Aleatory uncertainties with global resistance safety factors for non-linear analyses of slender reinforced concrete columns

ALEATORY UNCERTAINTIES WITH GLOBAL RESISTANCE SAFETY FACTORS FOR NON-LINEAR ANALYSES OF SLENDER REINFORCED CONCRETE COLUMNS

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ABSTRACT

The present study aims to investigate and evaluate the influence of aleatory uncertainties (i.e., material and geometric uncertainties) when non-linear numerical analyses are developed to assess the global resistance of slender reinforced concrete columns. For instance, a set of 40 experimental tests on slender reinforced concrete columns with different slenderness values known from the literature is considered. The experimental tests have been reproduced by means of non-linear numerical simulations adopting specific modelling hypotheses able to minimize the model uncertainty. Successively, the calibrated non-linear numerical models of the 40 columns are used to perform a probabilistic analysis of the global structural resistance (i.e., ultimate axial load) including material uncertainties only and, then, both geometric and material uncertainties. The two groups of the probabilistic investigation are useful to characterize the influence of both material and geometric uncertainties on the global structural response of slender RC columns. Finally, global resistance safety factors are proposed as a function of the slenderness discussing also the procedure to incorporate the proposed results within the use of the global resistance safety formats.

KEYWORDS: non-linear numerical analyses; global resistance safety format; slender columns; reinforced concrete; safety factor; slenderness.

1. INTRODUCTION

Non-linear numerical analyses (NLNAs) are one of the most important developments of the last decades in the field of structural design. In particular, the use of NLNAs allows to practitioner and researchers to investigate and reproduce the actual response of reinforced concrete (RC) members and systems accounting for both mechanical and geometric non-linearities. Over the years, many investigations and guidelines have been focused on the application of NLNAs in structural engineering field, with particular reference to RC structures [1]-[7]. Moreover, NLNAs can be an efficient instrument in order to assess RC existing structures and infrastructures [8]-[9] also considering the influence of deterioration process [10]-[12]. With reference to the reliability analysis of RC structures, NLNAs can be used efficiently within the global resistance methods [13]. In line to the limit states approach [14] for design or assessment of RC structures, the safety verifications may be performed through two different philosophies which are able to include both aleatory (i.e., mechanical and geometric) and epistemic sources of uncertainties [15]: the local or cross-sectional approach [14] and the global one [1],[13].

The “local approach”, is widely adopted by the design codes of practice [16] and it is very efficient in most of the cases for practitioners and designers [14],[17]. With regard to the partial factor method [14], by means of structural linear analyses, cross-sectional verifications are performed comparing the design value of the internal actions E_d with the related design value of the local resistances R_d. However, the local approach may fail if the structural system response is extremely close to the limit state configuration, as it does not take into account the overall response of the structure, the possible internal forces redistribution and the actual deformation capability. This eventuality is very common concerning the assessment of existing RC structures [2].

The global resistance approach [13] permits to define proper safety formats [1]-[2],[13],[18] which allow to compare the design values of the external actions F_d with the related design global resistance R_d of the structural members or system. The global structural resistance R can be estimated using a

NLNA and its design value, R_d, can be computed accounting for the influence of aleatory (i.e., materials, geometric) and epistemic (i.e., numerical model) uncertainties [13] through the definition of appropriate partial safety factors. These partial safety factors can be evaluated in line with pre-determined target levels of reliability distinguishing between new and existing structures [2],[13],[17],[19]-[20]. In fact, NLNAs take into account the global response of a structural member or system accounting for evolution of damaging due to the specific loading combination, concrete/reinforcements non-linear behavior and even local or global instabilities. In the global approach, the design value of external actions F_d, evaluated according to [14], can be compared to the design value of the global structural resistance R_d. The latter one can be evaluated using NLNAs according to the global resistance methods [13] with the following expression:

$$R_d = \frac{R_{NLNA}\left(f_{rep};a_{rep}\right)}{\gamma_R \gamma_{Rd}} \tag{1}$$

In Eq.(1), $R_{NLNA}(x_{rep})$ represents the global structural resistance evaluated by means of NLNAs using pre-determined representative values for material properties f_{rep} and geometric a_{rep} (which, in general, are, respectively, the mean and the nominal values [1]: f_m and a_n). The appropriate level of structural reliability is accounted for by means of two different global safety factors: the *global resistance safety factor* γ_R which considers the aleatory uncertainties related to the material properties and geometry [1],[13]; the *resistance model uncertainty safety factor* γ_{Rd} which takes into account the epistemic uncertainty related to definition of the non-linear numerical model of the structural member or system [21]-[24]. For the mentioned above reasons, the global resistance approach is now becoming the method provided in the next generation of practice codes [13] in order to perform design and assessment of RC structures using NLNAs. The influence of the aleatory uncertainty combined with the sensitivity of the numerical model to properly predict the failure mode as well as the consequential effects on the NLN results in reliability terms have been investigated by [2]. The importance of the uncertainties related to definition of the non-linear numerical model with reference to RC structures (i.e., epistemic uncertainty) has been studied by [21]-[24]. Other investigations have been also devoted to the inclusion of the geometric non-linearities [25]-[26]. However, an in-depth evaluation of the influence of both materials and geometric uncertainties on slender RC columns subjected to materials/geometric non-linearities within the global resistance methods [13],[27] is missing.

For instance, the aim of this study is to evaluate the influence of aleatory uncertainties (i.e., material and geometric uncertainties) when NLNAs are developed to assess the global resistance of slender RC columns within the framework of the global resistance methods [1]-[2],[13]. First of all, a database composed of 40 experimental tests results on slender RC columns with different slenderness values, selected in [24] from the scientific literature [28]-[36] excluding tests realized with high strength concretes or affected by creep effects, is considered. The experimental database of [24] has been adopted as it respects the requirements of the design codes of practice [13]-[14],[16] which implement the global resistance approach [13] for safety verifications through NLNAs. Then, the experimental tests on the 40 RC columns have been reproduced by means of NLN models using *OpenSees* software platform [37] including both mechanical and geometric non-linearities adopting modelling hypotheses able to minimize the model uncertainty according to [24]. Next, the 40 validated NLN models are used to perform an extensive probabilistic analysis including both material and geometric uncertainties, statistically characterized according to [38]. The probabilistic investigation is performed through the Latin Hypercube Sampling (LHS) technique [39] defining 100 sampled NLN models for each one of the 40 RC columns [28]-[36]. The LHS is repeated twice: the first time, it has been performed including material uncertainties only (LHS - *group 1*) and, the second time, including both geometric and material uncertainties (LHS - *group 2*). The outcomes from both LHS - *group 1* and LHS - *group 2* are used to characterize the dependency of the global response of slender RC columns on materials and geometric uncertainties also as a function of the slenderness. Finally, the *global resistance safety factor* γ_R, according to Eq.(1), is evaluated as a function of the slenderness. The probabilistic results regarding the geometric aleatory influence as a function of the slenderness are also incorporated in the application of one of the most common and simple global resistance safety formats, that is the (Estimation of Coefficient of Variation) ECoV method [13].

2. EXPERIMENTAL TESTS OF SLENDER RC COLUMNS AND NLN MODELS

This section introduces briefly the main features of the experimental database related to tests on slender RC columns collected by [24] and of the related modelling hypotheses to perform NLNAs. The mentioned above experimental database includes results from several investigations [28]-[36] and has been used in [24] in order to compare numerical vs experimental results with the aim to evaluate the resistance model uncertainty safety factor (i.e., epistemic uncertainty) in NLNAs of slender RC columns [24]. Grounding on the results of [24], the modelling hypotheses that minimize the uncertainty related to the numerical model definition (i.e., epistemic uncertainty) have been adopted in this investigation in order to perform the probabilistic investigation [21]-[22].

In particular, the experimental results collected by [24] from [28]-[36] have been adopted as they respect the limitations and specifications of widely recognized design codes as EN1992 [16] and *fib* Model Code 2010 [13] excluding tests realized with high strength concretes or affected by creep effects (i.e., long-term tests).

Moreover, the assembled set of [24] also meets some specifications of other worldwide design codes, such as [40]. Specific details about the criteria adopted for selection of the 40 experimental tests from [28]-[36] can be acknowledged in [24]. With reference to [24], the static configurations of the different test sets can be classified in four different typologies denoted as *Type A*, *B*, *C* and *D* schemes. The representation of the different test configurations with the related numerical model idealization may be found in [24]. The modelling hypotheses to define the NLN models of the 40 RC columns are reported and summarized in Table 1 according to [24]. In compliance with [21],[22],[23], the modelling hypotheses concern assumptions for materials constitutive laws, kinematic compatibility and evaluation of equilibrium between internal and external forces. The platform *Opensees* [37] has been adopted using a fiber-modelling approach with distributed plasticity and accounting for both mechanical and geometric non-linearities [24]. Detailed information about numerical modelling of the 40 RC columns can be acknowledged in [24].

Table 1. Modelling hypotheses for the NLNAs of the 40 RC columns by [28]-[36] using Opensees [37].

Opensees platform [37]	
Equilibrium of forces	- Solution of non-linear system of equations: full Newton-Raphson iterative method [5]; - Equilibrium evaluated in each iteration with reference to the deformed configuration accounting for the second order effects (*P-delta geometric transformation* [37]); maximum number of iterations for each load step: 200; - Convergence criteria based on displacements (with tolerance set equal to 1%); - Sizes of the incremental load steps defined with reference to both experiments execution and numerical calibration in order to achieve numerical accuracy;
Kinematic compatibility	- *Force-based* approach for *fiber beam-column elements* [37] for cross sections with non-linear response; 40x40 fiber-grid subdivision of the cross section has been adopted; - *Elastic beam-column* [37] elements for cross sections with elastic response;
Constitutive relationships	*Concrete (non-linear cross sections): Concrete02 model* [37] - Mono-axial non-linear model for un-confined and confined concrete in compression defined according to [41]; - Tensile response of concrete is elastic with a post peak linear tension softening (i.e., LTS) law; the extension of the softening branch has been calibrated in order meet experimental results; *Reinforcements (non-linear cross sections): ReinforcingSteel model* [37] - elastic with curvilinear hardening according to *Chang and Mander, 1994* [42]; - Buckling model based on *Dhakal and Maekawa*, 2002 [43] The values of the material properties (i.e., strengths, Young's modulus, ultimate strains, etc.) have been defined in line with the data provided by the original research papers [28]-[36] and in compliance to [16] if missing.

According to [24], Figure 1 reports the comparison between the experimental results R_{exp} of [28]-[36] and the NLNAs outcomes $R_{NLNA}(f_{exp};a_{exp})$ determined using the experimental values of both materials (f_{exp}) and geometric (a_{exp}) properties. Figure 1 illustrates, respectively, the variation of the ratios ϑ between the experimental R_{exp} and numerical $R_{NLNA}(f_{exp};a_{exp})$ ultimate axial loads in relation to the slenderness and to the experimental concrete resistance. The resistance model uncertainty random variable ϑ is a measure of the epistemic uncertainty related to the definition of the non-linear numerical models [21]-[22],[24].

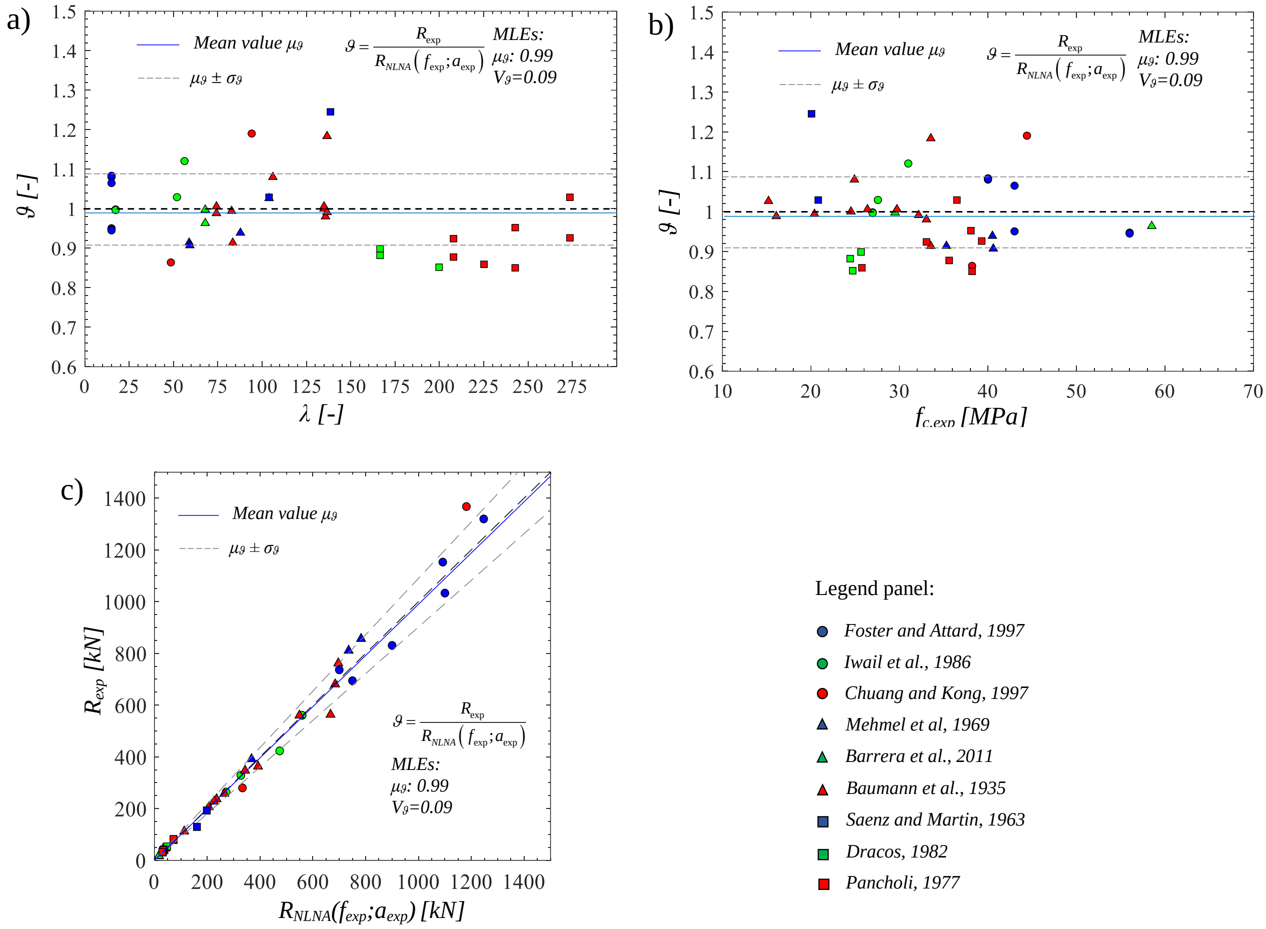

Fig. 1. Validation of the NLNAs with respect to experimental results of [28]-[36] according to [24] .

With the aim to validate the goodness of the adopted modelling hypotheses, the mean value and coefficient of variation of ϑ, computed for all the slenderness values, are also reported in Figure 1 and descend from the Maximum Likelihood Estimators (i.e., MLEs) of statistical parameters with assumption of lognormally distributed variable [22]. In particular, the coefficient of variation (V_ϑ) is equal to 9% and the mean value μ_ϑ turns out to be 0.99 (Figure 1(a)). Figure 1(c) shows that the results are mostly enclosed within the lines representing the mean value μ_ϑ plus/minus one standard deviation σ_ϑ demonstrating the goodness of the adopted modelling hypotheses [21]-[24]. Furthermore, it can also be noted that, for slenderness values lower than 150, the un-safe biased cases are low but increase for slenderness values higher than 150. As discussed in [24], the statistical characterization of the random variable ϑ associated to specific modelling hypotheses is useful to take into account the epistemic uncertainty and determine the suitable resistance model uncertainty safety factor γ_{Rd} for safety verifications through global resistance methods [13] (Eq.(1)). Within the herein adopted modelling hypotheses, the estimation of γ_{Rd} can be carried out differentiating two ranges of slenderness values: [15-150] and [151-275]. In this way, the influence of un-safe biased outcomes, especially, for slenderness values higher than 150 can be included appropriately to estimate the design value of the structural resistance R_d through Eq.(1). The values of γ_{Rd} are determined according to [24] assuming: ϑ as a lognormally distributed non-dominant variable, limited influence of the experimental uncertainty [24] and 50 years reference period [13]-[14],[17]. The resistance model uncertainty safety factors γ_{Rd} determined for the two different slenderness ranges are reported in Table 2.

Table 2. Characterization of the resistance model uncertainty random variable ϑ and related partial safety factor [24] for the two different slenderness ranges within the adopted modelling hypotheses.

Slenderness range [-]	**Mean value** $\mu_{\vartheta,act^{*1}}$ **[-]**	**Coefficient of variation** $V_{\vartheta,act^{*1}}$ **[-]**	**Probabilistic distribution [-]**	**Target reliability index** β_t **[-]**	γ_{Rd} **[-]**
15-150	1.01	0.08	Lognormal	3.8 (50 years reference period)	1.07
151-275	0.91	0.06			1.15

*1 actual values deprived of contribution of experimental uncertainty [24]

In the next, the values of γ_{Rd} depending on the slenderness of the RC column will be used to cover the epistemic uncertainties in the assessment of the design value of structural resistance R_d (Eq.(1)) according to pre-determined target reliability levels [13]. The results of Table 2 are specific for the adopted modelling hypotheses and less conservative than the outcomes of [24] achieved for more general applications.

The NLN models so far established are used in the following to perform an extensive probabilistic analysis investigating the influence of aleatory uncertainties (i.e., materials and geometry) on the structural response of the 40 slender RC columns.

3. PROBABILISTIC ANALYSIS OF THE GLOBAL STRUCTURAL RESPONSE

This section deals with the probabilistic analysis in terms of ultimate axial load (i.e., global resistance) related to NLN models of the 40 slender RC columns [24],[28]-[36]. Firstly, the probabilistic models adopted for materials and geometric uncertainties are described. Then, the probabilistic investigation of the global structural response of the 40 RC columns is performed through two different groups of 100 sampled NLN models. Finally, the outcomes from the probabilistic analysis are commented.

3.1 Definition of the probabilistic models

The probabilistic investigation of the global structural response of the 40 RC columns of [24],[28]-[36] has been performed including the relevant aleatory uncertainties. In particular, the aleatory influence of both materials and geometric properties has been considered as reported in Table 3. The sampling technique herein adopted for the probabilistic analysis is the Latin Hypercube Sampling (LHS) method [39],[44]-[46]. The LHS method has been used with the aim to define two different groups, each one composed of 100 sampled NLN models for every slender RC column. In particular, for each RC column, the *group 1* is represented by 100 sampled NLN models which include the randomness of material properties only, whereas, the *group 2* is represented by other 100 sampled NLN models which include aleatory variability of both materials and geometric properties. The comparison between the *group 1* and *group 2* is useful to discuss the influence of geometric and materials randomness in global non-linear numerical analyses of slender RC columns according to the safety formats [1],[13] for different slenderness values.

Note that the experimental values described in [24],[28]-[36] are herein associated to the mean values of the specific material or geometric properties, respectively.

Table 3. Probabilistic models for the aleatory uncertainties related to the 40 RC columns of [28]-[36].

Random variable	Probabilistic distribution	Mean value	Coefficient of variation [-]	Standard deviation	Statistical correlation	Ref.
Material properties						
Concrete cylinder compressive strength f_c [MPa]	*Lognormal*	$f_{c,exp}$	0.15	-	-	[13], [38]
Reinforcement tensile yielding strength f_y [MPa]	*Lognormal*	$f_{y,exp}$	0.05	-	f_u (0.75) ^{*2}, ε_u(-0.45)^{*2}	[13], [38]
Reinforcement ultimate tensile strength f_u [MPa]	*Lognormal*	$f_{u,exp^{*1}}$	0.05	-	f_y (0.75) ^{*2}, ε_u(-0.60)^{*2}	[38]
Reinforcement Young modulus E_s [MPa]	*Lognormal*	210000	0.03	-	-	[38]
Reinforcement ultimate strain in elongation ε_u [-]	*Lognormal*	0.075	0.09	-	f_y (-0.45) ^{*2}, f_u (-0.60)^{*2}	[38]

*1 when the experimental value $f_{u,exp}$ is not provided, the mean value of ultimate tensile strength f_u is evaluated assuming the experimental tensile yielding strength $f_{y,exp}$ increased by 15%.

*2 (-) coefficient of linear correlation with respect to other material variable.

Random variable	Probabilistic distribution	Mean value	Coefficient of variation [-]	Standard deviation	Statistical correlation	Ref.
Geometric properties						
Concrete cover (C) deviation $Y_C=C-C_{exp}$ [mm]	*Normal*	0	-	*5*	-	[38]
Cross section base (b) deviation $Y_b=b-b_{exp}$ [mm]	*Normal*	$0 \leq 0.003b_{exp} \leq 3$	-	$4+0.006\ b_{exp} \leq 10$	-	[38]
Cross section height (h) deviation $Y_h=h-h_{exp}$ [mm]	*Normal*	$0 \leq 0.003h_{exp} \leq 3$	-	$4+0.006\ h_{exp} \leq 10$	-	[38]
Column length (L) deviation $Y_L=L-L_{exp}$ [mm]	*Normal*	$0 \leq 0.003L_{exp} \leq 3$	-	$4+0.006\ L_{exp} \leq 10$	-	[38]
Axial load eccentricity e [mm]	*Normal*	e_{exp}	-	$L_{exp}/1000$	-	[47]

With reference to the mechanical aleatory, material properties have been modelled in compliance with [13],[38]. A unimodal lognormal distribution, with mean value and coefficient of variation in line to [13],[38] has been used to model the concrete cylinder compressive strength f_c (Table 3). The other properties, as concrete Young Modulus and tensile strength, have been derived as dependent random variables according to the relationships of [16]. The response of reinforcement steel has been defined according to the suggestions of [38] including statistical correlation between the different variables. The tensile yielding strength f_y, the tensile ultimate strength f_u, the Young modulus E_s and the ultimate strain ε_u have been modelled by means of unimodal lognormal distributions with mean value and coefficient of variation reported in Table 3. The statistical correlations have been included between such variables with the adoption of linear correlation coefficients as suggested by [38] and listed in Table 3. In particular, a positive linear correlation has been assumed between the tensile yielding strength f_y and tensile ultimate strength f_u. If the sampled value of the tensile ultimate strength f_u turns out to be lower than the related tensile yielding strength f_y, the value of f_u is set equal to f_y

assuming that strain-hardening does not occur with reference to the specific sample [39]. Moreover, negative linear correlation has been adopted between the ultimate strain in elongation ε_u with respect to both the tensile yielding strength f_y and the tensile ultimate strength f_u. The Young Modulus E_s has been considered as un-correlated random variable [38].

As for the geometry uncertainty, the geometric properties have been probabilistically modelled according to the approach of [38]. As listed in Table 3, concrete cover, column cross section properties and column length (intended as the length of the main body of the column in line with [24]) have been modelled associating a unimodal normal distribution to their deviation Y with respect to the experimental value (herein assumed as the nominal one). The mean value and standard deviation adopted to account for the statistical variability of the mentioned above geometric properties are adopted in compliance with [38] as listed in Table 3. The last geometric property, which has been probabilistically treated, is the axial load eccentricity e, modelled by means of a unimodal normal distribution with mean value equal to the experimental one $e_{\exp}$ and standard deviation equal to 1/1000 multiplied by the experimental length L_{exp} of the main body of the RC column in agreement with [47]. All the random variables representative of the geometric properties have been considered as statistically un-correlated variables. Note that, when in the following, the probabilistic results, including the geometric aleatory, are described in relation to the slenderness λ, it is referring to the experimental deterministic slenderness [24] corresponding to the experimental test of that specific slender RC column.

In the next sub-section, the outcomes from the LHS of both *group 1* and *group 2* are commented.

3.2 Discussion of the probabilistic results from the sampled NLN simulations

The present sub-section discusses the results of the probabilistic analysis on the global response in terms of ultimate axial load. For each RC column of [24],[28]-[36], the summary of the results related to the 100 sampled NLN models for both *group 1* and *group 2* are reported in Table 4. The Anderson-Darling statistical tests [48] have been conducted on the 100 results in terms of global ultimate axial load for each RC column of the both sampled groups. The null hypothesis of lognormally distributed variable for the global ultimate axial load has been checked and confirmed with 5% significance level for all the 40 selected RC columns of [24],[28]-[36] both not including (*group 1)* and including (*group 2*) the geometric uncertainties within the probabilistic models. This result is reflected by the P-values listed in Table 4. Moreover, Figures 2 and 3 report the relative frequency histograms of the 100 sampled results superimposed to the lognormal fitting distributions, as example, for 8 columns of [24],[28]-[36]. The statistical tests, combined to the plots of Figures 2 and 3, are able to demonstrate the goodness of the unimodal lognormal probabilistic models to describe the aleatory response in terms of global ultimate axial load of the slender RC columns for both *group 1* and *group 2*. Table 4 also reports a further comparison between the experimental ultimate axial loads R_{exp} and results of NLNAs using the experimental values for both materials and geometric properties $R_{NLNA}(f_{exp};a_{exp})$ according to Section 3. In addition, Table 4 lists the mean value and coefficient of variation estimated using the ML criteria [49] to determine the statistical parameters according to the assumption of lognormally distributed random variable for the global ultimate axial load of the slender RC columns. The statistical parameters have been estimated separately for the two groups: μ_{Rm} and V_{Rm} for *group 1* (the subscript *m* means that only material uncertainty is considered); μ_R and V_R for *group 2*. As already stated, the slenderness λ of Table 4 is the experimental deterministic one [24].

With reference to *group 1* (Table 4), the mean value μ_{Rm} and the coefficient of variation V_{Rm} (i.e., σ_{Rm}/μ_{Rm}) relate, for each RC column (i.e., slenderness), to the statistical parameters of the lognormal probabilistic distribution of the global ultimate axial load. Figure 4(a) shows the trend with the associated linear regression of the bias factors δ_{Rm} as a function of the RC column slenderness λ. The value of δ_{Rm}, for each RC column, is defined as the ratio between the mean value μ_{Rm} of the 100 global ultimate axial loads and the resistance achieved through a NLN simulation using the experimental values of both materials and geometric properties $R_{NLNA}(f_{exp};a_{exp})$. The linear trend of

δ_{Rm} is slightly increasing moving from around 0.90 up to 1.00 for very high slenderness values λ (i.e., >200).

Table 4. Outcomes from the probabilistic analysis using LHS with 100 sampled NLN models for each one of the 40 RC columns [24] regarding to both *group 1* and *group 2*.

Ref. [*]	**Exp. test**	**Type**	**λ [-]**	***R_{exp}* *[kN]***	***$R_{NLNA}(f_{exp};a_{exp})$* *[kN]***	*Probabilistic distribution*	**LHS - *group 1***			**LHS - *group 2***		
							*P-value**2	*μ_{Rm} [kN]* *3	*V_{Rm}* [-]*3	*P-value**2	*μ_R [kN]* *3	*V_R* [-]*3
[30]	2L20-30	*B*	15	750.0	694.3	Lognormal	0.97	616.5	0.124	0.65	627.0	0.136
	2L20-60			700.0	736.4		0.69	680.6	0.122	0.41	688.9	0.131
	2L8-120R			1092.0	1152.7		0.63	1077.1	0.138	0.05	1083.0	0.146
	4L8-30			1100.0	1032.9		0.27	960.8	0.111	0.25	969.1	0.120
	4L20-120			900.0	830.7		0.87	766.6	0.096	0.81	775.4	0.102
	4L8-120R			1247.0	1319.5		0.71	1240.2	0.123	0.65	1254.0	0.141
[33]	C000	*A*	17	559.6	560.6		0.96	504.9	0.103	0.20	512.8	0.107
	C020			327.3	328.5		0.93	303.0	0.093	0.06	307.7	0.112
	B020	*B*	52	271.5	263.7		0.91	246.0	0.093	0.25	246.5	0.130
	RL300		56	474.3	423.3		0.99	382.4	0.112	0.05	384.7	0.134
[34]	A-17-0.25	*B*	48	1181.4	1367.4		0.97	1245.2	0.088	0.53	1244.6	0.115
	C-31.7-0.25		94	333.4	280.1		0.33	260.1	0.061	0.20	265.4	0.200
[28]	3.3	*B*	59	782.6	856.4		0.99	776.1	0.133	0.87	787.5	0.153
	5.1			735.5	810.8		0.89	794.6	0.090	0.05	796.0	0.133
	4.1		88	367.7	391.7		0.99	426.5	0.104	0.20	428.7	0.164
[35]	N30-10.5-C0-3-30	*C*	21	16.6 (280)*1	16.6 (280)*1		0.41	13.1 (280)*1	0.123	0.28	13.1 (280)*1	0.211
	H60-10.5-C0-1-30			17.2 (412)*1	17.9 (412)*1		0.05	11.8 (412)*1	0.080	0.13	12.3 (412)*1	0.121
[36]	III	*A*	74	343.2	347.3		0.83	294.6	0.101	0.22	272.9	0.125
	Va			684.5	680.7		0.98	589.0	0.110	0.17	546.9	0.128
	2		83	235.4	762.0		0.99	643.7	0.134	0.66	428.7	0.140
	I		104	264.8	258.0		0.98	215.4	0.094	0.52	196.4	0.145
	VI		106	392.3	363.2		0.97	306.0	0.100	0.75	276.0	0.150
	15		136	549.2	560.3		0.98	469.6	0.111	0.15	416.9	0.176
	3		137	666.9	563.4		0.95	475.2	0.117	0.09	418.8	0.178
	8		83	235.4	236.8		0.97	209.1	0.111	0.33	210.1	0.167
	9	*B*	135	205.9	205.9		0.96	185.3	0.092	0.11	190.6	0.197
	12			112.8	112.2		0.78	159.1	0.086	0.66	162.4	0.154
	6		137	225.6	227.6		0.99	215.7	0.097	0.46	219.9	0.181
[29]	24D-2	*D*	104	198.4	192.8		0.96	175.1	0.102	0.55	158.5	0.162
	15E-2	*A*	139	161.0	129.3		0.93	119.3	0.086	0.05	105.6	0.176
[32]	S28	*B*	167	44.0	49.9		0.81	55.5	0.088	0.31	58.7	0.255
	S30			48.0	53.4		0.46	57.5	0.086	0.59	60.6	0.262
	S25		200	36.0	42.3		0.75	42.5	0.083	0.27	45.3	0.245
[31]	5	*A*	208	72.7	78.7		0.99	77.1	0.084	0.24	65.9	0.209
	6			72.2	82.3		0.99	81.0	0.087	0.17	70.5	0.225
	17A		225	31.9	37.1		0.81	36.0	0.082	0.52	31.9	0.312
	20		243	37.9	39.8		0.97	38.8	0.082	0.11	33.7	0.280
	18			33.9	39.8		0.95	38.8	0.088	0.61	33.5	0.258
	8		274	31.9	31.0		0.98	30.4	0.080	0.24	25.7	0.262
	7			29.9	32.3		0.50	31.8	0.082	0.17	27.5	0.274

(-)*1: constant value of the axial load applied to the column during the experimental test.
*2 the P-value is estimated testing the null hypothesis of lognormal distribution for the 100 outcomes from NLNAs for each column using the Anderson-Darlin test [48]. The null hypothesis can be accepted if P-value ≥0.05 with 5% level of significance.
*3 the statistical parameters have been estimated using the ML criteria [49] with the assumption of lognormal distributed variable.

This regression presents a low effectiveness (i.e., a low R-square value). In general, it can be observed that the mean value μ_{Rm} is slightly overestimated by the NLN simulations results *$R_{NLNA}(f_{exp};a_{exp})$* for low-medium levels of slenderness. Figure 4(b) illustrates, for each RC column of LHS - *group 1*, the similar plot in terms of the coefficient of variation V_{Rm}. The decreasing trend with the slenderness λ highlights that the bigger is the slenderness, the smaller is the influence of materials aleatory uncertainties on the global response of the RC columns. In fact, the values of V_{Rm} ranges between 0.08 and 0.15 until slenderness around 150.

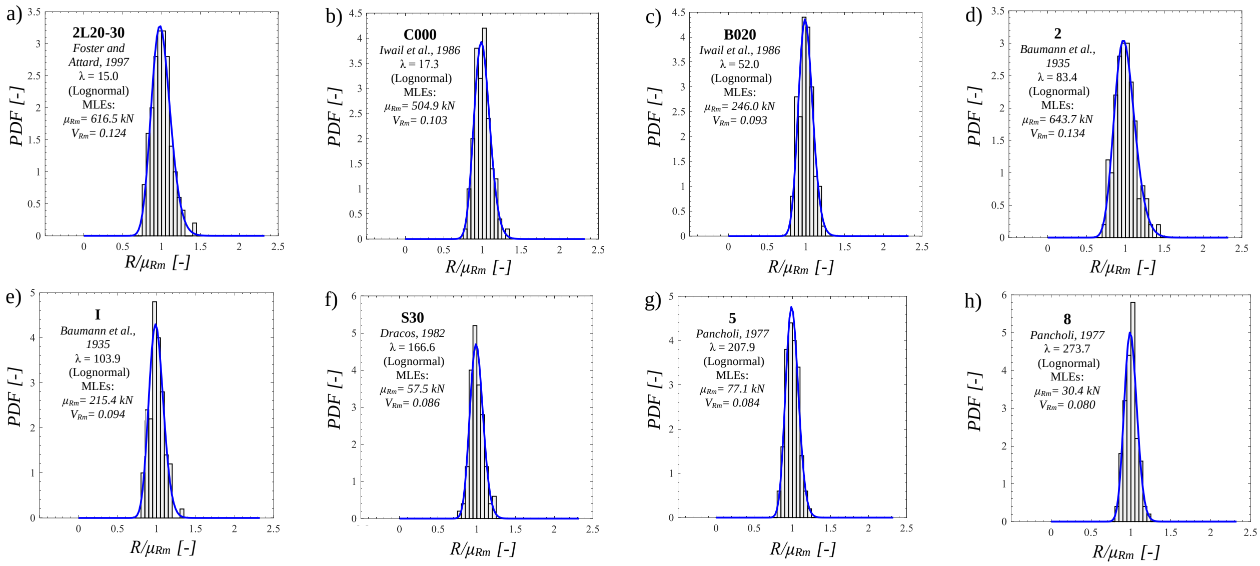

Fig. 2. Frequency histograms and lognormal fitting distributions for 8 columns [28]-[36] considering the LHS – *group 1* (only materials uncertainty).

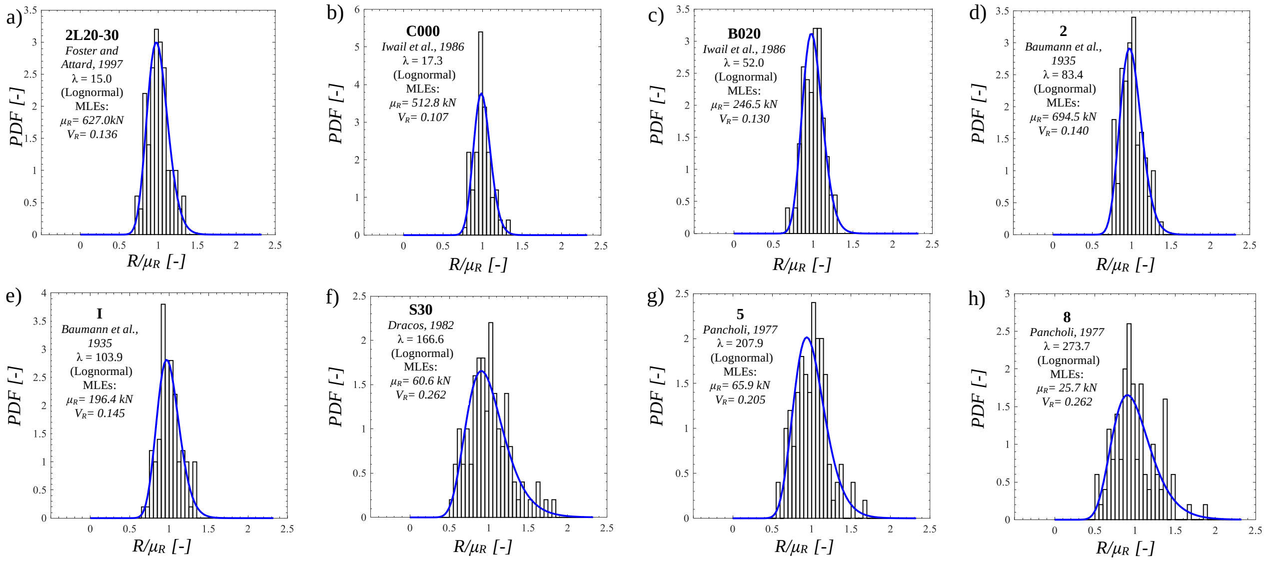

Fig. 3. Frequency histograms and lognormal fitting distributions for 8 columns [28]-[36] considering the LHS – *group 2* (both materials and geometric uncertainties).

Beyond this threshold, the failure mode is due to pure buckling of the column in elastic filed with a minimum influence of material resistances and their uncertainties. In fact, the random variability of the response is mainly governed by the randomness of concrete and reinforcement steel Young modulus.

Concerning *group 2* (Table 4), the mean value μ_R and the coefficient of variation V_R (i.e., σ_R / μ_R) relate, for each RC column (i.e., slenderness), to the statistical parameters of the lognormal probabilistic distribution of the global ultimate axial load *R* including both materials and geometric uncertainties, modelled according to Table 3. Figure 4(c) shows the variation of the bias factor δ_R as a function of the slenderness λ for the 40 RC columns. The trend of δ_R is almost constant and may be assumed as set equal to 0.9 even though scattering is recognized especially referring to [28],[32],[36] columns as achieved for the results of *LHS* - *group 1*. If compared to Figure 4(a), the additional influence of the geometric uncertainties in the probabilistic model is appreciable for $\lambda > 150$ where the bias factor δ_R does not increase up to 1.00 as happens for δ_{Rm}. Figure 4(d) depicts the evolution of the coefficient of variation V_R as a function of the slenderness λ, showing a linear increase as the slenderness increases. The linear regression approximates the outcomes of the 40 RC columns with a good effectiveness as demonstrated by the R-square value, equal to 0.8. The outcomes in terms of V_R do not show a significant difference with respect to LHS - *group 1* until slenderness λ lower than 50: the values of V_R do not fall below 0.10. In fact, in this part of the slenderness domain, the material uncertainties (in particular the concrete compressive strength) are still dominating the global structural response with respect to geometric ones. With increasing slenderness values, the influence of the geometric uncertainties becomes progressively dominant with respect to material uncertainties leading to a significant different values of V_R with respect to V_{Rm}.

In the following, these probabilistic results are adopted in order to derive the global resistance safety factors γ_R for NLNAs of slender RC columns within the global safety formats [13].

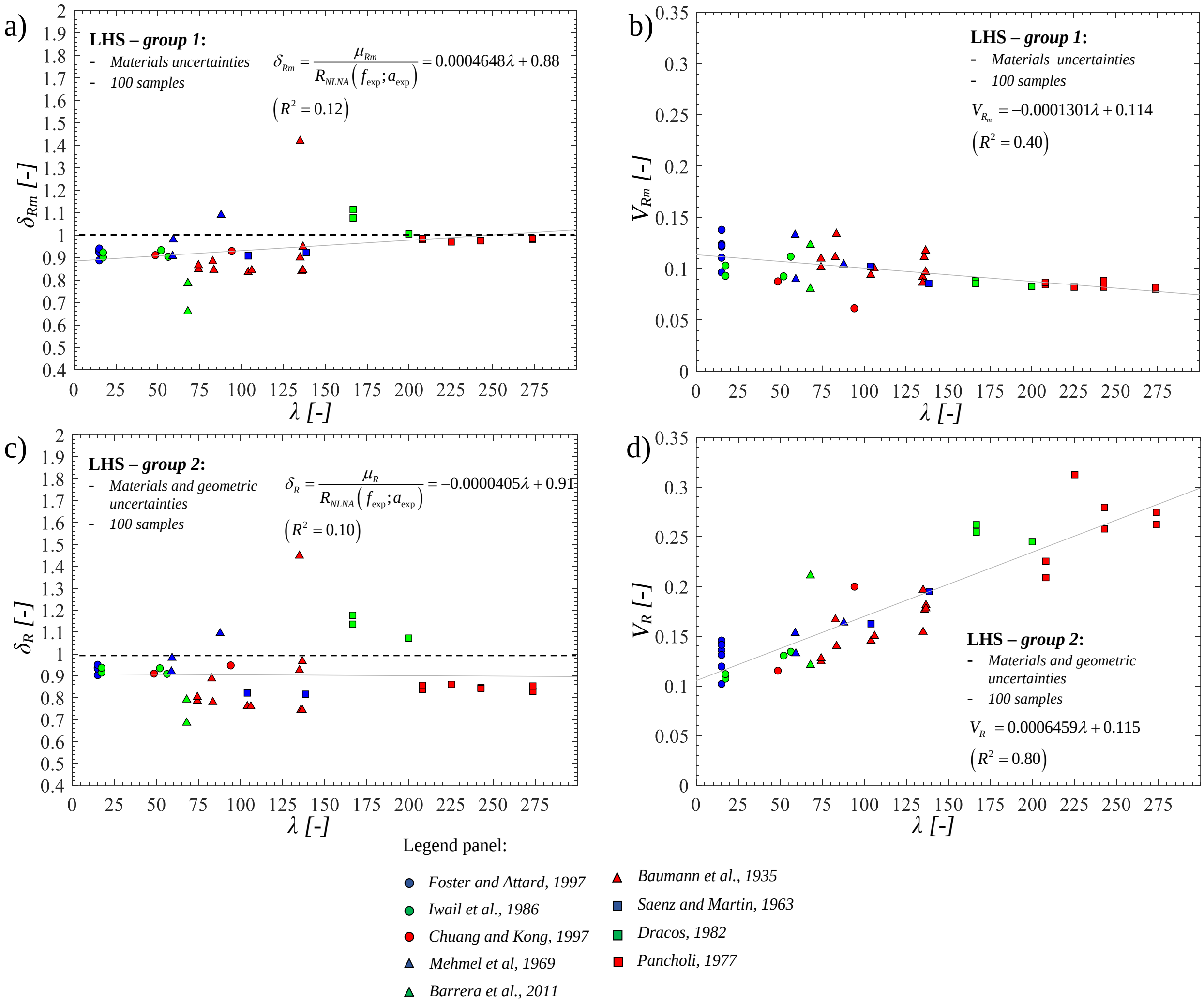

Fig. 4. Outcomes from probabilistic analysis of the 40 RC columns [24] with reference to *group 1* (a)-(b) and *group 2* (c)-(d): (a)-(c) ratios between μ_{Rm} or μ_R and $R_{NLNA}(f_{exp};a_{exp})$; (b)-(d) coefficients of variation V_{Rm} - V_R.

4. GLOBAL RESISTANCE SAFETY FACTORS FOR SLENDER RC COLUMNS

In this section, the previous results are used to discuss the influence of both materials and geometric uncertainties in the determination of the global resistance safety factor γ_R within the global resistance method [13]. Firstly, both material and geometric uncertainties are statistically characterized. Then, the global resistance safety factor γ_R is properly determined for each slender RC column. Finally, the probabilistic results regarding the geometric aleatory influence are also incorporated in the implementation of a global resistance safety format, the ECoV method [1],[13],[50]-[51], for practical applications.

4.1 Statistical characterization of both materials and geometric uncertainties

In this sub-section, the influence of materials and geometric uncertainties on the global response of slender RC columns is discussed within the global resistance method [13]. The global resistance random variable *R(f;a)* [52] (Eq.(1)) with reference to the ultimate axial load of the slender RC columns, depending on the material *(f)* and geometric *(a)* random variables, can be represented by a unimodal lognormal distribution with mean value μ_R and coefficient of variation V_R, as previously demonstrated. According to FORM approximation [53]-[54], the mean value μ_R and coefficient of variation V_R of the global resistance random variable *R(f;a)* can be expressed as follows:

$$\mu_R = \mu_{Rm} \cdot \delta_{Rg} \tag{2}$$

$$V_R = \sqrt{V_{Rm}^2 + V_{Rg}^2} \tag{3}$$

where μ_{Rm} is the mean value of the global structural resistance including materials uncertainty only; δ_{Rg} is the bias factor accounting for the influence of the geometric uncertainties on the global structural resistance; V_{Rm} is the coefficient of variation of the global resistance random variable including materials uncertainties only; V_{Rg} is the coefficient of variation which accounts for the influence of the geometric uncertainties on the global structural response. In particular, Eq.(3) turns out to be valid if the coefficient of variation V_R is lower than 0.2-0.3 as it happens for the present investigation (Figure 4(d)). In the previous section, the mean value μ_{Rm} and the coefficient of variation V_{Rm} have been evaluated for the 40 RC columns of [28]-[36] through the LHS - *group 1*; while, the mean value μ_R and the coefficient of variation V_R have been estimated by means of the LHS - *group 2*.

Employing the results of the previous section and reversing Eq.s(2)-(3), it is possible to derive an estimate of the bias factor δ_{Rg} and of the coefficient of variation V_{Rg} related to the influence only of the geometric uncertainties on the global structural response as follows:

$$\delta_{Rg} = \frac{\mu_R}{\mu_{Rm}} \tag{4}$$

$$V_{Rg} = \sqrt{V_R^2 - V_{Rm}^2} \tag{5}$$

Figures 5(a-b) depict the variation of the bias factor δ_{Rg} and of the coefficient of variation V_{Rg} as a function of the slenderness λ for the 40 RC columns of [28]-[36]. The linear regressions derived through the least square method are also reported with the associated R-square values. Precisely, Figure 5(a) reports the outcomes in terms of bias factor δ_{Rg} depending on the slenderness of the RC columns. The influence of the geometric uncertainty on the mean value of the global resistance *R* can be considered negligible up to values of λ around 75-100. So, in the interval 0-100 of slenderness λ, the bias factor δ_{Rg} may be, reasonably, assumed as equal to 1.00. For increasing slenderness, the bias factor δ_{Rg} turns out to decrease up to values slightly below 0.90 approaching extremely high λ (i.e., higher than 200). The regression for δ_{Rg} is reported in Figure 5(a) with the R-square value (i.e., 0.40).

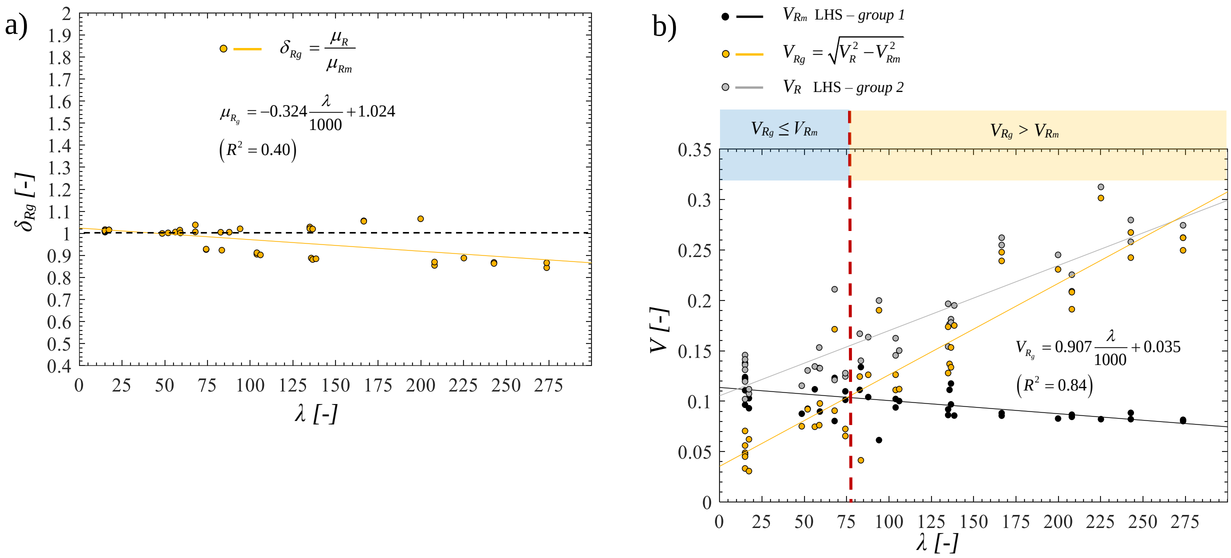

Fig. 5. Bias factor δ_{Rg} of the geometric uncertainties with respect to the slenderness (a). Comparison between V_{Rm}, V_R and V_{Rg} (b).

Figure 5(b) illustrates the coefficient of variation of the global resistance related to the geometric uncertainties V_{Rg} as a function of the slenderness λ also in comparison to the coefficients of variation V_{Rm} and V_R. It can be recognised that up to slenderness value of 50, the coefficient of variation V_{Rg} can be assumed ranging between 0.05-0.08 according to the assumptions for non-slender RC members [13]-[14]. With increasing slenderness, the influence of the geometric uncertainties becomes progressively more and more important on the coefficient of variation of the global structural resistance V_R. In particular, for slenderness values lower than 75, the global variability of the resistance *R* is dominated by materials uncertainties while, for slenderness values higher than 75, the geometric uncertainties become dominant with respect to the materials ones as shown in Figure 5(b). For extremely high values of slenderness (i.e., $\lambda > 200$), the geometric uncertainties fully govern the response in terms of ultimate resistance of the RC columns. The regression expression related to V_{Rg} is reported in Figure 5(b) with the R-square value (i.e., 0.84).

In the next, the global resistance safety factors are proposed together with the procedure to use the proposed results within the global safety formats for NLNAs of slender RC structures.

4.2 Evaluation of the global resistance safety factors for NLNAs of slender RC columns

In the following, the evaluation of the global resistance safety factors for NLNAs of the slender RC columns is investigated grounding on the results presented in the previous sub-section. In line with the global resistance methods [1],[13],[51], the global design resistance can be derived through Eq.(1). Moreover, according to the FORM approximation [53]-[54] and the assumption of lognormal distribution to describe the statistical variability of the global structural resistance, its design value R_d can be derived as follows:

$$R_d = \frac{\mu_R}{\gamma_{Rd}} \exp\left(-\alpha_R \beta_t \cdot V_R\right) \tag{6}$$

where β_t is the target reliability index (3.8 for a reference period of 50 years) [13]-[14],[17]; α_R is the FORM sensitivity factor assumed equal to 0.8 in the hypothesis of dominant resistance variable [13],[53] with respect to the modelling uncertainties; V_R is the coefficient of variation of the global resistance; μ_R is the mean value of the global structural resistance; γ_{Rd} is the partial safety factor accounting for the model uncertainty as deeply described in [24]. In relation to the specific modelling hypotheses, the values of γ_{Rd} proposed in Table 2 will be adopted in the next sub-section.

Accounting for the expression of Eq.(2) [51], Eq.(6) may be rewritten separating the influence of materials and geometric uncertainties on the design value of the global resistance R_d as follows:

$$R_d = \frac{\mu_{Rm} \cdot \delta_{Rg}}{\gamma_{Rd}} \exp\left(-\alpha_R \beta_t \cdot V_R\right) \tag{7}$$

where μ_{Rm} is the mean value of the global structural resistance evaluated including materials uncertainties only and δ_{Rg} is the bias factor for the global structural resistance accounting for influence of the geometric uncertainties (as previously shown).

In general, according to the global resistance methods [1],[13],[18],[50]-[51] and FORM analysis [53]-[54], the following first-order approximation can be performed (i.e., the bias factor δ_{Rm} is approximately unit):

$$\mu_{Rm} = \delta_{Rm} \cdot R_{NLNA}\left(f_{rep}; a_{rep}\right) \simeq R_{NLNA}\left(f_{rep}; a_{rep}\right) \tag{8}$$

Eq.(8) is accepted for practical applications of the global resistance methods and related safety formats for NLNAs [1],[13],[18],[50]-[51]. As illustrated in Figure 4(a) in terms of the bias factor δ_{Rm}, Eq.(8) is satisfied in the present analysis because the mean value of the global structural resistance considering only material uncertainties μ_{Rm} can be confused with the global structural resistance derived herein from a NLNA using the experimental values as the representative (i.e., mean/nominal) values for both materials and geometric properties $R_{NLNA}(f_{rep};a_{rep})=R_{NLNA}(f_{exp};a_{exp})$

[1]. Grounding on the mentioned above results, the design global structural resistance R_d can be evaluated according to [1],[13],[18],[50]-[51], as expressed in Eq.(7) or Eq.(1).

Next, in agreement with both Eq.(8) and the assumption of lognormal distribution for the global response in terms of axial load, the global resistance safety factor γ_R can be determined as follows:

$$\gamma_R = \frac{\exp\left(\alpha_R \beta_t \cdot V_R\right)}{\delta_{Rg}} \tag{9}$$

Note that Eq.s(6)-(9) are approximated expressions and turn out to be valid until that the value of V_R is equal to 0.20 and provide minimum discrepancies if compared to the exact equations until values of 0.30 as it happens in this investigation (Figure 5(b)). More details may be acknowledged in [55].

Considering the values of δ_{Rg} (Figure 5(a)) and of the coefficient of variation V_R, evaluated according to Eq.(3) or in Figure 5(b), corresponding to the outcomes of the LHS - *group 2*, the global resistance safety factors γ_R have been computed through Eq.(9). Figure 6 illustrates the variation of the global resistance safety factor γ_R for the 40 RC columns of [28]-[36] as a function of the main influencing parameters. Specifically, Figure 6(a) demonstrates the strong influence of the geometric uncertainties. In fact, the factor γ_R increases almost linearly with reference to the coefficient of variation V_{Rg} related to the geometric uncertainties. Figure 6(b) reports γ_R as a function of the slenderness λ of the 40 RC columns highlighting the strong relation with the coefficient of variation V_{Rg} .

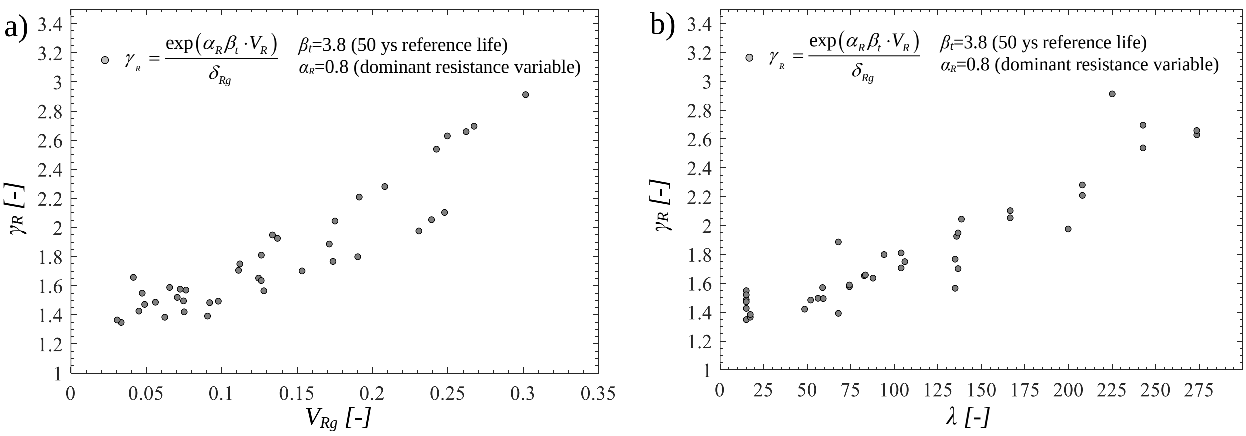

Fig. 6. Global resistance factor γ_R for the 40 RC columns of [28]-[36] with reference to V_{Rg} (a) and slenderness λ (b).

4.3 Procedure to include the geometric uncertainties within the global resistance safety formats for NLNAs of slender RC columns

The achieved global resistance safety factors (with the related design global structural resistance R_d) are compared with the ones derived with the application of one of the most used and common safety formats within the framework of global resistance methods, i.e., the ECoV method proposed by [13],[18]. The ECoV method [13],[18], allows to estimate the coefficient of variation of the global structural resistance related to the uncertainties of materials properties V_{Rm}^{ECoV} in the hypothesis of lognormal distribution as follows:

$$V_{Rm}^{ECoV} = \frac{1}{1.65} \ln\left(\frac{R_{NLNA}\left(f_m; a_n\right)}{R_{NLNA}\left(f_k; a_n\right)} \right) \tag{10}$$

where $R_{NLNA}(f_m;a_n)$ corresponds to the global structural resistance estimated by means of NLNAs using mean f_m and nominal a_n values for materials and geometric properties, respectively. The mean and nominal values are herein assumed as the experimental ones $R_{NLNA}(f_m;a_n)=R_{NLNA}(f_{exp};a_{exp})$. The term $R_{NLNA}(f_k;a_n)$ is the global structural resistance evaluated using characteristic values of material properties f_k and nominal a_n values for geometric ones. The characteristic values of material properties f_k have been derived from the probabilistic model adopted in Table 3 evaluating the 5% quantile. The geometric properties are the experimental ones. Then, other material properties necessary to define the NLN model have been derived according to [16], similarly to the discussion of Section 2.

As an improvement to the ECoV method [13],[18], the additional influence of the geometric uncertainties is herein proposed. In particular, the estimated coefficient of variation of the global structural resistance V_R^{ECoV} can be assessed as follows:

$$V_R^{ECoV} = \sqrt{\left(V_{Rm}^{ECoV}\right)^2 + V_{Rg}^2} \tag{11}$$

where V_{Rg} can be estimated according to the linear regression expression of Figure 5(b). Then, the global resistance factor in line to the ECoV method [13],[18] can be derived from Eq.(9) as follows:

$$\gamma_R^{ECoV} = \frac{\exp\left(\alpha_R \beta_t \cdot V_R^{ECoV}\right)}{\delta_{Rg}} \tag{12}$$

where δ_{Rg} can be evaluated according to the linear regression expression reported in Figure 6(a).

Finally, according to the ECoV method [13],[18], the design value of the global structural resistance can be evaluated as follows:

$$R_d^{ECoV} = \frac{R_{NLNA}\left(f_m; a_n\right)}{\gamma_R^{ECoV} \cdot \gamma_{Rd}} = \frac{R_{NLNA}\left(f_{\exp}; a_{\exp}\right)}{\gamma_R^{ECoV} \cdot \gamma_{Rd}} \tag{13}$$

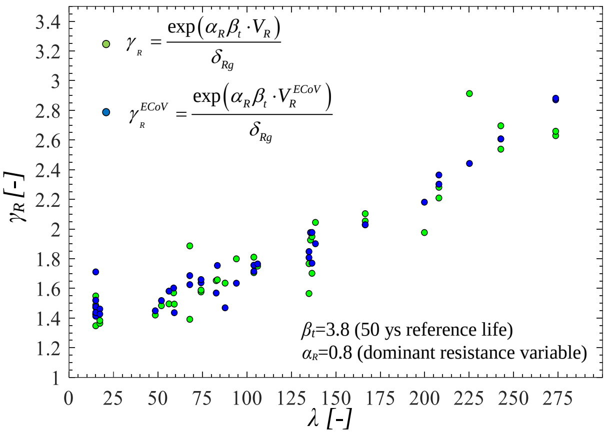

Fig. 7. Values of γ_R achieved from the probabilistic investigation (LHS - *group 2*) and ECoV method [13],[18] improved with the additional influence of the geometric uncertainties.

Table 5 reports the coefficients of variation, global resistance safety factors and design global resistances corresponding, respectively, to the LHS - *group 2* and the ECoV method [13],[18] improved with the additional influence of the geometric uncertainties. The comparison in terms of the global resistance safety factor, γ_R and γ_R^{ECoV}, is also depicted in Figure 7, showing a good agreement. Therefore, the results of Sub-section 4.1 can be used efficiently in combination to the global resistance safety formats [1],[13],[18],[50]-[51] for safety verifications of slender RC columns using NLNAs.

Table 5. Comparison between the results of the probabilistic analysis and ECoV method improved with the additional influence of the geometric uncertainties.

Ref. [*]	Exp. Test	Type	λ [-]	R_{exp} [kN]	$R_{NLNA}(f_{exp};a_{exp})$ [kN]	$R_{NLNA}(f_k;a_n)$ [kN]	γ_{Rd} [-]	LHS - *group 2* V_R [-]	γ_R [-]	R_d [kN]	*ECoV method* V_R^{ECoV} [-]	γ_R^{ECoV} [-]	R_d^{ECoV} [kN]
[30]	2L20-30			750.0	694.3	520.4		0.136	1.49	436.4	0.182	1.71	379.2
	2L20-60			700.0	736.4	600.5		0.131	1.47	467.6	0.133	1.48	466.2
	2L8-120R	*B*	15	1092.0	1152.7	924.5		0.146	1.55	695.5	0.143	1.52	709.2
	4L8-30			1100.0	1032.9	859.2	1.07	0.120	1.43	677.0	0.122	1.43	676.5
	4L20-120			900.0	830.7	695.0		0.102	1.35	575.9	0.119	1.41	549.4
	4L8-120R			1247.0	1319.5	1092.7		0.141	1.52	811.3	0.125	1.44	857.7
[33]	C000	*A*	17	559.6	560.6	460.9		0.107	1.36	383.9	0.130	1.46	358.6
	C020			327.3	328.5	273.8	1.07	0.112	1.38	221.9	0.122	1.43	215.1
	B020	*B*	52	271.5	263.7	220.7		0.130	1.48	166.1	0.136	1.52	162.4
	RL300		56	474.3	423.3	346.7		0.134	1.50	264.4	0.149	1.58	250.1
[34]	A-17-0.25	*B*	48	1181.4	1367.4	1175.4		0.115	1.42	899.5	0.122	1.45	881.9
	C-31.7-0.25		94	333.4	280.1	239.8	1.07	0.200	1.80	145.5	0.153	1.64	160.1
[28]	3.3		59	782.6	856.4	697.9		0.153	1.57	509.7	0.153	1.60	499.4
	5.1	*B*		735.5	810.8	716.2	1.07	0.133	1.49	507.2	0.117	1.44	527.6
	4.1		88	367.7	391.7	372.0		0.164	1.64	223.8	0.119	1.47	249.1
[35]	N30-10.5- C0-3-30	*C*	21	16.6 (280)^{*1}	16.6 (280)^{*1}	13.3 (280)^{*1}	1.07	0.211	1.89	8.2 (280)^{*1}	0.168	1.69	9.2 (280)^{*1}
	H60-10.5- C0-1-30			17.2 (412)^{*1}	17.9 (412)^{*1}	14.6 (412)^{*1}		0.121	1.39	12.0 (412)^{*1}	0.156	1.63	10.3 (412)^{*1}
[36]	III		74	343.2	347.3	285.5		0.125	1.58	205.9	0.157	1.64	198.1
	Va			684.5	680.7	554.4		0.128	1.59	400.5	0.162	1.66	383.4
	2		83	235.4	762.0	605.9		0.140	1.66	429.6	0.178	1.75	406.0
	I	*A*	104	264.8	258.0	216.9		0.145	1.71	141.3	0.167	1.71	140.6
	VI		106	392.3	363.2	299.6		0.150	1.75	194.0	0.176	1.76	192.4
	15		136	549.2	560.3	448.9	1.07	0.176	1.93	271.9	0.208	1.98	265.0
	3		137	666.9	563.4	452.1		0.178	1.95	270.2	0.208	1.98	266.4
	8		83	235.4	236.8	204.6		0.167	1.65	133.9	0.142	1.57	141.0
	9		135	205.9	205.9	179.3		0.197	1.77	108.9	0.179	1.81	106.5
	12	*B*		112.8	112.2	95.3		0.154	1.57	67.0	0.186	1.85	56.7
	6		137	225.6	227.6	204.9		0.181	1.70	125.0	0.172	1.77	120.2
[29]	24D-2	*D*	104	198.4	192.8	158.9	1.07	0.162	1.81	99.5	0.175	1.76	102.6
	15E-2	*A*	139	161.0	129.3	108.0		0.176	2.04	59.1	0.195	1.90	63.6
[32]	S28		167	44.0	49.9	42.4		0.255	2.05	21.1	0.211	2.03	21.4
	S30	*B*		48.0	53.4	45.4	1.15	0.262	2.10	22.1	0.211	2.03	22.9
	S25		200	36.0	42.3	37.5		0.245	1.98	18.6	0.229	2.18	16.8
[31]	5		208	72.7	78.7	64.7		0.209	2.21	31.0	0.254	2.36	29.0
	6			72.2	82.3	70.0		0.225	2.28	31.4	0.245	2.30	31.1
	17A		225	31.9	37.1	31.3	1.15	0.312	2.91	11.1	0.261	2.44	13.2
	20	*A*	243	37.9	39.8	33.1		0.280	2.70	12.8	0.279	2.61	13.3
	18			33.9	39.8	33.1		0.258	2.54	13.6	0.279	2.61	13.3
	8		274	31.9	31.0	25.8		0.262	2.63	10.2	0.305	2.87	9.4
	7			29.9	32.3	26.7		0.274	2.66	10.6	0.306	2.88	9.7

(-)^{*1}: constant value of axial load applied to the column during the experimental test.

Note 1: the partial safety factor accounting for model uncertainty γ_{Rd} has been derived assuming β_t=3.8 as target reliability index for 50 ys reference period and α_R=0.32 (non-dominant variable [24]) with a distinction of the slenderness ranges (Table 2).

Note 2: the global resistance safety factors has been derived assuming β_t=3.8 as target reliability index for 50 ys reference period and α_R=0.8 (dominant variable [24]).

5. CONCLUSIONS

The purpose of the present study is to understand the influence of the aleatory uncertainties (i.e., material and geometric) in the evaluation of the global resistance safety factor γ_R within the framework of the global resistance methods. Firstly, 40 experimental tests related to RC columns having slenderness ratio between 15 and 275 and different geometries in line with the international codes limitations have been introduced. The database excluded RC columns with high strength concretes or affected by creep effects. Then, the experimental tests have been reproduced by means of NLN simulations by minimizing the influence of the resistance model uncertainty which has been included in the reliability evaluation by means of the appropriate partial safety factor γ_{Rd} including the dependence from the slenderness value. Next, the calibrated NLN models have been used in order to define two different groups of sampled NLN models, i.e. LHS - *group* 1 and LHS - *group* 2, including, respectively, materials uncertainties only and both geometric and material uncertainties. From the outcomes of the mentioned above probabilistic investigation, several useful results can be derived. The global resistance of slender reinforced concrete columns can be modelled by means of unimodal lognormal distributions both with materials uncertainties only and with material and geometric uncertainties. The geometric uncertainties related to the global structural response become dominant with respect to materials ones for slenderness values higher than 75. In particular, the bias factor related to the geometric uncertainties δ_{Rg} turns out to be close to 1.00 up to slenderness values of 100, while linearly approaches the value of 0.90 for increasing slenderness of the RC columns. The coefficient of variation of the geometric uncertainties V_{Rg} related to the global structural response increases linearly with the slenderness and, for a slenderness higher than 75, becomes dominant in the definition of the global coefficient of variation of the structural resistance V_R. Successively, from the results of the two probabilistic groups, the global resistance safety factors γ_R are proposed highlighting that their evaluation is strongly affected by the influence of the geometric uncertainties for increasing slenderness of the RC columns. Then, the global resistance method as the ECoV method has been properly modified including also the contribution of the geometric uncertainties in the evaluation of γ_R. For instance, the results from the linear regressions for δ_{Rg} and V_{Rg} have been adopted in order to apply the ECoV method to the 40 RC columns leading to results comparable with respect to the ones provided by the probabilistic analysis including all the aleatory uncertainties. Therefore, the herein presented results are useful to include both material and geometric uncertainties in the global resistance safety formats for NLNAs of slender reinforced concrete columns.

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REFERENCES

[1] Allaix DL, Carbone VI, Mancini G. Global safety format for non-linear analysis of reinforced concrete structures. Structural Concrete 2013; 14(1): 29-42.

[2] P. Castaldo, D. Gino, G. Mancini (2019): Safety formats for non-linear analysis of reinforced concrete structures: discussion, comparison and proposals, Engineering Structures, 193, 136-153, https://doi.org/10.1016/j.engstruct.2018.09.041.

[3] NIST GCR 17-917-46v3. Guidelines for Nonlinear Structural Analysis for Design of Buildings, U.S. Department of Commerce, Engineering Laboratory National Institute of Standards and Technology, Gaithersburg, USA, 2017.

[4] DNV-RP-C208. Determination of structural capacity by non-linear FE analysis methods,

Recommended Practice, DET NORSKE VERITAS AS 2013, http://www.dnv.com.

[5] *fib* Bulletin N°45. Practitioner's guide to finite element modelling of reinforced concrete structures – State of the art report. Lausanne; 2008.

[6] Belletti B, Damoni C, Hendriks MAN. Development of guidelines for nonlinear finite element analyses of existing reinforced and prestressed beams. European Journal of Environmental and Civil Engineering 2011;15(9): 1361-1384.

[7] Belletti B., Scolari M., Vecchi F. PARC_CL 2.0 crack model for NLFEA of reinforced concrete structures under cyclic loadings. Computer and Structures 191: 165-179.

[8] Slobbe A, Rózsás _A, Allaix DL, Bigaj-van Vliet A. On the value of a reliability-based nonlinear finite element analysis approach in the assessment of concrete structures. Structural Concrete. 2019;1–16. https://doi.org/10.1002/suco.201800344

[9] Bagge N. Demonstration and examination of a procedure for successively improved structural assessment of concrete bridges. Structural Concrete. 2019;1–24.

[10] Biondini F., Vergani M. (2015) Deteriorating beam finite element for nonlinear analysis of concrete structures under corrosion, Structure and Infrastructure Engineering, 11:4, 519-532, DOI: 10.1080/15732479.2014.951863

[11] Castaldo, P., Palazzo, B., Mariniello, A. (2017) Effects of the axial force eccentricity on the time-variant structural reliability of aging r.c. cross-sections subjected to chloride-induced corrosion, Engineering Structures, 130, 261-274.

[12] Mohammed, A., Almansour, H. & Martín-Pérez, B. Simplified finite element model for evaluation of ultimate capacity of corrosion-damaged reinforced concrete beam-columns. Int J Adv Struct Eng 10, 381–400 (2018). https://doi.org/10.1007/s40091-018-0204-2.

[13] *fib* Model Code for Concrete Structures 2010. *fib* 2013. Lausanne.

[14] CEN. EN 1990: Eurocode – Basis of structural design. CEN 2013. Brussels.

[15] Kiureghian AD, Ditlevsen O. Aleatory or epistemic? Does it matter?. Structural Safety, 31:105-

112, 2009.

[16] CEN. EN 1992-1-1: Eurocode 2 – Design of concrete structures. Part 1-1: general rules and rules for buildings. CEN 2014. Brussels.

[17] ISO 2394. General principles on reliability for structures. Genéve. 2015.

[18] Cervenka V., Reliability –based non-linear analysis according to fib Model Code 2010, Structures Concrete, Journalof the fib, vol. 14, March 2013, ISSN1464-4177, p.p.19-28, 2011.

[19] *fib* Bulletin N°80. Partial factor methods for existing concrete structures, Lausanne, Switzerland; 2016.

[20] Gino D, Castaldo P, Bertagnoli G, Giordano L, Mancini G. Partial factor methods for existing structures according to fib Bulletin 80: Assessment of an existing prestressed concrete bridge. Structural Concrete. 2020;21:15–31. https://doi.org/10.1002/suco.201900231.

[21] P. Castaldo, D. Gino, G. Bertagnoli, G. Mancini (2018): Partial safety factor for resistance model uncertainties in 2D non-linear analysis of reinforced concrete structures, Engineering Structures, 176, 746-762. https://doi.org/10.1016/j.engstruct.2018.09.041.

[22] P. Castaldo, D. Gino, G. Bertagnoli, G. Mancini (2020): Resistance model uncertainty in non-linear finite element analyses of cyclically loaded reinforced concrete systems, Engineering Structures, 211(2020), 110496, https://doi.org/10.1016/j.engstruct.2020.110496

[23] Engen M, Hendriks MAN, Köhler J, Øverli JA, Åldtstedt E. A quantification of modelling uncertainty for non-linear finite element analysis of large concrete structures. Structural Safety 2017; 64: 1-8.

[24] Gino, D., Castaldo, P., Giordano, L., Mancini, G. Model uncertainty in non-linear numerical analyses of slender reinforced concrete members, Structural Concrete, 2021.

[25] Dashti, F., Dhakal, R.P., Pampanin, S. A parametric investigation on applicability of the curved shell finite element model to nonlinear response prediction of planar RC walls. Bulletin of Earthquake Engineering. Volume 17, Issue 12, 1 December 2019, Pages 6515-6546.

[26] Belletti B., Scolari M., Almeida J., Beyer K. Validation of NLFEA to simulate the instability of thin RC walls subjected to bidirectional loading. Lecture Notes in Civil Engineering 10: 32-48.

[27] Cervenka V. Global safety formats in *fib* Model Code 2010 for design of concrete structures. Proceedings of the 11th International Probabilistic Workshop, Brno, 2013.

[28] A Mehmel, H Schwarz, KH Karperek, and J Makovi. Tragverhalten ausmittig beanspruchter stahlbetondruckglieder. institut für baustatik, eht, deutscherausschuss für stahlbeton, heft 204, 1969.

[29] Luis P Saenz and Ignacio Martin. Test of reinforced concrete columns with high slenderness ratios. In Journal Proceedings, volume 60, pages 589–616, 1963.

[30] Stephen J Foster and Mario M Attard. Experimental tests on eccentrically loaded high strength concrete columns. Structural Journal, 94(3):295–303, 1997.

[31] VR Pancholi. The Instability of Slender Reinforced Concrete Columns. PhD thesis, University of Bradford, 1977.

[32] Adriana Dracos. Long slender reinforced concrete columns. PhD thesis, University of Bradford, 1982.

[33] S.IwaiI, K. Minami, and M. Wakabayashi. Stability of slender reinforced concrete columns subjected to biaxially eccentric loads, 1986.

[34] PH Chuang and FK Kong. Large-scale tests on slender, reinforced concrete columns. Structural Engineer, 75(23):410–16, 1997.

[35] AC Barrera, JL Bonet, Manuel L Romero, and PF Miguel. Experimental tests of slender reinforced concrete columns under combined axial load and lateral force. Engineering Structures, 33(12):3676–3689, 2011.

[36] Oskar Baumann. Die Knickung der Eisenbeton-Säulen. PhD thesis, ETH Zurich, 1935.

[37] F. McKenna, G.L. Fenves, M.H. Scott, Open system for earthquake engineering simulation,

University of California, Berkeley, CA, 2000.

[38] JCSS. JCSS Probabilistic Model Code. 2001.

[39] Mckey MD, Conover WJ, Beckman RJ. A comparison of three methods for selecting values of input variables in the analysis from a computer code. Technometrics, 1979; 21:239-45.

[40] ACI318-19, Building Code Requirements for Structural Concrete, American Concrete Institute, Farmington Hills, MI, USA, 2019.

[41] Murat Saatcioglu and Salim R Razvi. Strength and ductility of confined concrete. Journal of Structural engineering, 118(6):1590–1607, 1992.

[42] Chang, G. and Mander, J. (1994). "Seismic Energy Based Fatigue Damage Analysis of Bridge Columns: Part I – Evaluation of Seismic Capacity." NCEER Technical Report 94-0006.

[43] Dhakal, R. and Maekawa, K. (2002). "Modeling for Postyield Buckled of Reinforcement" J. Struct. Eng., 128(9), 1139-1147.

[44] Olsson A, Sandberg G, Dahlblom O. On latin hypercube sampling for structural reliability analysis , Structural Safety, 25(1), pp. 47-68, 2003.

[45] Dolsek M. Incremental dynamic analysis with consideration of modelling uncertainties. Earthquake Engineering and Structural Dynamics, 2009;38:805–25.

[46] Vořechovský M, Novák D. Correlation control in small-sample Monte Carlo type simulations I: a simulated annealing approach, Probabilistic Engineering Mechanics, 24(3):452–62, 2009.

[47] M. Achenbach, T. Lahmer and G. Morgenthal (2016) Global Sensitivity Analysis of Reinforced Concrete Walls Subjected to Standard Fire—A Comparison of Methods, 14th International Probabilistic Workshop, 97-106.

[48] Anderson, T. W.*;* Darling, D. A. (1952). Asymptotic theory of certain "goodness-of-fit" criteria based on stochastic processes. *Annals of Mathematical Statistics. 23: 193–212.* doi*:*10.1214/aoms/1177729437.

[49] Faber, Michael Havbro Statistics and Probability Theory, Springer, 2012.

[50] Holicky, M. (2006) Global resistance factor for reinforced concrete members, ACTA POLITECHNICA, CTU, Prague, 2006.

[51] Shlune H, Gylltoft K, Plos M. Safety format for non-linear analysis of concrete structures. Magazine of Concrete Research 2012; 64(7): 563-574.

[52] Framework for definition of design formulations from empirical and semi-empirical resistance models, Structural Concrete, 19(4), 980-987, 2018. https://doi.org/10.1002/suco.201800083

[53] Hasofer AM, Lind NC. Exact and invariant second moment code format, Journal of the Engineering Division ASCE 1974; 100(EM1): 111-121.

[54] Haldar A, Mahadevan S. Probability, reliability and statistical methods in engineering design, John Wiley and Sons Inc., ISBN 0-471-33119-8, 2000.

[55] Taerwe, R.L.: Toward a consistent treatment of model uncertainties in reliability formats for concrete structures, CEB Bulletin d'Information N° 105-S17, pp. 5-34, 1993.