Weighted SPSA-based Consensus Algorithm for Distributed Cooperative Target Tracking

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Abstract— **In this paper, a new algorithm for distributed multi-target tracking in a sensor network is proposed. The main feature of that algorithm, combining the SPSA techniques and iterative averaging ("consensus algorithm"), is the ability to solve distributed optimization problems in presence of signals with fully uncertain distribution; the only assumption is the signal's boundedness. As an example, we consider the multi-target tracking problem, in which the unknown signals include measurement errors and unpredictable target's maneuvers; statistical properties of these signals are unknown. A special choice of weights in the algorithm enables its application to targets exhibiting different behaviors. An explicit estimate of the residual's covariance matrix is obtained, which may be considered as a performance index of the algorithm. Theoretical results are illustrated by numerical simulations.**

I. INTRODUCTION

Multi-agent systems are ubiquitous in real-world applications, and algorithms for cooperative multi-agent control and coordination attract a great deal of attention. Many such algorithms are inspired by behaviors of social and biological agents [1], [2]. Teams of relatively simple agents can solve complex mathematical problems [3]–[8].

Wireless sensor networks (WSN) constitute an important class of multi-agent systems whose applications vary from robotics [9], [10] to personalized medicine [11]. In particular, WSNs can be used for locating and tracking maneuvering targets. Multisensor-multitarget tracking problems have been widely studied in many practical applications such as adaptive mobile networks, cognitive radio systems, target localization in biological networks, fish schooling, bee swarming, and bird flight (see, e.g., [12]). As the number of targets is growing, centralized algorithms for estimation of their parameters and trajectories become inefficient [13], and distributed techniques are typically applied. Distributed algorithms assume that the agents (sensors) can exchange their estimates; such interactions are, however, local in the sense that each sensor communicates only to few neighbors. The latter requirement can e.g. arise due to communication constraints: a communication channel or bandwidth has to be shared between many pairs of agents, and only few of them can communicate simultaneously.

To estimate unknown parameters of multiple targets, distributed optimization is often used. Since unknown disturbances and noises are usually measured as random signals, the performance index to be optimized is the expectation of some cost function; such functionals are often referred to as *mean-risk* functionals. The paper of optimal target tracking is a special case of stochastic optimization with non-stationary (time-varying) mean-risk functional [14], [15], which is naturally defined as the mean-square estimation error. To cope with such problems, maximum likelihood estimator and stochastic approximation (SA) algorithms are often used. As an extension of the SA, the simultaneous perturbation stochastic approximation (SPSA) was proposed by [16]. The important feature of SPSA is the gradient approximation that requires only two loss function measurements and does not depend on the number of variables d (vs. $2d$ measurements in the conventional SA). On average, the SPSA uses the same number of iterations as the usual SA. In [17], the SPSA algorithm has been applied to an unconstrained problem of (mean-square) optimal target tracking. It has been shown that SPSA converges even in the presence of an arbitrary unknown-but-bounded noise (typically, tracking algorithms are suitable only for noises with zero mean [18]).

This paper is based on the results of our previous works [19]–[23]. In [22], a consensus-based distributed SPSA algorithm for multi-target tracking was proposed. An important feature of this algorithm is a randomized gossip-based communication protocol, designed in such a way that each agent communicates only with a small group of its neighbors. In [23], stronger communication constraints are introduced, and parameter optimization is provided. In the papers mentioned above, we consider the variance of the tracking error as a performance index of our algorithm, and assume that dynamics of the targets are identical. In this paper, we propose a weighted version of the SPSA-based consensus algorithm, taking the heterogeneity of targets into account, and analyze its convergence. In practice, one may need to track a group of targets consisting of fixed-wing drones and rotor ones, which have different dynamics and speed. Furthermore, we examine the covariance matrix of the residuals, extending the approaches proposed in [24], [25].

The rest of this paper is organized as follows. Section II provides notations used in the paper. Section III is devoted to problem statement; the new SPSA-based tracking algorithm is introduced in Section IV. The main result concerning the

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estimation of the covariance matrix is given in Section V and illustrated in Section VI. Section VII concludes the paper.

II. PRELIMINARIES

Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space corresponding to sample space Ω, set of all events $\mathcal{F}$, and probability measure P. $\mathbb{E}$ denotes mathematical expectation.

Let $[\cdot]^{\mathrm{T}}$ be vector or matrix transpose operation, $[\cdot]^{-1}$ be matrix inversion. $\|A\|$ is the Frobenius norm: $\|A\| = \sqrt{\sum_i \sum_j (a^{i,j})^2}$. $\mathbf{1}_d = [1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^d$ is the vector of all ones. $\mathbf{e}_i = [\ldots, 0, 1, 0, \ldots]^{\mathrm{T}} \in \mathbb{R}^d$ is the canonical basis vector from $\mathbb{R}^d$, where i-th entry is equal to 1. $I_d \in \mathbb{R}^{d\times d}$ is the identity matrix, $\mathbb{J}_d = \mathbf{1}_d \mathbf{1}_d^\top \in \mathbb{R}^{d\times d}$ is the matrix of all ones. $A \otimes B$ is the Kronecker product defined for any matrices A and B. The following notation $A \leq B$ means that matrices are ordered in the sense of quadratic forms.

For a sequence of column vectors $x_1, \ldots, x_k$, let $\mathrm{col}\{x_1, \ldots, x_k\}$ denote the column vector obtained by stacking x_i on top of one another.

A network consisting of n nodes is described by a directed graph $\mathcal{G}_A = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, n\}$ is a set of vertices and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is a set of edges. Denote by $i \in \mathcal{N}$ an identifier of i-th node and $(j, i) \in \mathcal{E}$ if there is a directed edge from node j to node i. The latter means that node j is able to transmit data to node i. For a node $i \in \mathcal{N}$, the set of *neighbors* is defined as $\mathcal{N}^i = \{j \in \mathcal{N} : (j, i) \in \mathcal{E}\}$. The *in-degree* of $i \in \mathcal{N}$ equals $|\mathcal{N}^i|$. Hereinafter, $|\cdot|$ is the cardinality of a set, and superscripts stand for agents' indices.

Let $a^{i,j} > 0$ be the weight associated with the edge $(j, i) \in \mathcal{E}$ and $a^{i,j} = 0$ whenever $(j, i) \notin \mathcal{E}$. Let $A = [a^{i,j}]$ be the *weighted adjacency matrix*, or simply *connectivity matrix*, associated with graph $\mathcal{G}_A$. The *weighted in-degree* of $i \in \mathcal{N}$ is defined as $\deg_i^+(A) = \sum_{j=1}^n a^{i,j}$, the maximum in-degree among all nodes contained in graph $\mathcal{G}_A$ as $\deg_{\max}^+(A)$. Introducing the diagonal matrix $\mathcal{D}(A) = \mathrm{diag}_n(\deg_1^+(A), \ldots, \deg_n^+(A))$, matrix $\mathcal{L}(A) = \mathcal{D}(A) - A$ is referred to as the *Laplacian* of graph $\mathcal{G}_A$.

Definition 1. A directed graph $\mathcal{G}_A$ is said to be strongly connected if for every pair of nodes $j, i \in \mathcal{N}$, there exists a path of directed edges that goes from j to i.

Denote the eigenvalues of Laplacian $\mathcal{L}(A)$ by $\lambda_1, \ldots, \lambda_n$ and arrange them in ascending order of real parts: $0 \leq Re(\lambda_1) \leq Re(\lambda_2) \leq \ldots \leq Re(\lambda_n)$. It is known, that if the graph is strongly connected then $\lambda_1 = 0$ and all other eigenvalues of $\mathcal{L}$ are in the open right half of the complex plane (see, e.g., [26]). The eigenvalue of matrix A with maximum absolute magnitude is defined as $\lambda_{\max}(A)$.

III. MULTI-SENSOR MULTI-TARGET PROBLEM

We consider a network of n spatially-distributed sensors in a field, namely, agents, capable of measuring parameters (e.g., distance, heading, etc), performing local computations, and exchange information with neighboring nodes. In this field, there are m moving targets. Each sensor $i \in \mathcal{N} = \{1, \ldots, n\}$ has its own hypothesis regarding the state of the targets (i.e., their positions) or more simply an estimate of the states. The goal of the network is to accurately estimate the unknown parameters of the targets. The sensors must also act together as a team to achieve this goal.

Let $\mathbf{s}_t^i = [s_t^{i,1}, \ldots, s_t^{i,d}]^{\mathrm{T}} \in \mathbb{R}^d$ be the state of sensor i at time instant t, $\mathbf{r}_t^l = [r_t^{l,1}, \ldots, r_t^{l,d}]^{\mathrm{T}} \in \mathbb{R}^d$ be the state of target $l \in \mathcal{M} = \{1, \ldots, m\}$, and $\theta_t = \mathrm{col}\{\mathbf{r}_t^1, \ldots, \mathbf{r}_t^m\}$ be the vector consisting of all states to be estimated. Suppose that each sensor measures a scalar quantity, which is the distance between its own position and position of a target:

$$\rho(\mathbf{s}_t^i, \mathbf{r}_t^l) = ||\mathbf{r}_t^l - \mathbf{s}_t^i||^2, \quad \forall i \in \mathcal{N}, l \in \mathcal{M}. \tag{1}$$

Note that the proposed approach can be used for other types of measuring parameters (e.g., bearing/azimuth).

In general, the problem is to find an estimate $\hat{\theta}_t$ of an unknown parameter θ_t:

$$\hat{\theta}_t^\star = \arg\min_{\hat{\theta}_t} ||\hat{\theta}_t - \theta_t||^2. \tag{2}$$

In this paper, we consider a more difficult problem setting. First, the solution of the optimization problem (2) needs to be found in a distributed way. Second, we impose the following *communication constraints*: at time instant t, each sensor $i \in \mathcal{N}$ is able to measure the squared distance to not more than one target. In practice, due to certain constraints, the number of communication channels that can be used is usually less than the dimension of space or equal to it. Without loss of generality, in this paper, we assume that each sensor is able to collect data only from d neighbors. In this case and if there is no noise, we can use standard triangular approaches to determine the target position. However, if positions of all m targets need to be computed, then we have to simultaneously collect $m(d-1)$ measurements, and it is often impossible in practice. Third, we assume that there is the *unknown-but-bounded* noise involved in the measuring process, which is considered in the next subsection.

A. Measurements

Suppose sensor i estimates the state of target $\mathbf{r}_t^l$ at time instant t. The sensor is able to collect the distances to the same target measured by its neighbors $j \in \bar{\mathcal{N}}_t^i \subset \mathcal{N}^i$, $|\bar{\mathcal{N}}_t^i| = d$. Let $\mathbf{u}_t^i = [j_1, \ldots, j_d, l]^{\mathrm{T}}$, $j_1, \ldots, j_d \in \bar{\mathcal{N}}_t^i$, be a vector defining a set of neighbors used to collect measurements associated with target l at time instant t. Denote by

$$\bar{\rho}_t^j(\mathbf{u}_t^i) = \rho(\mathbf{s}_t^i, \mathbf{r}_t^{h(\mathbf{u}_t^i)}) - \rho(\mathbf{s}_t^j, \mathbf{r}_t^{h(\mathbf{u}_t^i)}) \; \forall j \in \bar{\mathcal{N}}_t^i, \tag{3}$$

a residual between a measurement of sensor i and its neighbors. Here and after, $h(\mathbf{u}_t^i) : \mathbb{R}^{d+1} \to \mathbb{R}$ gives the last element of $\mathbf{u}_t^i$.

In this case, using the square difference formula we get d equations

$$\bar{\rho}_t^j(\mathbf{u}_t^i) = (\mathbf{s}_t^j - \mathbf{s}_t^i)^{\mathrm{T}}(2\mathbf{r}_t^{h(\mathbf{u}_t^i)} - \mathbf{s}_t^j - \mathbf{s}_t^i), j \in \bar{\mathcal{N}}_t^i.$$

This allows us to derive

$$C_t^{\mathbf{u}_t^i} \mathbf{r}_t^{h(\mathbf{u}_t^i)} = D_t^{\mathbf{u}_t^i}, \quad \mathbf{r}_t^{h(\mathbf{u}_t^i)} = [C_t^{\mathbf{u}_t^i}]^{-1} D_t^{\mathbf{u}_t^i}, \tag{4}$$

where

$$C_t^{\mathbf{u}_t^i} = 2\begin{bmatrix}(\mathbf{s}_t^{j_1}-\mathbf{s}_t^i)^{\mathrm{T}}\\ \cdots \\ (\mathbf{s}_t^{j_d}-\mathbf{s}_t^i)^{\mathrm{T}}\end{bmatrix}, D_t^{\mathbf{u}_t^i} = \begin{bmatrix}\bar\rho_t^1(\mathbf{u}_t^i)+\|\mathbf{s}_t^{j_1}\|^2-\|\mathbf{s}_t^i\|^2\\ \cdots \\ \bar\rho_t^d(\mathbf{u}_t^i)+\|\mathbf{s}_t^{j_d}\|^2-\|\mathbf{s}_t^i\|^2\end{bmatrix}.$$

Using the introduced notations, we define the measurements of sensor $i \in \mathcal{N}$ at time instant t as follows:

$$y_t^i = F_t^i(\mathbf{u}_t^i, \mathbf{x}_t^i) + v_t^i = \|\hat{\mathbf{r}}_t^{h(\mathbf{u}_t^i)} - \mathbf{r}_t^{h(\mathbf{u}_t^i)}\|^2 + v_t^i, \quad (5)$$

where v_t^i is the unknown-but-bounded additive noise, $\mathbf{x}_t^i$ is the measurement point depending on currently available estimate $\hat{\mathbf{r}}_t^{h(\mathbf{u}_t^i)}$ at time instant t. For example, $\mathbf{x}_t^i = \hat{\mathbf{r}}_t^{h(\mathbf{u}_t^i)}$.

B. Distributed Optimization

Denote by $\mathcal{F}_{t-1}$ the σ-algebra of all probabilistic events, which happened up to time instant t. $\mathbb{E}_{\mathcal{F}_{t-1}}$ denotes the conditional expectation with respect to the σ-algebra $\mathcal{F}_{t-1}$. This σ-algebra is generated by the values of all random variables (i.e., position of targets, noise, changes in communication topology) at time instants $\tau = \{1, 2, \ldots, t\}$.

Let $\mathbf{u}_t = [\mathbf{u}_t^1, \ldots, \mathbf{u}_t^n]^{\mathbf{T}}$ be the common vector defining the sets of neighbors used to collect measurements from each sensor. The multi-sensor multi-target problem can be formulated as the following minimization problem: to find estimate $\hat\theta_t = \mathrm{col}\{\hat{\mathbf{r}}_t^{h(\mathbf{u}_t^1)}, \ldots, \hat{\mathbf{r}}_t^{h(\mathbf{u}_t^m)}\}$ that minimizes the following loss function

$$\bar{F}_t(\hat\theta_t, \mathbf{u}_t) = \mathbb{E}_{\mathcal{F}_{t-1}} \sum_{i\in\mathcal{N}} F_t^i(\mathbf{u}_t^i, \hat{\mathbf{r}}_t^{h(\mathbf{u}_t^i)}) \to \min_{\hat\theta_t}. \quad (6)$$

Usually, during optimization, each sensor fuses the needed information from all available neighboring nodes. In our problem setting, we mentioned the communication constraints that prohibit such communication strategy of the sensors. These communication constraints arise due to hardware and physical limitations since the bandwidths of communication channels is not unlimited. When a large number of sensors send and receive messages at the same time, communication becomes a bottleneck. To deal with this, we propose to choose communication links between sensors randomly. More formally, for each sensor $i \in \mathcal{N}$, we randomize the communication topology described by graph $\mathcal{G}_A$ at each time instant t to satisfy topology constraints such as the maximum number of links equals to d. We use a randomly chosen subgraph $\mathcal{G}_{B_t} \subset \mathcal{G}_A$ associated with adjacency matrix $B_t = [b_t^{i,j}]$, where the rows contain no more than d nonzero entries. Afterwards, the observable target at time instant t contained in $\mathbf{u}_t^i$ is generated from a uniform distribution independently for each sensor $i \in \mathcal{N}$ as in gossip algorithm [4]. We randomize the communication topology described by graph $\mathcal{G}_A$ based on the strategy similar to one presented in [27].

IV. WEIGHTED SPSA-BASED CONSENSUS ALGORITHM

Let $\mathbf{u}_k^i$ and $\mathbf{\Delta}_k^i \in \mathbb{R}^d$, $k = 1, 2, \ldots,$ $i \in \mathcal{N}$, be independent random variables. We generate $\mathbf{\Delta}_k^i$ called the *simultaneous test perturbation* from Bernoulli distribution with each component independently taking values $\pm\frac{1}{\sqrt{d}}$ with probabilities $\frac{1}{2}$. Let $\mathbf{e}_{h(\mathbf{u}_k^i)} \in \mathbb{R}^m$ be the sparse vector corresponding to the current target that sensor i observes, then $\hat\Delta_k^i = \mathbf{e}_{h(\mathbf{u}_k^i)} \otimes \mathbf{\Delta}_k^i$. In this case, $\hat\Delta_k^i$ is the vector of all zeros except for the rows that corresponds to $h(\mathbf{u}_k^i)$.

Let $\mathbb{U}^{i,l}$ be a set containing all possible subsets $\bar{\mathcal{N}}_t^i$ for target l. The neighborhood of sensor i at time instant t is defined by the i-th row of matrix B_t associated with graph $\mathcal{G}_{B_t}$. This row is defined by subset $\bar{\mathcal{N}}_t^i$ generated from the uniform distribution on the set $\mathbb{U}^{i,l}$.

Next, we introduce a weighted version of SPSA-based consensus algorithm. We define diagonal matrix $\Psi = [\psi_{ij}]$, where $\psi_{ij} > 0$ if $i = j$ and $\psi_{ij} = 0$ otherwise. At initialization step, for each $i \in \mathcal{N}$, we choose initial vector $\widehat\theta_0^i \in \mathbb{R}^{md}$, positive step-size α, matrix Ψ, gain coefficient γ, and the scale of perturbation $\beta > 0$.

In order to get estimates $\{\widehat\theta_t^i\}$ of overall state vectors $\{\theta_t^i\}$ based on measurement points $\{\mathbf{x}_t^i\}$, we propose to use the weighted algorithm with two measurements of distributed sub-functions $F_t^i(\mathbf{u}_t^i, \mathbf{x}_t^i)$:

$$\begin{cases}\mathbf{x}_{2k}^i = \widehat\theta_{2k-2}^i + \beta\hat\Delta_k^i,\ \mathbf{x}_{2k-1}^i = \widehat\theta_{2k-2}^i - \beta\hat\Delta_k^i,\\ \widehat\theta_{2k-1}^i = \widehat\theta_{2k-2}^i,\\ \widehat\theta_{2k}^i = \widehat\theta_{2k-1}^i - \alpha\Psi\Big[\hat\Delta_k^i \frac{y_{2k}^i - y_{2k-1}^i}{2\beta} + \\ \qquad\qquad \gamma\sum_{j\in\bar{\mathcal{N}}_{2k-1}^i} b_{2k-1}^{i,j}(\widehat\theta_{2k-1}^i - \widehat\theta_{2k-1}^j)\Big].\end{cases} \quad (7)$$

Consider the last equation of the algorithm (7): the first part is similar to SPSA-like algorithm from [17] and the second one coincides with Local Voting Protocol (LVP) from [28], where it was studied for stochastic networks in the context of load balancing problem. The SPSA part represents a stochastic gradient descent of sub-functions $F_t^i(\mathbf{u}_t^i, \mathbf{x}_t^i)$, and LVP part is determined for each agent i by the weighted sum of differences between the information about the current estimate $\widehat\theta_{2k-1}^i$ of agent i and available information about the estimates of its neighbors.

Further, we use notation $\bar\theta_t = \mathrm{col}\{\widehat\theta_t^1, \ldots, \widehat\theta_t^n\}$ for the common vector of estimates of all agents at time instant t. Also, we introduce the following:

$$\bar{\mathbf{y}}_t = \mathrm{col}\{y_t^1, \ldots, y_t^n\},$$

$$\bar\Delta_{t\div 2} = \mathrm{diag}_{nmd}(\mathrm{col}\{\hat\Delta_{t\div 2}^1, \ldots, \hat\Delta_{t\div 2}^n\}).$$

Using these notations, the algorithm (7) can be rewritten in the following form

$$\bar\theta_{2k} = \bar\theta_{2k-1} - \alpha\bar\Psi\left[\bar\Delta_k\left(\frac{\bar{\mathbf{y}}_{2k} - \bar{\mathbf{y}}_{2k-1}}{2\beta}\otimes\mathbf{1}_{md}\right) + \gamma(\mathcal{L}(B_{2k-1})\otimes I_{md})\bar\theta_{2k-1}\right]. \quad (8)$$

The algorithm (7) runs in parallel at each sensor to estimate $\widehat\theta_t$. This means that we have n parallel sequences of estimates. In the next section, we show that all these n sequences converge to the neighborhood of true vector θ_t.

V. Main Result

In this section, we provide a convergence analysis of the proposed algorithm. First, let us formulate assumptions about the dynamics of the targets, noise, and network topology.

Assumption 1: Let $\xi_t^l = \mathbf{r}_t^l - \mathbf{r}_{t-1}^l$, $l \in \mathcal{M}$, denotes the difference between two successive states of target l, and $\tilde{\xi}_t$ be the common vector containing such differences for all targets, i.e., $\tilde{\xi}_t = \mathrm{col}(\{\xi_t^1, \dots, \xi_t^m\}) \in \mathbb{R}^{md}$. We assume that
a) $\forall l \in \mathcal{M}$, the successive differences of states are bounded: $\|\xi_t^l\| \le \delta^l < \infty$ or, if sequence $\{\tilde{\xi}_t\}$ is random, $\mathbb{E}\|\xi_t^l\|^2 \le (\delta^l)^2$, $\mathbb{E}[\tilde{\xi}_t \tilde{\xi}_t^{\mathrm{T}}] \le Q_\delta$, $\mathbb{E}[\tilde{\xi}_t \tilde{\xi}_{t-1}^{\mathrm{T}}] \le Q_\delta$.
b) $\forall i \in \mathcal{N}$, $k = 1, 2, \dots$, matrices $C_{2k}^{\mathbf{u}_k^i}$, $C_{2k-1}^{\mathbf{u}_k^i}$ are invertible.

Assumption 2: For $k = 1, 2, \dots$, the successive differences $\tilde{v}_k^i = v_{2k}^i - v_{2k-1}^i$ of measurement noise are bounded: $|\tilde{v}_k^i| \le c_v < \infty$, or $\mathbb{E}(\tilde{v}_k^i)^2 \le c_v^2$ if sequence $\{\tilde{v}_t^i\}$ is random.

Assumption 3: For all $k = 1, 2, \dots,\ i \in \mathcal{N}, l \in \mathcal{M}$:
a) vectors $\mathbf{u}_k^i$, $\mathbf{\Delta}_k^i$, are mutually independent;
b) if $\mathbf{u}_k^i$, $\mathbf{\Delta}_k^i$, ξ_{2k-1}^l, ξ_{2k}^l, and $\mathbf{s}_{2k-1}^i$, $\mathbf{s}_{2k}^i$ are random, they do not depend on the σ-algebra $\mathcal{F}_{2k-2}$;
c) if $\xi_{2k-1}^l, \xi_{2k}^l, \tilde{v}_k^i$ are random, then random vectors $\mathbf{u}_k^i$, $\mathbf{\Delta}_k^i$, and elements $\mathbf{s}_{2k-1}^i$, $\mathbf{s}_{2k}^i$, $\xi_{2k-1}^l, \xi_{2k}^l, \tilde{v}_k^i$ are independent;
d) $\mathbb{E}\|\mathbf{\Delta}_k^i\|^2 \le \sigma_\Delta^2$, $\mathbb{E}[\mathbf{\Delta}_k^i (\mathbf{\Delta}_k^i)^{\mathrm{T}}] \le \sigma_\Delta^2 I_{md}$.

Assumption 4: a) For all $i \in \mathcal{N},\ j \in \bar{\mathcal{N}}_t^i$ weights $b_t^{i,j}$ are independent random variables with mean $\mathbb{E} b_t^{i,j} = b_{av}^{i,j}$, and $\mathbb{E}[(\mathcal{L}(B_t) - \mathcal{L}(B_{av}))(\mathcal{L}(B_t) - \mathcal{L}(B_{av}))^{\mathrm{T}}] \le Q_B$, where $B_{av} = [b_{av}^{i,j}]$.
b) Graph $\mathcal{G}_{B_{av}}$ is strongly connected.

Our analysis of the proposed algorithm applied to the problem (6) relies on the following definition.

Definition. A covariance matrix of residual has *an asymptotically efficient upper bound* $\Sigma > 0$ if $\exists \bar{k}$ such that $\forall k > \bar{k}$

$$\mathbb{E}[(\bar{\theta}_{2k} - \mathbf{1}_n \otimes \theta_{2k})(\bar{\theta}_{2k} - \mathbf{1}_n \otimes \theta_{2k})^{\mathrm{T}}] \le \Sigma + E_k,$$

where $E_k \to 0$.

The following theorem shows the asymptotically efficient upper bound of the covariance matrix of residual provided by the algorithm (7).

Theorem 1: If Assumptions *1–4* hold and $\alpha > 0$ *then* the covariance matrix of residual provided by the algorithm (7) has asymptotically efficient upper bound Σ, which is the solution of the following equation

$$\Sigma = c_1([I_{nmd} - \alpha C_6]\Sigma[I_{nmd} - \alpha C_6^{\mathrm{T}}]) - \alpha c_2 C_2 \Sigma C_2^{\mathrm{T}} + \alpha^2 \gamma^2 \bar{\Psi} \Sigma (Q_B \otimes I_{md}) \bar{\Psi}^{\mathrm{T}} + \alpha^2 C_3 + \alpha C_4, \quad (9)$$

where $c_1 = 1 - \varepsilon_1 - \alpha \sigma_\Delta^2 \varepsilon_2, c_2 = \alpha \frac{4\sigma_\Delta^4}{c_1} + 2\sigma_\Delta^2 \varepsilon_3, C_1 = \gamma \bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md}), C_2 = \bar{\Psi} \bar{R}_k, C_3 = \frac{1}{\beta}\sigma_\Delta^2[(4\beta^2\sigma_\Delta^2 + \delta^2)(4\nu_{k-1}^{\mathrm{T}} R_k \nu_{k-1} - 12\delta e \nu_{k-1} + 9\delta^2) - 2c_v\delta(2e\nu_{k-1} - 3\delta) + c_v^2] I_{nmd}, C_4 = \sigma_\Delta^2[(-\frac{1}{\varepsilon_1} + 1)\bar{\Psi}(\mathrm{diag}_{nmd}(e)(9\mathbb{J}_n \otimes Q_\delta)\mathrm{diag}_{nmd}(e)\bar{\Psi}^{\mathrm{T}} + (-2\frac{1}{\varepsilon_2} + 1)(4\mathbb{J}_n \otimes Q_\delta)], C_5 = (1 - \frac{1}{\varepsilon_3})4\mathbb{J}_n \otimes Q_\delta, C_6 = C_1 + \frac{2\sigma_\Delta^2}{c_1} C_2$.

See the proof of Theorem 1 in Appendix.

VI. Simulation

In this section, we present a numerical experiment, which illustrates the performance of the suggested algorithm (7).

Given a distributed network of 3 sensors monitoring an area of interest. Let $\mathcal{N} = \{1, 2, 3\}$ be the set of sensors. Each sensor has no more than two active communication channels at each time instant, i.e., $|\bar{\mathcal{N}}_t^i| = 2$. The communication channels are used to collect data from the neighbors. Within the area of interest, there are 6 moving targets. The sensors have to estimate their states. At time instant t, $\mathbf{s}_t^i = [s_t^{i,1}, s_t^{i,2}]^{\mathrm{T}} \in \mathbb{R}^2$ is the current state of sensor $i \in \mathcal{N}$, $\mathbf{r}_t^l = [r_t^{l,1}, r_t^{l,2}]^{\mathrm{T}} \in \mathbb{R}^2$ is the state of target $l \in \mathcal{M} = \{1, 2, \dots, 6\}$, $\theta_t = \mathrm{col}\{\mathbf{r}_t^1, \dots, \mathbf{r}_t^6\}$ is the common state of all targets.

We consider three types of noise: uniformly distributed random variable falling within the interval $[-1; 1]$, an unknown constant, and hybrid noise which is uniformly distributed around constants that change with time, e.g. $v_k^i = \pm 1 + 0.1 * sin(k)$, where the sign in front of 1 switches each 50-th iteration. In the simulation presented in the paper, we provide the estimations that are common for each type of noise.

The algorithm (7) working on each node has the following parameters: $\alpha = 0.05$, $\beta = 0.1$, $\gamma = 1.0$. The targets start their motion at the position consisting of randomly chosen components from the interval $[0; 100]$. Dynamics of the targets motion is as follows: $\mathbf{r}_t^l = \mathbf{r}_{t-1}^l + \chi_{t-1}^l$. Let χ_{t-1}^l be a random vector uniformly distributed on the ball of radius equal to 0.2 for targets with odd numbers and 0.6 for targets with even numbers. This means that some targets possess different from other targets behaviour and complicates the suggested simulation in comparison with [23]. The sensors don't move and their coordinates are random values uniformly distributed in interval $[100; 120]$.

Let us consider for every target l and sensor i at each time instant t the covariance matrix of residuals $\tilde{\Sigma}_t^{i,l} \in \mathbb{R}^{d \times d}$, which is represented as a part of the common covariance matrix.

In this simulation, the new algorithm is compared with the previous one from [23]. In order to illustrate how matrix Ψ influences the convergence of the algorithm, we consider two different cases:
$\Psi^* = \mathrm{diag}_6(\mathrm{col}\{2, 10, 2, 10, 2, 10\})$,
$\Psi^{**} = \mathrm{diag}_6(\mathrm{col}\{0.2, 100, 0.2, 100, 0.2, 100\})$.

Fig. **??** show how the average last diagonal entry of the covariance matrix of residuals $\tilde{\Sigma}_t^{i,l}$ evolve over time. It is well seen that the new algorithm with $\Psi = \Psi^*$ converges faster than the previous algorithm. However, the convergence rate depends on Ψ. The choice of optimal parameters will be done in the future works.

VII. Conclusion

In this paper, we studied and estimated the covariance matrix of residual provided by the weighted version of the combined algorithm of Simultaneous Perturbation Stochastic Approximation and the consensus algorithm. We also applied the algorithm on the multisensor-multitarget problem and validated it through simulation. It is well seen that this new algorithm converges faster than the one from our previous works with targets possessing different behaviours. In future

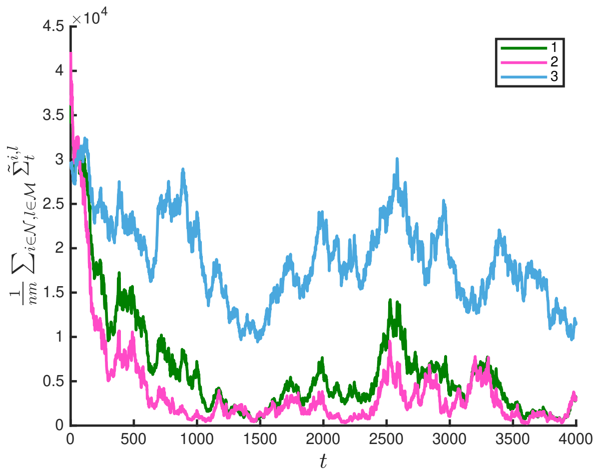

Fig. 1. The average value over all sensors and targets of the first entry of covariance matrix of residuals $\tilde{\Sigma}_t^{i,l}$, where (1) is the previous algorithm with $\Psi = I_m$, (2) and (3) are the new algorithms with Ψ equals Ψ^* and Ψ^{**} correspondingly

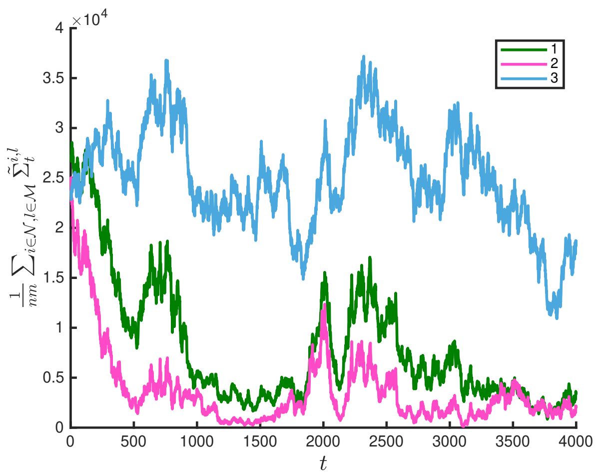

Fig. 2. The average value over all sensors and targets of the last entry of covariance matrix of residuals $\tilde{\Sigma}_t^{i,l}$, where (1) is the previous algorithm with $\Psi = I_m$, (2) and (3) are the new algorithms with Ψ equals Ψ^* and Ψ^{**} correspondingly

works, we are planning to optimize the step-size of the algorithm as it was made in [29].

APPENDIX

The proof of Theorem 1:

Denote $\mathbf{d}_t^i = \widehat{\theta}^i_{2\lceil \frac{t-1}{2} \rceil} - \theta_t$, $\bar{\mathbf{d}}_t = \mathrm{col}\{\mathbf{d}_t^1, \ldots, \mathbf{d}_t^n\}$, where $\lceil \cdot \rceil$ is a ceiling function, $\nu_k = \bar{\mathbf{d}}_{2k}$, $\Sigma_k = \mathbb{E}[\nu_k \nu_k^{\mathrm{T}}]$, $\bar{\mathbf{s}}_k = \frac{\alpha}{2\beta} \bar{\Delta}_k ((\bar{\mathbf{y}}_{2k} - \bar{\mathbf{y}}_{2k-1}) \otimes \mathbf{1}_{md})$, $\bar{\mathbf{v}}_t = \mathrm{col}\{\tilde{v}_t^1, \ldots, \tilde{v}_t^n\}$, $\bar{\mathbf{u}}_{t \div 2} = \mathrm{col}\{\mathbf{u}^1_{t \div 2}, \ldots, \mathbf{u}^n_{t \div 2}\}$, $\bar{\Psi} = I_n \otimes \Psi$.

Let $\bar{\mathcal{F}}_{k-1} = \sigma\{\mathcal{F}_{k-1}, \bar{\mathbf{v}}_{2k-1}, \bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}, \bar{\mathbf{u}}_k, \bar{\Delta}_k\}$ be the σ-algebra of probabilistic events generated by $\mathcal{F}_{k-1}$, $\bar{\mathbf{v}}_{2k-1}$, $\bar{\mathbf{v}}_{2k}$, ξ_{2k-1}, ξ_{2k}, $\bar{\mathbf{u}}_k$, $\bar{\Delta}_k$, $\tilde{\mathcal{F}}_{k-1} = \sigma\{\mathcal{F}_{k-1}, \bar{\mathbf{v}}_{2k-1}, \bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}, \bar{\mathbf{u}}_k\}$, and $\hat{\mathcal{F}}_{k-1} = \sigma\{\mathcal{F}_{k-1}, \bar{\mathbf{v}}_{2k-1}, \bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}\}$: $\mathcal{F}_{k-1} \subset \hat{\mathcal{F}}_{k-1} \subset \tilde{\mathcal{F}}_{k-1} \subset \bar{\mathcal{F}}_{k-1} \subset \mathcal{F}_k$.

Using that $\bar{\theta}_{2k-1} = \bar{\theta}_{2k-2}$ and $\mathcal{L}(B_{2k-2})\mathbf{1}_n = 0$, we get

$$\nu_k = \bar{\theta}_{2k} - \mathbf{1}_n \otimes \theta_{2k} = \\ \bar{\mathbf{g}}_k - \bar{\Psi}\bar{\mathbf{s}}_k - \alpha\gamma\bar{\Psi}[(\mathcal{L}(B_{2k-2}) - \mathcal{L}(B_{av})) \otimes I_{md}]\nu_{k-1},$$

where $\bar{\mathbf{g}}_k = [I_{nmd} - \alpha\gamma\bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md})]\nu_{k-1} + \mathbf{1}_n \otimes (\theta_{2k-2} - \theta_{2k})$. Then,

$$D_k = \nu_k \nu_k^{\mathrm{T}} = \bar{\mathbf{g}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} - \bar{\mathbf{g}}_k \bar{\mathbf{s}}_k^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}} - \bar{\Psi}\bar{\mathbf{s}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} + \bar{\Psi}\bar{\mathbf{s}}_k \bar{\mathbf{s}}_k^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}} - \\ \alpha\gamma(\bar{\mathbf{g}}_k - \bar{\Psi}\bar{\mathbf{s}}_k)[(\mathcal{L}(B_{2k-2}) - \mathcal{L}(B_{av})) \otimes I_{md}]^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}} - \\ \alpha\gamma\bar{\Psi}[(\mathcal{L}(B_{2k-2}) - \mathcal{L}(B_{av})) \otimes I_{md}]\nu_{k-1}(\bar{\mathbf{g}}_k^{\mathrm{T}} - \bar{\mathbf{s}}_k^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}}) + \\ \alpha^2\gamma^2\bar{\Psi}[(\mathcal{L}(B_{2k-2}) - \mathcal{L}(B_{av})) \otimes I_{md}]D_{k-1}[(\mathcal{L}(B_{2k-2}) - \\ \mathcal{L}(B_{av})) \otimes I_{md}]^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}}.$$

Now, we take the conditional expectation over σ-algebra $\bar{\mathcal{F}}_{k-1}$ and apply Assumption *4*:

$$\mathbb{E}_{\bar{\mathcal{F}}_{k-1}}[D_k] \le \bar{\mathbf{g}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} - \bar{\mathbf{g}}_k \bar{\mathbf{s}}_k^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}} - \bar{\Psi}\bar{\mathbf{s}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} + \bar{\Psi}\bar{\mathbf{s}}_k \bar{\mathbf{s}}_k^{\mathrm{T}} \bar{\Psi}^{\mathrm{T}} + \\ \alpha^2\gamma^2 b_{\max} \bar{\Psi} D_{k-1} \bar{\Psi}^{\mathrm{T}}, \qquad (10)$$

where $b_{\max}$ is the maximum element of Q_{B}.

After we take the conditional expectation over σ-algebra $\tilde{\mathcal{F}}_{k-1}$ step by step:

$$\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[D_k] \le \bar{\mathbf{g}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} - \bar{\mathbf{g}}_k \mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k^{\mathrm{T}}] \bar{\Psi}^{\mathrm{T}} - \bar{\Psi}\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k] \bar{\mathbf{g}}_k^{\mathrm{T}} + \\ \bar{\Psi}\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k \bar{\mathbf{s}}_k^{\mathrm{T}}] \bar{\Psi}^{\mathrm{T}} + \alpha^2\gamma^2 b_{\max} \bar{\Psi} D_{k-1} \bar{\Psi}^{\mathrm{T}}. \qquad (11)$$

Under Assumption *4b*, we have $\bar{\lambda}_2 = Re(\lambda_2(\mathcal{L}(B_{av}))) > 0$ and $\bar{\lambda}_{\max} = Re(\lambda_{\max}(\mathcal{L}(B_{av}))) > 0$ (see [30]). Hence, for the first term in (11) and using Assumption *1a* we derive

$$\bar{\mathbf{g}}_k \bar{\mathbf{g}}_k^{\mathrm{T}} \le (1 - \varepsilon_3)([I_{nmd} - \alpha\gamma\bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md})]\Sigma_{k-1} \cdot \\ \cdot [I_{nmd} - \alpha\gamma\bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md})]^{\mathrm{T}}) + (1 - \frac{1}{\varepsilon_3})4\mathbb{J}_n \otimes Q_\delta.$$

Denote $\mathbf{r}_t^i = \mathbf{e}_{h(\mathbf{u}^i_{t \div 2})} \otimes [C_t^{\mathbf{u}^i_{t \div 2}}]^{-1} D_t^{\mathbf{u}^i_{t \div 2}}$, $\widehat{\mathbf{r}}_t^i = \mathrm{diag}_{md}(\mathbf{e}_{h(\mathbf{u}^i_{t \div 2})} \otimes I_d)\widehat{\theta}_t^i$, and $\Xi_t^i = \mathbf{r}_t^i - \mathbf{r}_{t-1}^i$, then $\forall i \in \{1, \ldots, n\}$:

$$y_{2k}^i - y_{2k-1}^i = (2\beta\hat{\Delta}_k^i - \Xi_{2k}^i)^{\mathrm{T}}(2\mathrm{diag}_{md}(\mathbf{e}_{h(\mathbf{u}_k^i)} \otimes I_d)\mathbf{d}_{2k-2}^i \\ -2\Xi_{2k-1}^i - \Xi_{2k}^i) + v_{2k}^i - v_{2k-1}^i.$$

Under Assumption *3d*, multiplying by $\hat{\Delta}_k^i$ and taking the conditional expectation over σ-algebra $\tilde{\mathcal{F}}_{k-1}$, we get

$$\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[(y_{2k}^i - y_{2k-1}^i)\hat{\Delta}_k^i] = 2\beta\sigma_\Delta^2 \mathbf{q}_{2k-2}^i,$$

where $\mathbf{q}_{2k-2}^i = 2\mathrm{diag}_{md}(\mathbf{e}_{h(\mathbf{u}_k^i)} \otimes I_d)\mathbf{d}_{2k-2}^i - 2\Xi_{2k-1}^i - \Xi_{2k}^i$.

Here and after, $\sigma_\Delta = \frac{1}{\sqrt{d}}$. Denote $\bar{R}_t = \mathrm{diag}_{nmd}(\mathrm{col}\{\mathbf{e}_{h(\mathbf{u}^1_{t \div 2})} \otimes I_d, \ldots, \mathbf{e}_{h(\mathbf{u}^n_{t \div 2})} \otimes I_d\})$, $\bar{\Xi}_t = \mathrm{col}\{\Xi_t^1, \ldots, \Xi_t^n\}$, $e = \mathrm{col}\{e_{h(u_t^1)}, \ldots, e_{h(u_t^n)}\}$. By Assumption *1a*, taking the conditional expectation over σ-algebra $\hat{\mathcal{F}}_{k-1}$ and using $AB^{\mathrm{T}} + BA^{\mathrm{T}} \le AA^{\mathrm{T}} + BB^{\mathrm{T}}$

we derive the following for the second and third terms:

$$-\bar{\mathbf{g}}_k \mathbb{E}_{\hat{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k^{\mathrm{T}}]\bar{\Psi}^{\mathrm{T}} - \bar{\Psi}\mathbb{E}_{\hat{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k]\bar{\mathbf{g}}_k^{\mathrm{T}} \leq$$
$$\alpha\sigma_\Delta^2[-2(I_{nmd} - 2\alpha\gamma\bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md}))\Sigma_{k-1}\bar{R}_k\bar{\Psi}^{\mathrm{T}} -$$
$$2\bar{\Psi}\bar{R}_k\Sigma_{k-1}(I_{nmd} - 2\alpha\gamma(\mathcal{L}(B_{av})^{\mathrm{T}} \otimes I_{md})\bar{\Psi}^{\mathrm{T}}) -$$
$$\varepsilon_1(I_{nmd} - 2\alpha\gamma\bar{\Psi}(\mathcal{L}(B_{av}) \otimes I_{md}))\Sigma_{k-1}(I_{nmd} -$$
$$2\alpha\gamma(\mathcal{L}(B_{av})^{\mathrm{T}} \otimes I_{md})\bar{\Psi}^{\mathrm{T}}) + (-\frac{1}{\varepsilon_1} + 1)\bar{\Psi}(\mathrm{diag}_{nmd}(e)$$
$$(9\mathbb{J}_n \otimes Q_\delta)\mathrm{diag}_{nmd}(e)\bar{\Psi}^{\mathrm{T}} -$$
$$2\varepsilon_2\bar{\Psi}\bar{R}_k\Sigma_{k-1}^{\mathrm{T}}\bar{R}_k\bar{\Psi}^{\mathrm{T}} + (-2\frac{1}{\varepsilon_2} + 1)(4\mathbb{J}_n \otimes Q_\delta)].$$

By Assumption *3* and *1a*, using that Δ_k^i is drawn from the symmetric distribution, for the fourth term in (11), we successively take the the conditional expectation over σ-algebra $\tilde{\mathcal{F}}_{k-1}$ and after that over σ-algebra $\hat{\mathcal{F}}_{k-1}$. In the result, we obtain

$$\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}[\bar{\mathbf{s}}_k\bar{\mathbf{s}}_k^{\mathrm{T}}] \leq \frac{\alpha^2}{\beta}\sigma_\Delta^2[(4\beta^2\sigma_\Delta^2 + \delta^2)(4\nu_{k-1}^{\mathrm{T}}R_k\nu_{k-1} -$$
$$12\delta e\nu_{k-1} + 9\delta^2) - 2c_v\delta(2e\nu_{k-1} - 3\delta) + c_v^2]I_{nmd}.$$

Summing up the bounds and taking the unconditional expectation, we derive the following

$$\Sigma_k \leq c_1([I_{nmd} - \alpha C_6]\Sigma_{k-1}[I_{nmd} - \alpha C_6^{\mathrm{T}}]) -$$
$$\alpha c_2 C_2\Sigma_{k-1}C_2^{\mathrm{T}} + \alpha^2\gamma^2 b_{\max}\bar{\Psi}\Sigma_{k-1}\bar{\Psi}^{\mathrm{T}} +$$
$$\alpha^2 C_3 + \alpha C_4 + C_5.$$

Note that the constants are defined in the theorem.

Let us make the substitution: $\Sigma_k = \Sigma + E_k$, where Σ is the solution of (9).

Consider

$$E_k \leq c_1([I_{nmd} - \alpha C_6]E_{k-1}[I_{nmd} - \alpha C_6^{\mathrm{T}}]) -$$
$$\alpha c_2 C_2 E_{k-1}C_2^{\mathrm{T}} + \alpha^2\gamma^2 b_{\max}\bar{\Psi}E_{k-1}\bar{\Psi}^{\mathrm{T}} + C_5.$$

According to Theorem from [24] $E_k \to 0$. This completes the proof of Theorem 1.

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