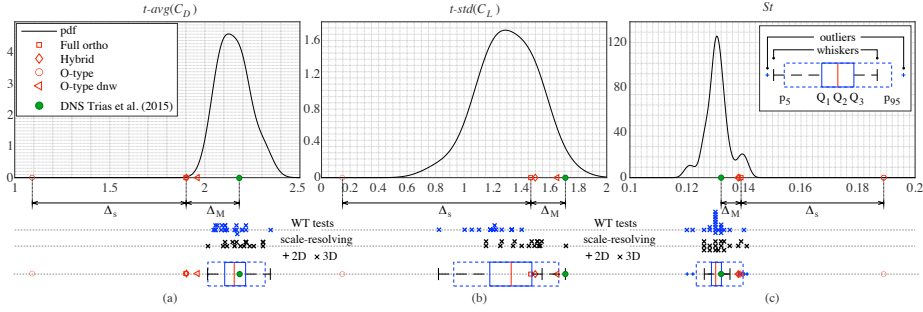


Figure 12: Bulk parameters: statistics of time-averaged drag coefficient (a), standard deviation of the lift coefficient (b), Strouhal number (c)

565
 566 are moderately skewed and unimodal. The three-mode PDF of the St number
 567 seems not due to the selected approach: for instance, WT realizations contribute
 568 to all modes. The coefficient of variation is small for the time-averaged drag
 569 ($c.o.v.(t - avg(C_D)) = 0.04$) and Strouhal number ($c.o.v.(St) = 0.03$), while
 570 it is significant for lift fluctuations ($c.o.v.(t - std(C_L)) = 0.15$). The *a priori*
 571 high fidelity model closely fits the ensemble average value of $t - avg(C_D)$ and
 572 St . Conversely, DNS significantly overestimates the $t - std(C_L)$ in respect to
 573 the whole ensemble, and to the experimental measurements in particular. We
 574 conjecture this can be ascribed to the unsatisfactory simulation of the spanwise
 575 pressure correlation, probably due to too short spanwise domain size, and/or to

576 poor grid density in the same direction (Bruno et al., 2012). Both modelling and
 577 skewness errors imply the reduction of $t - avg(C_D)$ and $t - std(C_L)$, while the
 578 Strouhal number increases under the effects of both Δ_M and Δ_S . The skewness
 579 error is much higher than the modelling one for all bulk metrics: 2.9 times for
 580 $t - avg(C_D)$, 5.4 times for $t - std(C_L)$, 7.2 times for St . Let us consider the
 581 drag coefficient as an example (Fig. 12-a). The ensemble median is $Q_2 \approx 2.15$,
 582 and DNS returns a reference value $t - avg(C_D)_{rep} \approx 2.18$ (71-th percentile in in
 583 probabilistic terms), while Full ortho and Hybrid GBL predict $t - avg(C_D) \approx 1.9$
 584 (corresponding to the 0.01-th percentile, and resulting in $\Delta_M \approx 0.28$), and O-
 585 type GBL $t - avg(C_D) \approx 1.1$ (corresponding to a full outlier, and resulting in
 586 $\Delta_s \approx 1.08$). It follows that both modelling and skewness errors lead to the
 587 unsafe estimation of the drag force. However, the design could be effectively
 588 secured versus Δ_M by adopting a partial safety factor γ_Q for variable actions,
 589 also accounting for model errors/uncertainties (e.g. $\gamma_Q = 1.5$ in EN 1990, 2002;
 590 EN 1991-1-4, 2005, for wind action in structural engineering). The same usual
 591 value of the partial safety factor is not effective face to the large skewness error.
 592 Analogous comments are relevant to $t - std(C_L)$ and St , if the deviation of the
 593 Strouhal number has unfavourable effects on the structural response.

594 5. Conclusions

595 The flow around a square cylinder at $Re=22,000$ is simulated under four
 596 types of Grid Boundary Layers, the other components of the computational
 597 model being unchanged. GBLs differ in cell quality, while they share common
 598 other features, such as the geometry of their boundaries, their size, the grid den-
 599 sity at wall, the grid in the outer part of the computational domain. Thanks to
 600 this, the effects of the cell skewness are separately discussed with respect to the
 601 ideal full orthogonal GBL. The attention paid in the recent past to the adopted
 602 benchmark by the scientific community allowed a huge number of previous re-
 603 sults to be collected and statistically analyzed. Hence, the obtained results are
 604 compared not only among them, but also with such a huge database.

605 Some conclusions can be summarized:

- 606 • The local error induced by few, moderately skewed, near-wall cells upwind
607 the cylinder propagates windward because of the convection-dominated
608 problem. The error globally and significantly affects the boundary layer
609 separation and the vortex shedding in the wake. The same skewed cells
610 around the trailing edge only affect the flow to a lower extent;
- 611 • The skewness error on bulk aerodynamic coefficients may largely prevails
612 on the overall modelling error, despite a very simple turbulence model and
613 a second-order accurate numerical approach are deliberately adopted in
614 the study;
- 615 • O-type, block structured, skewed GBL results in the qualitative simulation
616 of the instantaneous vortex street in the wake, and in the concurrent
617 dramatic underestimation of the lift fluctuating component by a factor
618 5. Such results call to mind the well renowned quote by [Ferziger \(1993\)](#):
619 *"The greatest disaster one can encounter in computation is not instability*
620 *or lack of convergence, but results that are simultaneously good enough to*
621 *be believable, but bad enough to cause trouble"*;
- 622 • Hybrid GBL with unstructured patterns around the cylinder edges can
623 provide results analogous to the ones obtained with a full orthogonal grid,
624 provided that from 2 to 4 cell rows nearly orthogonal and skewness-free are
625 generated near the wall (corresponding to a width from $1/12D$ to $1/6D$),
626 while clusters of skewed cells are kept further from the wall;
- 627 • Hybrid GBL is judged to be a fine balance between accuracy and flexibility
628 in grid generation, with the above due care and attention to detail. In
629 particular, hybrid GBLs are nearly unescapable for real-world engineering
630 applications with intricate geometries, e.g. with multiple obtuse or acute
631 edges.

632 Analogous studies on grid quality effects are recommended for other paradig-
633 matic high-Re flows in bluff body aerodynamics, such as the one around elon-

634 gated rectangular cylinders with separated and reattached boundary layer, and
635 around real world structures, such as bridge decks and tall buildings. The rela-
636 tive significance of skewness viscosity with respect to other modelling approaches
637 to turbulence such as wall resolved LES would be worth to be investigated as
638 well.

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