Compressed stochastic macromodeling of electrical systems via rational polynomial chaos and principal component analysis

Compressed Stochastic Macromodeling of Electrical Systems via Rational Polynomial Chaos and Principal Component Analysis

Paolo Manfredi and Stefano Grivet-Talocia
Department of Electronics and Telecommunications
Politecnico di Torino, Turin, Italy 10129
Email: paolo.manfredi@polito.it

***Abstract*—This paper introduces a compression strategy to speed-up the calculation of frequency-domain stochastic models based on rational polynomial chaos expansions. Principal component analysis is used to remove redundancy in the data, thus leading to a considerable reduction in the number of model coefficients to estimate. Compared to the state-of-the-art techniques, the proposed solution turns out to be a good trade-off between accuracy and processing efficiency. As a validation, the method is applied to the uncertainty quantification of the scattering responses of a nine-port distributed network.**

***Index Terms*—Multiport systems, polynomial chaos, principal component analysis, rational modeling, variability analysis, uncertainty quantification.**

I. Introduction

The first-time-right design of electrical and electronic products often involves uncertainty quantification to assess the impact of manufacturing tolerances, as well as of free or uncontrollable design parameters. In this field, various methods based on polynomial chaos expansion (PCE) [1] have recently emerged as a powerful and efficient alternative to traditional Monte Carlo-like techniques [2], [3].

The PCE is basically a surrogate model in which stochastic outputs are represented in terms of suitable orthogonal polynomials, which are functions of the uncertain parameters. While conventional approaches make use of linear PCE models [4], a *rational* PCE model was recently proposed as a more suitable alternative for the frequency-domain characterization of stochastic linear systems [5]. Indeed, a rational model turns out to be more accurate for distributed circuits, especially at high frequency, and even exact for lumped ones. Nevertheless, compared to the conventional linear PCE, it suffers from a reduced model-building efficiency that is exacerbated when the number of ports and frequency points is large.

This paper aims at mitigating this issue by introducing a compression strategy based on principal component analysis (PCA) [6]. The PCA allows removing the redundancy in the model data, thus leading to a large reduction of the model coefficients to be estimated. The advocated approach is applied to the uncertainty quantification of the scattering parameters of a nine-port distributed circuit.

II. Conventional and Rational PCEs

Consider a stochastic, linear, P-port electrical system affected by d uncertain parameters defined by vector $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_d)$. According to conventional approaches (see, e.g., [4]), any frequency-domain port (e.g., scattering) representation is approximated with the following PCE:

$$\mathbf{S}(s, \boldsymbol{\xi}) \approx \sum_{\ell=1}^{L} \mathbf{S}_\ell(s) \varphi_\ell(\boldsymbol{\xi}), \tag{1}$$

where the basis functions φ_ℓ are a set of orthogonal polynomials w.r.t. the joint probability density function (PDF) of the uncertain parameters $\boldsymbol{\xi}$, whereas the model coefficients $\mathbf{S}_\ell$ are computed by means of linear regression based on a set of random responses ("training samples"). The number of expansion terms L depends on the truncation strategy [3], [5].

The model (1) is approximate also for lumped circuits, and rapidly becomes inaccurate for distributed and/or strongly resonant systems. For this reason, a rational model of the form

$$\mathbf{S}(s, \boldsymbol{\xi}) \approx \frac{\sum_{\ell=1}^{L} \mathbf{N}_\ell(s) \varphi_\ell(\boldsymbol{\xi})}{1 + \sum_{\ell=2}^{L} \mathbf{D}_\ell(s) \varphi_\ell(\boldsymbol{\xi})} \tag{2}$$

was put forward, and proven to be exact for lumped circuits (with a suitable choice of the basis functions) and much more accurate for distributed ones [5]. The model coefficients $\mathbf{N}_\ell$ and $\mathbf{D}_\ell$ are computed via an iterative re-weighted linearized regression. Unfortunately, this method requires to solve the regression problem separately for each matrix entry and frequency of interest, thus rapidly becoming inefficient for large datasets.

III. PCA Compression

To alleviate the aforementioned burden, PCA is here exploited, leveraging on its intrinsic compression properties. The following derivations are inspired by the method documented in [7], [8] (details on this are deferred to a future report).

Assume a set of K random samples of the $P \times P$ network response $\mathbf{S}$ are available at M frequency points. First of all, this $P \times P \times M \times K$ dataset is reorganized into a matrix $\boldsymbol{\mathcal{B}} \in \mathbb{C}^{MP^2 \times K}$. Next, an "economy-size" singular value decomposition (SVD) of the zero-mean matrix $\widetilde{\boldsymbol{\mathcal{B}}} = \boldsymbol{\mathcal{B}} - \boldsymbol{\mu}$,

where $\boldsymbol{\mu}$ is the mean value of $\boldsymbol{\mathcal{B}}$, calculated rowwise and subtracted columnwise, is computed:

$$\widetilde{\boldsymbol{\mathcal{B}}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{H}} \tag{3}$$

(the superscript $^{\mathsf{H}}$ stands for the complex conjugate transpose).

In (3), $\boldsymbol{\Sigma}$ is a diagonal matrix containing the singular values $\{\sigma_i\}_{i=1}^{K}$ of $\widetilde{\boldsymbol{\mathcal{B}}}$, in descending order. By selecting $\bar{n}$ singular values, such that $\sigma_i/\sigma_1 \leq \epsilon$, $\forall i > \bar{n}$, a truncation of (3) is defined:

$$\widetilde{\boldsymbol{\mathcal{B}}} \approx \widehat{\boldsymbol{\mathcal{B}}} = \widehat{\mathbf{U}}\widehat{\boldsymbol{\Sigma}}\widehat{\mathbf{V}}^{\mathsf{H}} = \widehat{\mathbf{U}}\widehat{\boldsymbol{\mathcal{Z}}} \tag{4}$$

where $\widehat{\mathbf{U}}$ and $\widehat{\mathbf{V}}$ are matrices consisting of only the first $\bar{n}$ columns of $\mathbf{U}$ and $\mathbf{V}$ in (3), and $\widehat{\boldsymbol{\Sigma}}$ is a matrix corresponding to the $\bar{n} \times \bar{n}$ upper-left sub-block of $\boldsymbol{\Sigma}$.

It is important to note that, in (4), $\widehat{\boldsymbol{\mathcal{Z}}} \in \mathbb{C}^{\bar{n}\times K}$, and that $\bar{n} < K \ll MP^2$. Such a matrix, much smaller than $\widetilde{\boldsymbol{\mathcal{B}}}$, can be interpreted as a reduced dataset collecting K samples of $\bar{n}$ stochastic variables $\{Z_i\}_{i=1}^{\bar{n}}$ that depend, just like the original data, on the uncertain parameters $\boldsymbol{\xi}$. These stochastic variables are also modeled using the rational PCE (2). The fundamental difference, however, is that now only $\bar{n}$ models have to computed, instead of MP^2. See also [9]. Once a model like (2) is available for the compressed variables, it can be used to inexpensively generate new data samples thereof. Corresponding samples for the original variables are recovered as $\widehat{\boldsymbol{\mathcal{B}}}+\boldsymbol{\mu}$ through (4). It should be noted that the approximation error is controlled by the threshold ϵ for the PCA-cut.

IV. Numerical Results

As a validation example, the nine-port distributed network investigated in [5] is considered, which includes three coupled microstrip lines and $d = 2$ uncertain parameters (namely, the microstrip trace gap and length, with Gaussian distribution).

As a reference, 10000 samples of the pertinent 9×9 S-parameter matrix are evaluated at 401 frequency points by means of HSPICE simulations, using a Latin hypercube design. For all the PCE-based models, *total degree* expansions of order $p = 3$ are used[1], leading to $L = 10$. Another set of $K = 76$ samples is generated for training the PCE models, drawn again with a Latin hypercube design. Figure 1 shows the plot of the first 40 normalized singular values of the zero-mean training dataset $\widetilde{\boldsymbol{\mathcal{B}}}$. By choosing a threshold of $\epsilon = 10^{-2}$ for the PCA-cut, indicated by the blue line, there are $\bar{n} = 8$ singular values above it, shown by green dots.

Figure 2 shows the variability of a meaningful selection of S-parameters, either in magnitude or phase. The gray lines are a subset of 100 samples from the reference data, providing a visual idea of the variability of the scattering responses due to the uncertainty in the microstrip gap and length. The solid blue lines are the mean and standard deviation of the reference samples. The same statistical information is also estimated using the conventional single PCE model (1) (dashed red lines), the standard rational PCE model (2) (dash-dotted green lines), and the proposed PCA-compressed rational PCE model (dotted orange lines).

[1]The Reader is referred to [5] for further details on the truncation schemes.

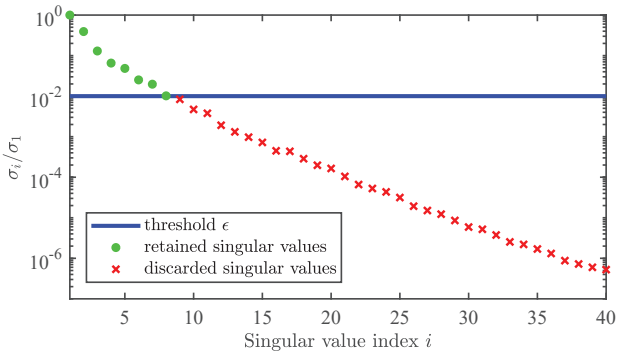

Fig. 1. Singular values of the zero-mean training dataset $\widetilde{\boldsymbol{\mathcal{B}}}$. Singular values that are above and below the selected threshold for the PCA-cut ($\epsilon = 10^{-2}$, blue line) are shown by green $\circ$ and red $\times$ markers, respectively.

As already observed in [5], the rational models exhibit superior accuracy compared to the single PCE, especially at high frequency. Nonetheless, it is interesting to note that, while generally providing very high accuracy, the standard rational model exhibits large local errors at some of the frequencies. This is probably due to a convergence issue of the iterative re-weighted regression. On the contrary, the PCA-compressed rational model is very accurate over the entire frequency range.

Figure 3 shows the error on the standard deviation of the S-parameters shown in Fig. 2. It is observed that the error of the PCA-compressed model is much lower than the error of the single PCE, and comparable to that of the standard rational model, but it is distributed more evenly over frequency. This is expected, as the model is computed for the reduced variables, rather than frequency by frequency.

Finally, Table I collects the relevant information about the accuracy and the efficiency of the various PCE-based techniques. The accuracy in reproducing the reference samples is first assessed by calculating the root-mean-square (RMS) error over all the 10000 random samples, as well as over frequency and matrix entries (port variables). The maximum error on the standard deviation over frequency is also considered. It is noted that the RMS accuracy of the proposed PCA-compressed method is much higher than that of the conventional single PCE, yet lower compared to the standard rational model. Nevertheless, the processing time, inclusive of the SVD as well as of the construction and iterative solution of the regression problem, is 21.8× faster than for the conventional rational model, and even slightly faster than for the single PCE model. Moreover, the PCA-based model exhibits the lowest maximum error on the standard deviation since, unlike the standard rational model, it does not exhibits error spikes. It is important to mention that no significant difference is found by using $\epsilon = 10^{-3}$ in the SVD truncation. Therefore, the main contribution to the RMS error increase, compared to [5], is to be ascribed to a lower accuracy in modeling the reduced variables Z_i for a given expansion order.

V. Conclusions

This paper presented a PCA-based compression strategy to speed-up the calculation of rational PCE models for the frequency-domain responses of stochastic linear circuits. The

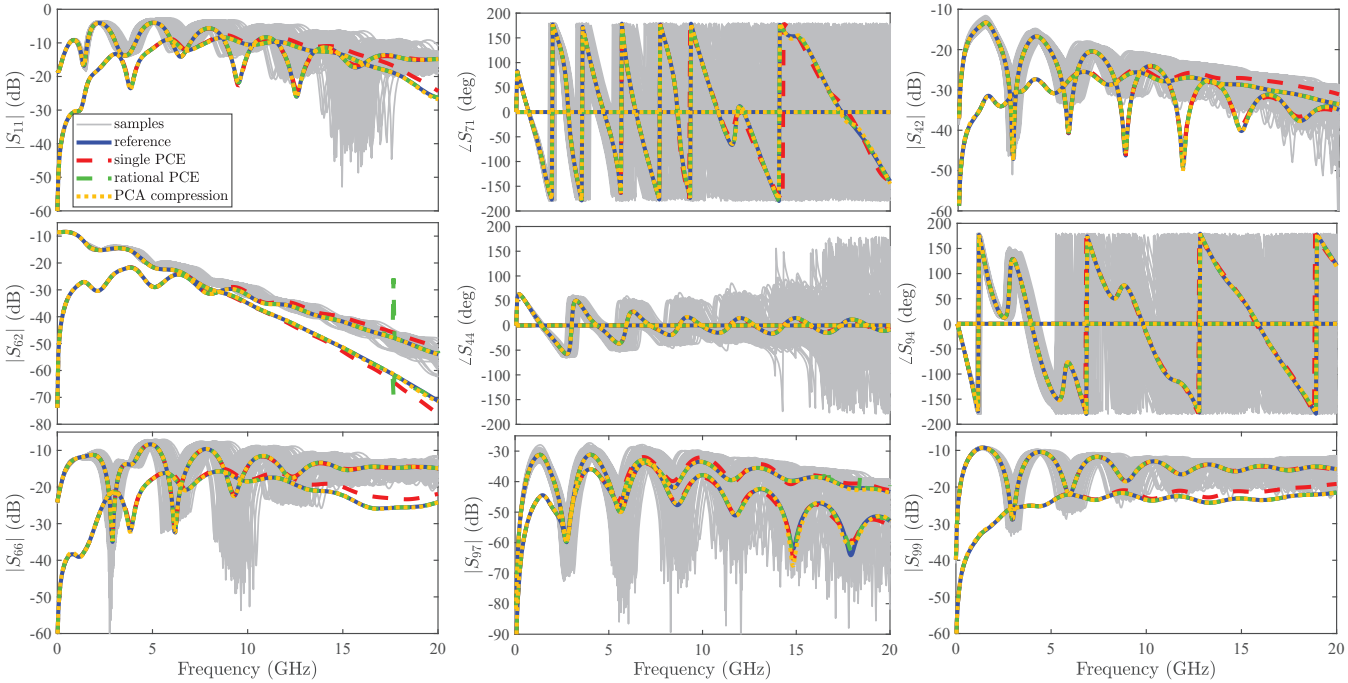

Fig. 2. Variability of some of the S-parameters for the considered test case. Gray lines: subset of reference samples; blue lines: mean and standard deviation of the reference samples; dashed red, dash-dotted green, and dotted orange lines: mean and standard deviation obtained with the conventional single PCE [4], with the rational PCE [5], and with the proposed PCA-compressed rational PCE, respectively.

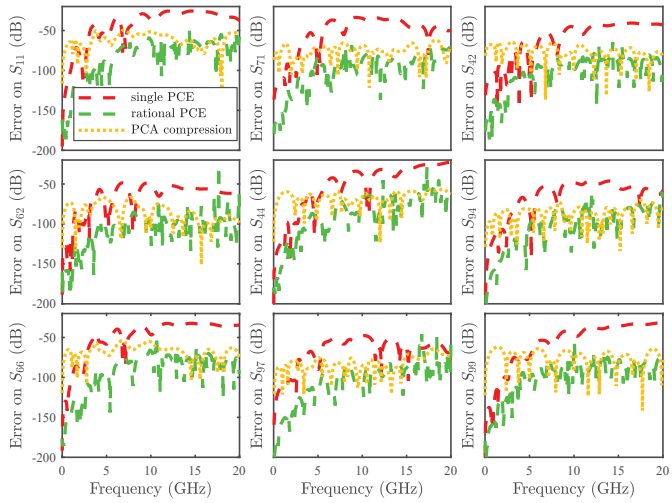

Fig. 3. Error on the standard deviation of the S-parameters in Fig. 2 for the considered PCE-based methods.

advocated method reduces the modeling to a much smaller set of stochastic variables, corresponding to the most significant singular values of the original data. Compared to the state-of-the-art rational model approach, the proposed method offers a trade-off between accuracy and processing efficiency for large datasets in terms of number of ports and frequency points. The techniques has been validated based on a nine-port network with distributed elements.

TABLE I
PERFORMANCE OF THE VARIOUS PCE-BASED TECHNIQUES IN TERMS OF ACCURACY AND MODEL CALCULATION TIME.

Method	RMS error	MAX error on std	Time
single PCE [4]	0.0718	0.0279	4.05 s
rational PCE [5]	0.0074	0.0165	88.38 s
PCA-compressed rational PCE (proposed)	0.0131	0.0044	3.69 s

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