Torelli Control Box Method Application for Zeta Buckboost and Cuk Boost Combination Converters...

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I. CASE STUDIES

A. Dynamic Behavior of a Single Input Multi-output Zeta BuckBoost and Cuk Boost Combination Converters

In this section, the application of the TCB approach is simulated on a Single Input Multi-output (SIMO) Zeta Buck-Boost (ZBB) and Cuk Boost Combination (CBC) converter topologies as proposed in [1]. With their ability to provide more than one output voltages, the SIMO converters have found various applications in the field of renewable energy, microelectronics, lighting, Hybrid or battery electric vehicles etc [2], [3]. One example of a SIMO converters is the ZBB converter which is a combination of Zeta and Buckboost converter topologies and has been designed in such a way that both of these converters share a switch and an inductor at the input. It is able to provide bipolar output voltage as well as Buck-boost ability using only a single switch. Because of the use of a single switch, it is simpler to design its controller and the reduced components used also means it is inexpensive and has higher efficiency [2]. Similarly, the CBC converter topology is a combination of boost and Cuk converters and provides two different output voltages but with the same polarity. One issue that arises in the modeling of ZBB and CBC converters is that their dynamics are governed by Differential Algebraic Equations (DAE) instead of Ordinary differential equations (ODE) as is the case for conventional converters. This means we cannot use traditional averaging methods to derive its steady state model. In this paper, we have applied the Torelli Control Box (TCB) approach to simulate the averaged model of both of these converters.

The Zeta Buck-boost converter is shown in Figure 1. The small signal model for this converter has already been derived in [3], [4] using quasi-Weierstrass transformation and the use of consistency projectors. In this example, however, we have modeled both ZBB and CBC converters using the TCB approach instead. Mathematical modeling of the CBC converter has not been done in the literature to the best of our knowledge.

B. Steady State Modeling of ZBB Converter

The ZBB converter shown in Figure 1 has two modes of operations. Let the steady state parameters be defined as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ v_{C1} \\ v_{C2} \\ v_C \end{bmatrix}$$

The converter has two modes of operation. In Mode 1 switch S is closed and the diodes are reverse biased. If we denote the total switching period by T_s and duty ratio D as the ratio for which switch S is ON, then the time interval for mode 1 can defined as $0 \leq t \leq DT_s$. By applying Kirchhoff's voltage and current laws we can write the following equations for mode 1:

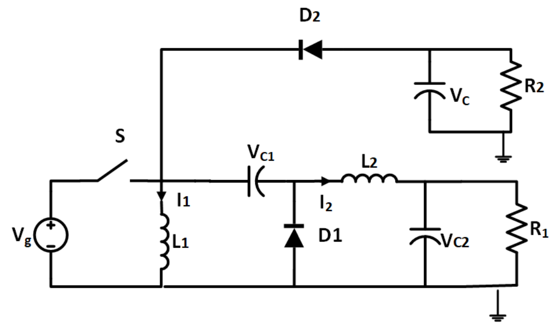


Fig. 1. Zeta BuckBoost Converter

$$
\begin{aligned}
\dot{x}_1 &= \frac{V_g}{L_1} \\
\dot{x}_2 &= \frac{V_g - x_3 - x_4}{L_2} \\
\dot{x}_3 &= \frac{x_2}{C_1} \\
\dot{x}_4 &= \frac{x_2}{C_2} - \frac{x_4}{R_1 C_2} \\
\dot{x}_5 &= \frac{-x_5}{R_2 C}
\end{aligned} \tag{1}
$$

Similarly in Mode 2, for the interval $DTs \leq t \leq T_s$, switch S is open and diodes D_1 and D_2 are forward biased. Again using Kirchhoff's voltage and current laws we can derive the following equations:

$$
\begin{aligned}
\dot{x}_1 &= \frac{x_3}{L_1} \\
\dot{x}_2 &= \frac{-x_4}{L_2} \\
\dot{x}_3 &= \frac{-x_1}{C_1 + C} - \frac{x_3}{R_2(C_1 + C)} \\
\dot{x}_4 &= \frac{x_2}{C_2} - \frac{x_4}{R_1 C_2} \\
x_5 &= x_3
\end{aligned} \tag{2}
$$

C. Application of Torelli Control Box Approach

As seen from equation of the converter in mode 2, the dynamics of the converter are constrained by a DAE which does not allow us to use the traditional Volt-second or Charge-second balance to obtain the averaged model of the converter. A possible approach of solving this issue is by first converting the system equations in mode 2 into simple ODE form. This is done through the process of TCB method described above and system equations have been derived using both sensitivity and without sensitivity parameters.

1) TCB with Sensitivity Parameters: Using the procedure described in the previous section, we can define our algebraic constraint as $g(x) : x_5 - x_3 = 0$; and we can define our sensitivity parameters as follows: $w_1 = \frac{dx_1}{dx_5}$; $w_2 = \frac{dx_2}{dx_5}$; $w_3 = \frac{dx_3}{dx_5}$; $w_4 = \frac{dx_4}{dx_5}$. Using the TCB approach, we can chose x_5 as our variable y. Then using equation (*eqn 12 of TCB paper mentioned in section above?*) mentioned above, we can derive the following equations for mode 2 of our system:

$$
\begin{aligned}
\dot{x}_1 &= \frac{x_3}{L_1} \\
\dot{x}_2 &= \frac{-x_4}{L_2} \\
\dot{x}_3 &= \frac{-x_1}{C_1 + C} - \frac{x_3}{R_2(C_1 + C)} \\
\dot{x}_4 &= \frac{x_2}{C_2} - \frac{x_4}{R_1 C_2} \\
\dot{x}_5 &= -K(1 - w_3)(x_5 - x_3)
\end{aligned} \tag{3}
$$

The sensitivity parameter equations can be derived as follows:

$$
\begin{aligned}
\dot{w}_1 &= \frac{w_3}{L_1} \\
\dot{w}_2 &= \frac{-w_4}{L_2} \\
\dot{w}_3 &= \frac{-w_1}{C_1 + C} - \frac{w_3}{R_2(C_1 + C)} \\
\dot{w}_4 &= \frac{w_2}{C_2} - \frac{w_4}{R_1 C_2}
\end{aligned} \tag{4}
$$

The sensitivity parameters $\dot{w}_2$ and $\dot{w}_4$ are not used in the modeling of our system. As we can see from equation (3) and (4) our converter dynamics in Mode 2 are no longer constrained by DAE. Hence, we can now apply volt-second and charge-second balance to derive the averaged model of our converter as follows:

$$
\begin{aligned}
<\dot{x}_1> &= \frac{DV_g}{L_1} + \frac{(1-D)x_3}{L_1} \\
<\dot{x}_2> &= \frac{D(V_g - x_3)}{L_2} - \frac{x_4}{L_2} \\
<\dot{x}_3> &= \frac{x_2 D}{C_1} - \frac{x_1(1-D)}{C_1 + C} - \frac{x_3(1-D)}{(C_1 + C)R_2} \\
<\dot{x}_4> &= \frac{x_2}{C_2} - \frac{x_4}{R_1 C_2} \\
<\dot{x}_5> &= \frac{-x_5 D}{R_2 C} - K(1-D)(1-w_3)(x_5 - x_3)
\end{aligned} \tag{5}
$$

2) TCB without Sensitivity Parameters: Again following methods described we can derive the averaged equations for the parameters of our converter as follows:

$$
\begin{aligned}
<\dot{x}_1> &= \frac{DV_g}{L_1} + \frac{(1-D)x_3}{L_1} \\
<\dot{x}_2> &= \frac{D(V_g - x_3)}{L_2} - \frac{x_4}{L_2} \\
<\dot{x}_3> &= \frac{x_2 D}{C_1} - \frac{x_1(1-D)}{C_1 + C} - \frac{x_3(1-D)}{(C_1 + C)R_2} \\
<\dot{x}_4> &= \frac{x_2}{C_2} - \frac{x_4}{R_1 C_2} \\
<\dot{x}_5> &= \frac{-x_5 D}{R_2 C} - K(1-D)(x_5 - x_3)
\end{aligned} \tag{6}
$$

The steady state values of the converter are given by the following equations:
$x_1 = \frac{D(1+D)V_g}{(1-D)^2 R}$; $x_2 = \frac{DV_g}{(1-D)R1}$; $x_3 = \frac{-DVg}{(1-D)}$; $x_4 = \frac{DV_g}{1-D}$; $x_5 = \frac{-DVg}{(1-D)}$

D. Simulation Results

The simulation of the model was done in simulink using an ODE 15s solver with a step time of 0.001s. The specifications for the converter are shown in Table I. The simulation results are given in Figure 5 for the all the parameters except V_c. To test the validity of the model derived, a step up of duty ratio from D = 0.5 to 0.6 was given at time t = 4s followed by a step down from D = 0.6 to 0.5 at t = 6s. The resulting output of the converter model was compared with the theoretical values of the steady state equations derived above.

1) Response of V_c due to change in value of K: The response of V_c depends directly on the value

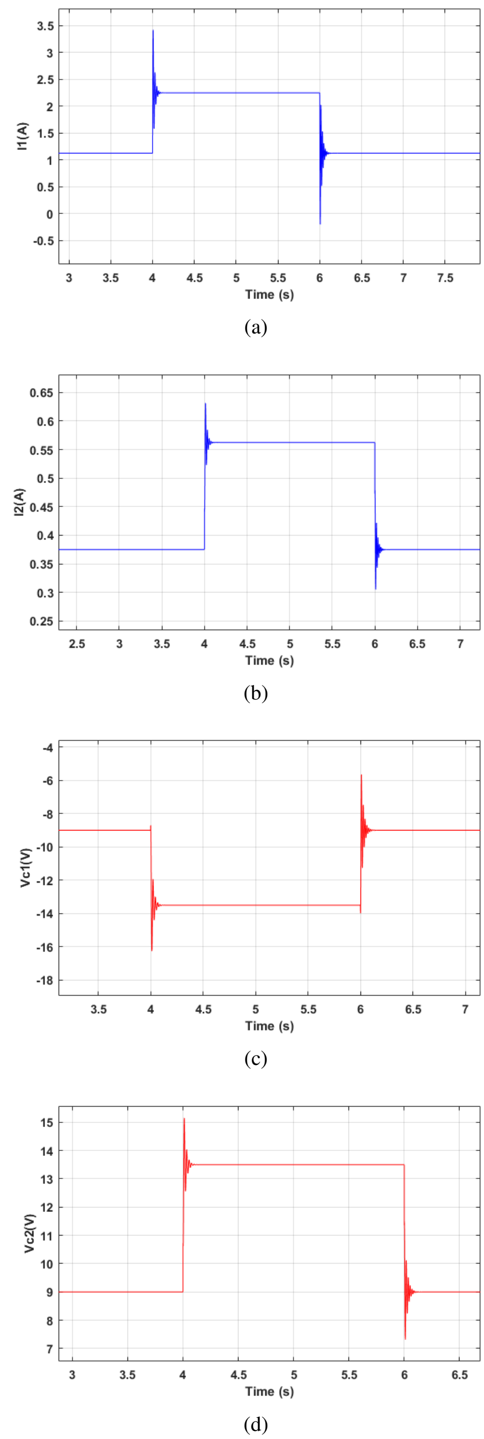
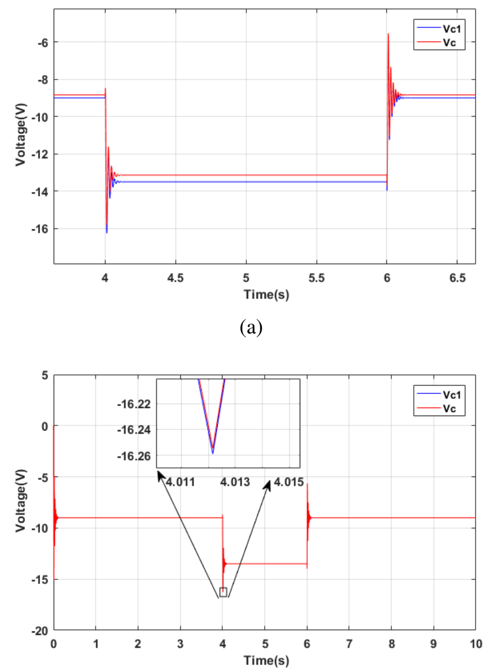


Fig. 2. Voltage and Current response of ZBB converter parameters (a) I_1 (b) I_2 (c) V_1 (d) V_2 in face of step change in duty ratio from 0.5 to 0.6

TABLE I
SPECIFICATIONS OF ZETA BUCKBOOST CONVERTER

Description of Parameters	Nominal Value
Capacitor, C_1	0.2280 mF
Capacitor, C_2	2.5024 uF
Capacitor, C	0.2280 mF
Inductor, L_1	4.1 mH
Inductor, L_1	4.1 mH
Resistance, R_1	24 Ω
Resistance, R_2	24 Ω
Switching Frequency, f_s	50 kHz
Gain, K	10^6

of gain K. It is only for the high value of gain K = 10^6 that the value of V_c tracks its expected value perfectly as shown in Figure 3(b). At lower values of K, say 10^4, the error between the reference value and the original value increases sufficiently. This is shown in Figure 3(a).



(a)

(b)

Fig. 3. Response of V_c (a) With $K > 10^4$ (b) With $K > 10^6$

E. Steady State modeling of a Single Input Multi-output Cuk Boost Combination Converter

The Cuk Boost Combination converter is shown in Figure 4. Again we can define our steady state parameters as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ v_C \\ v_1 \\ v_2 \end{bmatrix}$$

Following the same procedure as in section 1, we can derive the values of steady state parameters in mode 1 and 2. In Mode 1, switch S is closed and the diodes D1 and D2 are reverse biased. By applying Kirchhoff's voltage and current laws we can write the following equations:

$$\begin{aligned} \dot{x}_1 &= \frac{V_g}{L_1} \\ \dot{x}_2 &= \frac{x_4 - x_3}{L_2} \\ \dot{x}_3 &= \frac{x_2}{C} \\ \dot{x}_4 &= \frac{-x_2}{C_1} - \frac{x_4}{R_1 C_1} \\ \dot{x}_5 &= \frac{-x_5}{R_2 C_2} \end{aligned} \tag{7}$$

In Mode 2, switch S is open and diodes D_1 and D_2 are forward biased. Again using Kirchhoff's voltage and current laws we can derive the following equations:

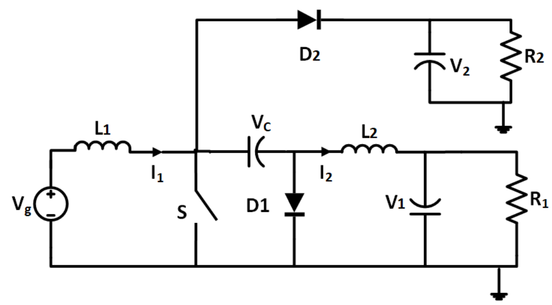


Fig. 4. Cuk Boost Converter

$$
\begin{aligned}
\dot{x}_1 &= \frac{V_g - x_3}{L_1} \\
\dot{x}_2 &= \frac{x_4}{L_2} \\
\dot{x}_3 &= \frac{x_1}{C + C_2} - \frac{x_5}{R_2(C + C_2)} \\
\dot{x}_4 &= \frac{-x_2}{C_1} - \frac{x_4}{R_1 C_1} \\
x_5 &= x_3
\end{aligned} \tag{8}
$$

F. Application of Torelli Control Box Approach

To apply Volt-sec and Charge-sec balance, we need to convert the DAE equation in mode 2 to standard ODE form.

1) TCB with Sensitivity Parameters: We can define our algebraic constraint as $g(x) : x_5 - x_3 = 0$; and our sensitivity parameters as follows: $w_1 = \frac{dx_1}{dx_5}$; $w_2 = \frac{dx_2}{dx_5}$; $w_3 = \frac{dx_3}{dx_5}$; $w_4 = \frac{dx_4}{dx_5}$. Using the TCB approach, we can derive the following equations for mode 2 of our system:

$$
\begin{aligned}
\dot{x}_1 &= \frac{V_g - x_3}{L_1} \\
\dot{x}_2 &= \frac{x_4}{L_2} \\
\dot{x}_3 &= \frac{x_1}{C + C_2} - \frac{x_5}{R_2(C + C_2)} \\
\dot{x}_4 &= \frac{-x_2}{C_1} - \frac{x_4}{R_1 C_1} \\
\dot{x}_5 &= -K(x_5 - x_3)(1 - w_3)
\end{aligned} \tag{9}
$$

The sensitivity parameter equations can be derived as follows:

$$
\begin{aligned}
\dot{w}_1 &= \frac{-w_3}{L_1} \\
\dot{w}_2 &= \frac{w_4}{L_2} \\
\dot{w}_3 &= \frac{w_1}{C_2 + C} - \frac{1}{R_2(C_2 + C)} \\
\dot{w}_4 &= \frac{w_2}{C_1} - \frac{w_4}{R_2 C_2}
\end{aligned} \tag{10}
$$

In this case as well, the sensitivity parameters $\dot{w}_2$ and $\dot{w}_4$ are not being used in the modeling of our system. We can now apply Volt-sec and Charge-sec balance since the dynamics in mode 2 are no longer constrained by a DAE. The averaged model of our converter can be derived as follows:

$$
\begin{aligned}
<\dot{x}_1> &= \frac{V_g}{L_1} - \frac{(1-D)x_3}{L_1} \\
<\dot{x}_2> &= \frac{x_4}{L_2} - \frac{Dx_3}{L_2} \\
<\dot{x}_3> &= \frac{x_2 D}{C} - \frac{x_5(1-D)}{(C_2 + C)R_2} + \frac{x_1(1-D)}{(C_2 + C)} \\
<\dot{x}_4> &= \frac{-x_2}{C_1} - \frac{x_4}{R_1 C_1} \\
<\dot{x}_5> &= \frac{-x_5 D}{R_2 C_2} - K(1-D)(1-w_3)(x_5 - x_3)
\end{aligned} \tag{11}
$$

2) TCB without Sensitivity Parameters: Again following methods described we can derive the averaged equations for the parameters of our converter as follows:

$$
\begin{aligned}
<\dot{x}_1> &= \frac{V_g}{L_1} - \frac{(1-D)x_3}{L_1} \\
<\dot{x}_2> &= \frac{x_4}{L_2} - \frac{Dx_3}{L_2} \\
<\dot{x}_3> &= \frac{x_2 D}{C} - \frac{x_5(1-D)}{(C_2 + C)R_2} + \frac{x_1(1-D)}{(C_2 + C)} \\
<\dot{x}_4> &= \frac{-x_2}{C_1} - \frac{x_4}{R_1 C_1} \\
<\dot{x}_5> &= \frac{-x_5 D}{R_2 C_2} - K(1-D)(x_5 - x_3)
\end{aligned} \tag{12}
$$

The steady state values of the parameters can be derived from the volt-sec and charge-sec balance equations as follows:
$x_1 = \frac{V_g}{(1-D)R_2} + \frac{2D^2 V_g}{(1-D)^2 R_1}$ $x_2 = \frac{-DV_g}{(1-D)R_1}$; $x_3 = \frac{Vg}{(1-D)}$; $x_4 = \frac{DV_g}{1-D}$; $x_5 = \frac{Vg}{(1-D)}$

TABLE II
SPECIFICATIONS OF CUK BOOST COMBINATION CONVERTER

Parameters	Nominal Value
Capacitor, C	0.11 mF
Capacitor, C_1	2.10 uF
Capacitor, C_2	0.11 mF
Inductor, L_1	8.1 mH
Inductor, L_1	8.1 mH
Resistance, R_1	24 Ω
Resistance, R_2	24 Ω
Switching Frequency, f_s	50 kHz
Gain, K	10^6

G. Simulation Results

The simulation was done in simulink using an ODE 15s solver with a step time of 0.001s. The specifications for the converter are shown in Table II. Again to validate the mathematical model derived, a step change of duty ratio D from 0.5 to 0.6 was given at time t = 0.4s followed by a step change of D from 0.6 to 0.5 at t = 0.6s. The result of simulation of the CBC converter is shown in Figure 5 for the all the parameters except V_2. The output values of the parameters correspond to the theoretical steady state values of the converter derived above.

1) Response of V_2 due to change in value of K: The response of x_5 depends directly on the value of gain K. It is only for the high value of gain K = 10^6 that the value of V_2 tracks its expected value perfectly as shown in Figure 6(b). At lower values of K, say 10^4, the error between the reference value and the original value increases sufficiently. This is shown in Figure 6(a).

H. TCB comparison with literature

The modeling of switched capacitor converters is nontrivial due to the presence of state jumps. The dynamic behavior of converter is then represented by both ODE and algebraic equations in a particular mode of operation. In literature, different methodologies have been proposed to rectify this issue. In [5], [6] switched capacitors have been modeled by

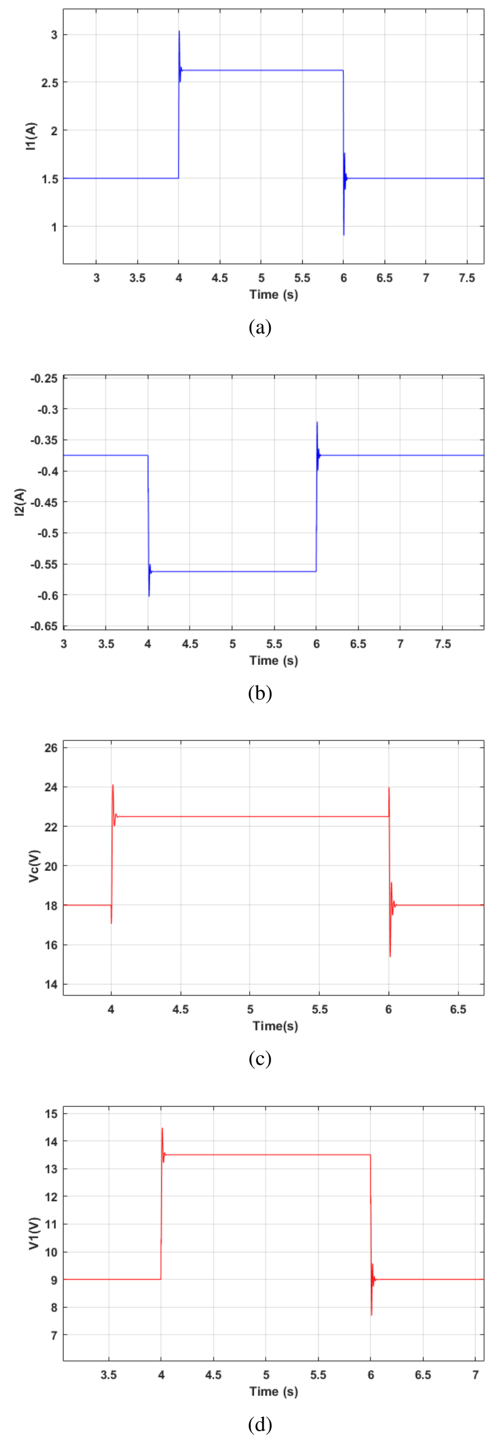
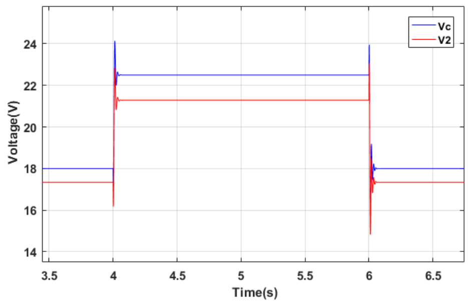
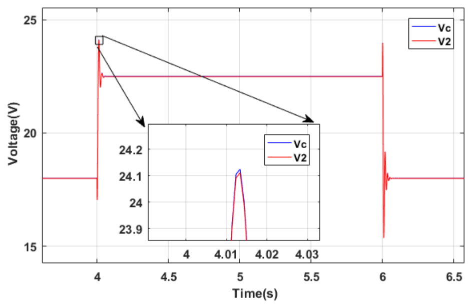


Fig. 5. Voltage and Current response of CBC converter parameters (a) I_1 (b) I_2 (c) V_1 (d) V_2 in face of step change in duty ratio from 0.5 to 0.6



(a)



(b)

Fig. 6. Response of V_2 (a) With $K > 10^4$ (b) With $K > 10^6$

considering the losses produced when capacitors are connected in parallel. In [7] an incremental graph approach has been employed for determining steady state voltage gains. In [8], a discrete time framework for the analysis of switched capacitors has been presented. Furthermore, switched dynamic averaged modeling approach has been used for systems with state jumps [4], [9], [10]. This involves the use of quasi-weirstrass transformation and the derivation of consistency projectors to find the equivalent model of the converter.

The use of Torelli control box method, in comparison to the techniques mentioned above, is much more straightforward and allows us to derive the averaged model of the converter using traditional volt and charge second balance methods even in the presence of algebraic equations. The dynamic model derived is asymptotically stable and is insensitive to parametric variations.

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