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Uncertainty Quantification of Cable Inductances and Capacitances via Mixed-Fidelity Models

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***Abstract*—In this paper, we investigate a mixed-fidelity approach for the uncertainty quantification of the per-unit-length (p.u.l.) capacitance and inductance of cables with random geometrical and material parameters. Polynomial chaos expansion is used to model uncertainty, whereas a numerical discretization technique is used to calculate p.u.l. inductances and capacitances. However, instead of using a model with high fidelity in both features, the results are obtained as a combination of two complementary models with mixed fidelity in each feature. Numerical examples concerning the statistical assessment of the p.u.l. inductance and capacitance matrices of two shielded cables show that similar accuracy is attained at a fraction of the computational cost compared to conventional approaches.**

***Index Terms*—Cables, multiconductor transmission lines, polynomial chaos, tolerance analysis, uncertainty quantification.**

I. Introduction

Uncertainty is becoming ubiquitous in electromagnetic compatibility (EMC) and signal integrity (SI) investigations. In this framework, uncertainty quantification is an increasingly important task in the early design phase. As opposed to traditional Monte Carlo techniques, polynomial chaos expansion (PCE) [1] recently gain wide consideration in engineering thanks to its remarkable accuracy and efficiency. The method consists in expanding stochastic quantities of interest into series of suitable orthogonal polynomials. This approach yields a convenient surrogate model from which statistical and sensitivity information is readily extracted [2].

This work focuses in particular on the statistical assessment of the per-unit-length (p.u.l.) inductance and capacitance of cables subject to uncertain geometrical and material parameters, which is often a preliminary step in stochastic EMC/SI analysis [2]. Some efficient numerical approaches were proposed based on PCE [3], [4]. In these methods, there are two key parameters that control the accuracy of the result: the number of PCE terms and the discretization of the charge distributions around metallic and dielectric interfaces. Setting both features to a sufficiently high value is an obvious option.

Nonetheless, inspired by some recent mathematical developments [5] and by a "predictor-corrector" algorithm for carbon nanotube interconnects [6], in this paper we propose a "mixed-fidelity" approach: instead of performing an expensive calculation of a model that has high fidelity both in terms of PCE approximation and geometrical discretization, we combine two models, one with high probabilistic fidelity but low geometrical fidelity, and vice versa. Since the calculation of this mixed-fidelity model is overall cheaper than evaluating the high-fidelity model, the proposed approach provides further computational speed-up with respect to state-of-the-art techniques. This is illustrated through the analysis of two shielded cables.

The paper is organized as follows. Section II briefly describes the numerical method for the calculation of the p.u.l. inductance and capacitance. A conventional high-fidelity approach is summarized in Section III, whereas the proposed mixed-fidelity method is outlined in Section IV. Numerical examples are discussed in Section V, while conclusions are drawn in Section VI.

II. Calculation of the Inductance and Capacitance Matrices for Circular Conductors

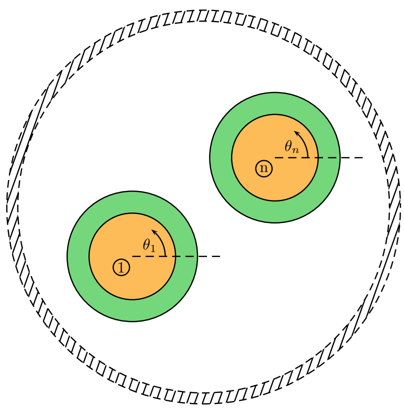

Fig. 1. Cable cross-section consisting of circular wires, possibly dielectric coated and/or enclosed within a metallic shield.

We consider a system of N circular conductors, possibly shielded and/or dielectric-coated, as shown in Fig. 1. The reference for the voltages and return path for the currents can be one of the conductors, a ground plane, or a shield. The p.u.l. capacitance and inductance of the wires are computed following the approach in [7] and [8], which starts by describing the charge distributions at the metal-dielectric and dielectric-background interfaces as the Fourier expansions

$$\rho_n(\theta_n) = a_{n0} + \sum_{m=1}^{A} a_{nm}\cos(m\theta_n) + \sum_{m=1}^{A} b_{nm}\sin(m\theta_n) \tag{1a}$$

and

$$\rho'_n(\theta_n) = a'_{n0} + \sum_{m=1}^{A} a'_{nm}\cos(m\theta_n) + \sum_{m=1}^{A} b'_{nm}\sin(m\theta_n), \tag{1b}$$

respectively, with $n = 1, \ldots, N$ and θ_n denoting the angular coordinate on wire n.

The charge distributions produce an electric potential and an electric field. By enforcing that 1) the potential on each conductor periphery is constant with the angular position and 2) the normal component of the electric displacement field is continuous across each dielectric periphery, an algebraic system of $2N(1+2A)$ equations in the unknown charge distribution coefficients is obtained. The p.u.l. capacitances are related to the constant terms a_{n0} and a'_{n0} in (1), which are obtained from the solution of the aforementioned system. The p.u.l. inductance matrix is obtained from the capacitance matrix as described in [8]. The overall accuracy is determined by the angular discretization parameter A. Wire proximity demands for larger values of A, with $A = 10$ typically being a good choice for closely-spaced wires.

III. Conventional Polynomial Chaos Approach

The PCE method amounts to approximating stochastic quantities of interest, p.u.l. inductances and capacitances in the context of this paper and here generically denoted with X, using the following model:

$$X(\boldsymbol{\xi}) \approx \sum_{k=1}^{K} X_k \varphi_k(\boldsymbol{\xi}), \tag{2}$$

where $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_d)$ is a set of d uncertain parameters which X depends on (typically, geometrical and material parameters), whereas $\{\varphi_k\}_{k=1}^{K}$ is a basis of multivariate orthogonal polynomials based on the distribution of $\boldsymbol{\xi}$ [1].

The multivariate basis functions φ_k are constructed as the product combination of univariate orthogonal polynomials:

$$\varphi_k(\boldsymbol{\xi}) = \prod_{i=1}^{d} \phi_{k_i}(\xi_i), \tag{3}$$

where ϕ_{k_i} is a polynomial of degree k_i in the variable ξ_i. There is a one-to-one mapping between each scalar index k and a corresponding vector of multi-indices $\boldsymbol{k} = (k_1, \ldots, k_d)$. Several schemes are available for the truncation of (2) based on a predefined order p, leading to increasing sparsity in the number of terms K. The three most common strategies are summarized in Table I. Usually, a total-degree truncation [9] with $p = 2$ or $p = 3$ yields satisfactory results in many practical applications [2]. Hyperbolic truncations [10] lead to sparser representations controlled by the parameter u.

The main difference among the various PCE implementations resides in the strategy for the calculation of the unknown coefficients X_k. In [3] and [4], two *intrusive* numerical techniques, respectively based on a Galerkin and on a stochastic testing approach, were proposed to solve for the inductance and capacitance coefficients. In general, however, non-intrusive and "black-box" sampling-based methods relying on suitable regression/interpolation strategies can be used [11]–[13]. For our purposes, we adopt here the interpolative method of [12], and we evaluate the PCE coefficients as a linear combination of suitable samples of the stochastic p.u.l. parameters:

TABLE I
Definition of common PCE truncation strategies.

Strategy	Truncation condition	Number of terms K
Tensor degree	$\max_i\{k_i\} = \|\boldsymbol{k}\|_\infty \leq p$	$(p+1)^d$
Total degree	$\sum_{i=1}^{d} k_i = \|\boldsymbol{k}\|_1 \leq p$	$\frac{(p+d)!}{p!d!}$
Hyperbolic	$\left(\sum_{i=1}^{d} k_i^u\right)^{1/u} = \|\boldsymbol{k}\|_u \leq p$	$\in \left(2d+1, \frac{(p+d)!}{p!d!}\right)$

$$X_k = \sum_{q=1}^{K} b_{kq} X(\boldsymbol{\xi}_q), \tag{4}$$

where $\{\boldsymbol{\xi}_q\}_{q=1}^{K}$ are the stochastic testing nodes [14] and b_{kq} are the elements of the inverse of the Vandermonde-like matrix containing the basis functions φ_k evaluated at these nodes. It is important to note that this approach requires a number of samples that equals the number of unknown PCE coefficients K.

In any case, there are two fundamental parameters that control the accuracy of the calculation: the number of PCE terms K in (2) and (4), and the discretization of the charge distribution A that is used in (1) for the evaluation of the samples $X(\boldsymbol{\xi}_q)$ in (4). An immediate, reasonable choice is to set both parameters to a sufficiently high value. Nevertheless, there exist theories suggesting that it is often unnecessary to simultaneously set both parameters to a large value, as we discuss in the next section.

IV. Mixed-Fidelity Polynomial Chaos Approach

Let us denote with $\mathcal{M}_{A,K}$ a model for X that has "high fidelity" in terms of both angular discretization (geometrical fidelity) and PCE approximation (probabilistic fidelity). By using a *telescopic identity*, this model can be expressed as [5]

$$\begin{aligned} \mathcal{M}_{A,K} = \;& \mathcal{M}_{A,K} \\ & +\mathcal{M}_{\alpha,K} - \mathcal{M}_{\alpha,K} \\ & +\mathcal{M}_{A,\kappa} - \mathcal{M}_{A,\kappa} \\ & +\mathcal{M}_{\alpha,\kappa} - \mathcal{M}_{\alpha,\kappa} \\ & +\mathcal{M}_{\alpha,\kappa} - \mathcal{M}_{\alpha,\kappa}, \end{aligned} \tag{5}$$

where lowercase subscripts α and κ denote low fidelity in geometry and probability, respectively. By mere rearrangement of the above identity as

$$\begin{aligned} \mathcal{M}_{A,K} = \;& \mathcal{M}_{\alpha,k} \\ & +\mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,\kappa} \\ & +\mathcal{M}_{\alpha,K} - \mathcal{M}_{\alpha,\kappa} \\ & +\mathcal{M}_{A,K} - \mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,K} + \mathcal{M}_{\alpha,\kappa}, \end{aligned} \tag{6}$$

we can interpret the second and third rows as corrections in geometrical discretization and probabilistic approximation, respectively, while the term in the fourth row plays the role of a mixed correction. Since this "higher-order" mixed correction is often negligible [5], the high-fidelity model can be approximated as

$$\begin{aligned}\mathcal{M}_{A,K} \approx \; & \mathcal{M}_{\alpha,\kappa} + \mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,\kappa} + \mathcal{M}_{\alpha,K} - \mathcal{M}_{\alpha,\kappa} \\ & = \mathcal{M}_{\alpha,K} + \mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,\kappa}.\end{aligned} \tag{7}$$

Practically speaking, the above result suggests that the high-fidelity model $\mathcal{M}_{A,K}$ can be approximated by the sum of a model computed with high probabilistic fidelity but low geometrical fidelity ($\mathcal{M}_{\alpha,K}$) and a low-fidelity probabilistic model of the difference between high- and low-fidelity models in geometrical discretization ($\mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,\kappa}$). The main advantage is that the calculation of these two models is usually computationally cheaper than the direct evaluation of $\mathcal{M}_{A,K}$.

A similar concept is found, under different names, in various uncertainty quantification or design optimization methodologies, including (but not limited to):

- multi-level Monte Carlo method [15];
- multi-index stochastic collocation [5];
- multi-fidelity design optimization [16];
- predictor-corrector polynomial chaos scheme [6].

In particular, the approach that we propose in this paper is closely inspired by the predictor-correct algorithm in [6].

We first start by computing a predictor model of the p.u.l. inductance and capacitance using a high-fidelity probabilistic approximation:

$$\mathcal{M}_p = \mathcal{M}_{\alpha,K} = \sum_{k=1}^{K} X_{p,k}\varphi_k(\boldsymbol{\xi}). \tag{8}$$

To introduce a computational compensation for the high accuracy in probability, we evaluate the PCE coefficients $X_{p,k}$ with (4) by using samples computed with a coarse geometrical discretization, which we denote as $X^{\alpha}(\boldsymbol{\xi}_q)$ meaning that $\alpha < A$ is used in (1).

The above predictor is complemented by a corrector that describes, with low probabilistic fidelity, the difference between high and low geometrical fidelities:

$$\mathcal{M}_c = \mathcal{M}_{A,\kappa} - \mathcal{M}_{\alpha,\kappa} = \sum_{k=1}^{\kappa} X_{c,k}\varphi_k(\boldsymbol{\xi}), \tag{9}$$

where $\kappa < K$ is obtained by using a lower PCE order p and/or a sparser truncation scheme (cfr. Table I). The coefficients $X_{c,k}$ are evaluated based on the difference between samples with high and low geometrical fidelity, the former being denoted with $X^{A}(\boldsymbol{\xi}_q)$. Since changing the probabilistic fidelity potentially leads to a different set of stochastic testing nodes, the samples with low geometrical fidelity $X^{\alpha}(\boldsymbol{\xi}_q)$ are effectively obtained by evaluating the predictor (8), rather than by solving again for the charge distributions with a coarser discretization. It should be noted that the computational cost of building (9) is relatively low because the higer cost of evaluating $X^{A}(\boldsymbol{\xi}_q)$ is offset by the lower number κ of required samples, and the evaluation of (8) is inexpensive.

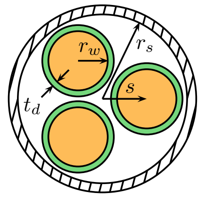

Fig. 2. Cross-section of the application test case consisting of three dielectric-coated wires within a metallic shield.

V. Numerical Results

The proposed method is applied to the shielded cable with the cross-section depicted in Fig. 2 [3]. The nominal geometrical dimensions are as follows: conductor radius $r_w = 0.52$ mm, dielectric thickness $t_d = 64$ μm, shield radius $r_s = 1.675$ mm. The three wires are placed at a radial distance of $s = 0.8575$ mm from the shield center. The relative permittivity of the dielectric coating is $\varepsilon_r = 2.6$. The uncertainty is provided by the conductor radius as well as by the thickness and relative permittivity of the dielectric coating of each wire, leading to $d = 9$ random parameters. The distribution is assumed to be Gaussian with a 5% relative standard deviation from the nominal value.

First of all, reference results are computed using a Monte Carlo analysis with 10000 samples of the uncertain parameters, which takes 153.4 s on a Lenovo ThinkPad X1 Yoga laptop with an Intel(R) Core(TM) i7-7500U, CPU running at 2.7 GHz and 16 GB of RAM. Next, a high-fidelity model $\mathcal{M}_{A,K}$ of the p.u.l. inductance and capacitance matrices is computed using a total-degree PCE with order $p = 3$, which has therefore $K = 220$ terms. The PCE coefficients are obtained via (4) using 220 samples of the inductance and capacitance matrices calculated with $A = 10$ in (1). The time required is 3.2 s, with a speed-up of 48× over the Monte Carlo analysis.

For the proposed mixed-fidelity approach instead, we first build a predictor model $\mathcal{M}_p$ with low geometrical fidelity and the same probabilistic features as the high-fidelity one. Therefore, we use $\alpha = 2$ instead of $A = 10$ for the calculation of the PCE coefficients. Given the much coarser geometrical discretization, this step now only requires 0.3 s. Then we construct a corrector model $\mathcal{M}_c$ with low probabilistic fidelity. To this end, we consider a hyperbolic PCE truncation with $u = 0.7$, while keeping the order to $p = 3$. This leads to a sparser PCE with $\kappa = 64$ terms. The coefficients of the corrector model are computed using 64 samples of the p.u.l. inductance and capacitance matrix, with the samples of high geometrical fidelity calculated using $A = 10$ and the samples of low geometrical fidelity estimated with the predictor model. This step takes 1.1 s. Hence, the proposed mixed-fidelity approach overall takes 1.4 s, with a further speed-up of 2.3× compared to the conventional high-fidelity approach.

Figure 3 shows the probability density functions of the 3×3 p.u.l. inductance and capacitance matrix entries. Symmetric elements are equal because of reciprocity and they are therefore omitted. The units are nH/m for inductances and

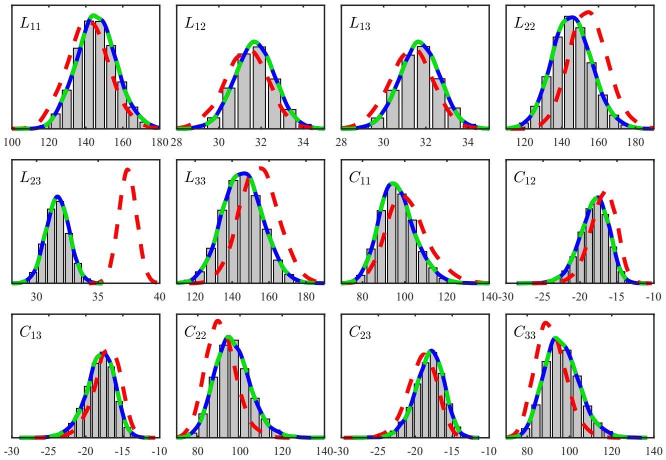

Fig. 3. Distributions of the p.u.l. inductance and capacitance matrix entries for the cable of Fig. 2, computed with Monte Carlo (gray bars), conventional high-fidelity model $\mathcal{M}_{A,K}$ (solid blue lines), predictor model $\mathcal{M}_p$ (dashed red lines), and mixed-fidelity model $\mathcal{M}_p + \mathcal{M}_s$ (dashed green lines).

pF/m for capacitances. The gray bars show the distribution of the Monte Carlo samples. The solid blue lines are the distributions obtained with the conventional high-fidelity model, exhibiting excellent agreement with the reference Monte Carlo distribution. The dashed red lines are the distributions of the predictor model (8), which highlight the inaccuracy due to the low geometrical fidelity. However, if the model is adjusted by including the corrector (9), the dashed green lines are obtained, which match the results of the high-fidelity model.

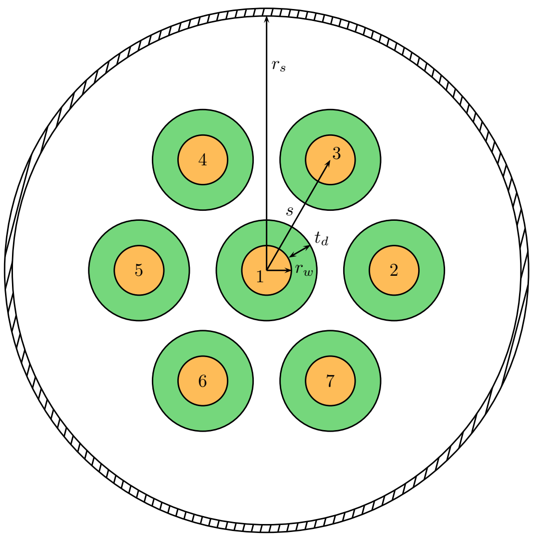

Fig. 4. Cross-section of the second application test case consisting of seven dielectric-coated wires within a metallic shield.

As a second example, we consider the shielded cable with the cross-section shown in Fig. 4 [4], [17]. Given the radius r_w of each wire, the shield radius is $r_s = 10\,r_w$ and the thickness of the dielectric coating is $t_d = r_w$. The outer wires are located evenly along the angular coordinate and at a radial distance of $s = 5\,r_w$ from the central wire. The relative permittivity of the dielectric coating is $\varepsilon_r = 4$. The uncertainty is provided by the Cartesian coordinates of the outer wires, which vary uniformly within $\pm 0.5\,r_w$ around their nominal center. The total number of random parameters is thus $d = 12$.

The calculation of 10000 reference Monte Carlo samples takes 337.6 s. A high-fidelity model is again computed using a third-order PCE with total-degree truncation, leading in this case to $K = 445$ terms. The calculation of the corresponding PCE coefficients by setting the geometrical discretization to $A = 10$ requires 15.8 s, i.e., 21× faster than the Monte Carlo analysis.

The same features as in the previous example are used to construct the mixed-fidelity model, i.e., $\alpha = 2$ for low geometrical fidelity and a hyperbolic truncation with $u = 0.7$, leading to $\kappa = 103$ terms, for low probabilistic fidelity. The calculations of the predictor and corrector models take 1.4 s and 3.6 s, respectively, with a further 3.2× speed-up compared

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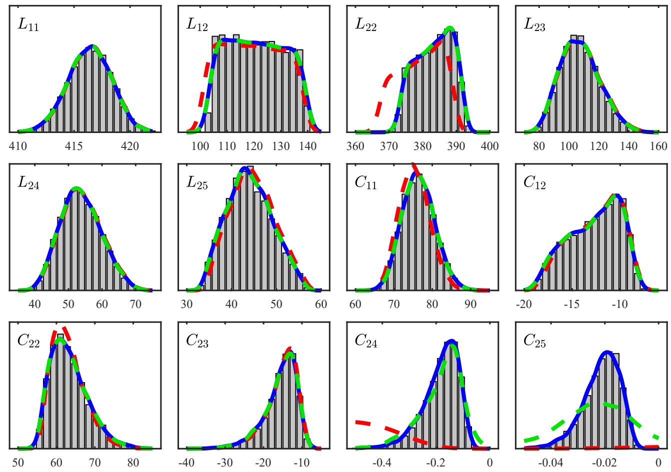

Fig. 5. Distributions of the p.u.l. inductance and capacitance matrix entries for the cable of Fig. 4. Curve identification is as in Fig. 3.

The probability density functions of some elements of the 7×7 p.u.l. inductance and capacitance matrices are shown in Fig. 5. The omitted entries have similar statistical behavior due structural symmetries. The curve identification is the same as in Fig. 2. Once again, an excellent agreement is observed between the proposed mixed-fidelity model (dashed green lines) and the conventional high-fidelity model (solid blue lines), except for C_{25}. However, it should be noted that this element is almost negligible as it takes extremely low values (cfr. the x-axis scale). This is readily explained by the fact that wires #2 and #5 are "shielded" from each other by wire #1, also causing the predictor to provide a very poor model. The same consideration applies to 3-6 and 4-7 pairs.

The main figures concerning the performance of the proposed mixed-fidelity method for the two application examples, and the speed-up versus the conventional high-fidelity approach, are summarized in Table II. The overall speed-up achieved versus the Monte Carlo analysis is 110× and 67× for the test cases of Figs. 2 and 5, respectively. The same parameters for geometrical discretization and PCE approximation turned out to provide a reasonable trade-off between accuracy and efficiency improvement for both application examples. An extensive investigation of the impact of such parameters on the performance is deferred to future works.

TABLE II
PERFORMANCE OF THE CONVENTIONAL HIGH-FIDELITY AND PROPOSED MIXED-FIDELITY MODELS FOR THE PROPOSED APPLICATION EXAMPLES.

Cable	High-fidelity	Mixed-fidelity			
		Predictor	Corrector	Total	Speed-up
Fig. 2	3.2 s	0.3 s	1.1 s	1.4 s	2.3×
Fig. 4	15.8 s	1.4 s	3.6 s	5.0 s	3.2×

VI. Conclusions

This paper presented a mixed-fidelity approach, based on polynomial chaos, for the uncertainty quantification of the p.u.l. inductance and capacitance matrices of cables with uncertain geometrical and material parameters. Accurate results were obtained by combining two models, one with high probabilistic fidelity but computed with low geometrical fidelity, and one with low probabilistic fidelity but computed with high geometrical fidelity. The proposed method was shown to provide comparable results with respect to a conventional high-fidelity approach, at a fraction of the computational cost. For this purpose, two shielded cables were considered. The development of similar methods for other EMC/SI problems is currently under investigation.

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