POLITECNICO DI TORINO
Repository ISTITUZIONALE

Eavesdropping on the Decohering Environment: Quantum Darwinism, Amplification, and the Origin of Objective Classical Reality

Original
Eavesdropping on the Decohering Environment: Quantum Darwinism, Amplification, and the Origin of Objective Classical Reality / Touil, A., Yan, B., Girolami, D., Deffner, S., Hubert Zurek, W.. - In: PHYSICAL REVIEW LETTERS. - ISSN 0031-9007. - ELETTRONICO. - 128:1(2022). [10.1103/PhysRevLett.128.010401]

Availability:
This version is available at: 11583/2949132 since: 2022-01-12T09:35:42Z

Publisher:
American Physical Society

Published
DOI:10.1103/PhysRevLett.128.010401

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)

26 November 2026

Eavesdropping on the Decohering Environment: Quantum Darwinism, Amplification, and the Origin of Objective Classical Reality

Akram Touil,[1,2,*] Bin Yan,[2,3] Davide Girolami,[4] Sebastian Deffner,[1,5] and Wojciech Hubert Zurek[3]

[1]Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250, USA
[2]Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[3]Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[4]Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino 10129, Italy
[5]Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas, 13083-859 Campinas, São Paulo, Brazil

(Received 12 July 2021; accepted 30 November 2021; published 6 January 2022)

"How much information about a system $\mathcal{S}$ can one extract from a fragment $\mathcal{F}$ of the environment $\mathcal{E}$ that decohered it?" is the central question of Quantum Darwinism. To date, most answers relied on the quantum mutual information of $\mathcal{SF}$, or on the Holevo bound on the channel capacity of $\mathcal{F}$ to communicate the classical information encoded in $\mathcal{S}$. These are reasonable upper bounds on what is really needed but much harder to calculate—the accessible information in the fragment $\mathcal{F}$ about $\mathcal{S}$. We consider a model based on imperfect C-NOT gates where all the above can be computed, and discuss its implications for the emergence of objective classical reality. We find that all relevant quantities, such as the quantum mutual information as well as various bounds on the accessible information exhibit similar behavior. In the regime relevant for the emergence of objective classical reality this includes scaling independent of the quality of the imperfect C-NOT gates or the size of $\mathcal{E}$, and even nearly independent of the initial state of $\mathcal{S}$.

DOI: 10.1103/PhysRevLett.128.010401

Quantum Darwinism [1–5] explains the emergence of objective classical reality in our quantum Universe: The decohering environment $\mathcal{E}$ is a "witness" who monitors and can reveal the state of the system $\mathcal{S}$. Agents like us never measure systems of interest directly. Rather, we access fragments $\mathcal{F}$ of $\mathcal{E}$ that carry information about them. Since its inception [1], Quantum Darwinism has advanced on both theory [6–25] and experimental fronts [26–29].

Quantum mutual information $I(\mathcal{S}:\mathcal{F})$ between an environment fragment and the system yields an upper bound on what $\mathcal{F}$ can reveal about $\mathcal{S}$. It has been used to estimate the capacity of the environment as a communication channel. We analyze a solvable model based on imperfect tunable C-NOT (or C-MAYBE) gates that couple $\mathcal{S}$ to the subsystems of $\mathcal{E}$. We compute the mutual information $I(\mathcal{S}:\mathcal{E})$ as well as the Holevo $\chi(\mathcal{S}:\mathcal{F})$ [30,31]—that characterize the accessible information in our C-MAYBE-based model. We also compute the quantum discord [1,32–37]—the difference of $I(\mathcal{S}:\mathcal{F})$ and $\chi(\mathcal{S}:\mathcal{F})$ that quantifies the genuinely quantum correlations between $\mathcal{S}$ and $\mathcal{F}$ [38–41].

We find that $I(\mathcal{S}:\mathcal{F})$ and $\chi(\mathcal{S},\mathcal{F})$ exhibit strikingly similar dependence on the size of $\mathcal{F}$, with the initial steep rise followed by the classical plateau where, at the level set by the entropy $H_{\mathcal{S}}$ of the system, the information $\mathcal{F}$ has about $\mathcal{S}$ saturates: Enlarging $\mathcal{F}$ only confirms what is already known. This behavior is universal and nearly independent of the initial state of $\mathcal{S}$ and the size of $\mathcal{E}$.

The model.—The system $\mathcal{S}$ is a qubit coupled to N independent noninteracting qubits of the environment $\mathcal{E}$ via a C-MAYBE gate,

$$U_{\oslash} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & c & -s \end{pmatrix}. \tag{1}$$

The parameters $c = \cos(a)$ and $s = \sin(a)$ (where a is the rotation angle of the target qubit) quantify the imperfection.

Our Quantum Universe $\mathcal{SE}$ starts in a pure state:

$$|\Psi^0_{\mathcal{SE}}\rangle = (\sqrt{p}|0_{\mathcal{S}}\rangle + \sqrt{q}|1_{\mathcal{S}}\rangle) \bigotimes_{i=1}^{N} |0^i\rangle, \tag{2}$$

where $p + q = 1$. The unitary $U_{\oslash}$ correlates each qubit in $\mathcal{E}$ with $\mathcal{S}$, and we obtain a branching state [42],

$$|\Psi^{\oslash}_{\mathcal{SE}}\rangle = \sqrt{p}|0_{\mathcal{S}}\rangle \bigotimes_{i=1}^{N} |0_{\mathcal{E}_i}\rangle + \sqrt{q}|1_{\mathcal{S}}\rangle \bigotimes_{i=1}^{N} |1_{\mathcal{E}_i}\rangle. \tag{3}$$

By construction $|0_{\mathcal{S}}\rangle$ and $|1_{\mathcal{S}}\rangle$ are the pointer states [43,44]. They are orthogonal and immune to decoherence. The corresponding record states of $\mathcal{E}$ are

$$|0_{\mathcal{E}_i}\rangle \equiv |0^i\rangle \quad \text{and} \quad |1_{\mathcal{E}_i}\rangle \equiv s|0^i\rangle + c|1^i\rangle, \tag{4}$$

 © 2022 American Physical Society

in terms of the orthogonal basis $|0^i\rangle$ and $|1^i\rangle$ of the ith qubit that defines $U_{\oslash}$, so that $\langle 0_{\mathcal{E}_i}|1_{\mathcal{E}_i}\rangle = s$.

We will be interested in the correlations between the fragment $\mathcal{F}$ and $\mathcal{S}$. The marginal states of $\mathcal{S}$, an m-qubit fragment $\mathcal{F}_m$, and a bipartition $\mathcal{S}F_m$ are rank-two density matrices [45]:

$$\rho_{\mathcal{S}} \equiv \mathrm{tr}_{\mathcal{E}}\{|\Psi^{\oslash}_{\mathcal{SE}}\rangle\langle\Psi^{\oslash}_{\mathcal{SE}}|\} = \begin{pmatrix} p & s^N\sqrt{pq} \\ s^N\sqrt{pq} & q \end{pmatrix}, \quad (5)$$

$$\rho_{\mathcal{F}_m} = \begin{pmatrix} p & s^m\sqrt{pq} \\ s^m\sqrt{pq} & q \end{pmatrix}, \quad (6)$$

$$\rho_{\mathcal{SF}_m} = \begin{pmatrix} p & s^{N-m}\sqrt{pq} \\ s^{N-m}\sqrt{pq} & q \end{pmatrix}. \quad (7)$$

Symmetric quantum mutual information is often used to estimate the accessible information in $\mathcal{F}$ in Quantum Darwinism [2–4,11–13,47–49]. It is defined using the von Neumann entropy, $H(\rho) = -\mathrm{tr}\{\rho \log_2(\rho)\}$ as

$$I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{S}} + H_{\mathcal{F}_m} - H_{\mathcal{S},\mathcal{F}_m}. \quad (8)$$

Joint entropy $H_{\mathcal{S},\mathcal{F}_m}$ quantifies the ignorance about the state of $\mathcal{SF}_m$ in the tensor product of the Hilbert spaces of $\mathcal{S}$ and $\mathcal{F}$.

In our model $I(\mathcal{S}:\mathcal{F}_m)$ can be computed exactly [46]:

$$I(\mathcal{S}:\mathcal{F}_m) = h(\lambda^+_{N,p}) + h(\lambda^+_{m,p}) - h(\lambda^+_{N-m,p}), \quad (9)$$

where $h(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ and $\lambda^{\pm}_{k,p}$ are the eigenvalues of the density matrices (5)–(7),

$$\lambda^{\pm}_{k,p} = \frac{1}{2}(1 \pm \sqrt{(q-p)^2 + 4s^{2k}pq}). \quad (10)$$

We thus have a closed expression for the mutual information $I(\mathcal{S}:\mathcal{F}_m)$.

As seen in Fig. 1, symmetric mutual information $I(\mathcal{S}:\mathcal{F}_m)$ exhibits a steep initial rise with increasing fragment size m, as a larger $\mathcal{F}_m$ provides more data about $\mathcal{S}$. This initial rise is followed by a long classical plateau, where the additional information imprinted on the environment is redundant.

Note that, when $\mathcal{SE}$ is in a pure state, the entropy of a fragment $\mathcal{F}$ is equal to $H_{\mathcal{S}d\mathcal{F}}$, that is the entropy $\mathcal{S}$ would have if it was decohered only by the fragment $\mathcal{F}$. When we further assume good decoherence [42,46]—i.e., that the off-diagonal terms of $\rho_{\mathcal{S}}$ and $\rho_{\mathcal{SF}_m}$ are negligible (which in our model corresponds to $s^{N-m} \ll s^m$)—we obtain an approximate equality;

$$I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{F}_m} = H_{\mathcal{S}d\mathcal{F}_m}, \quad (11)$$

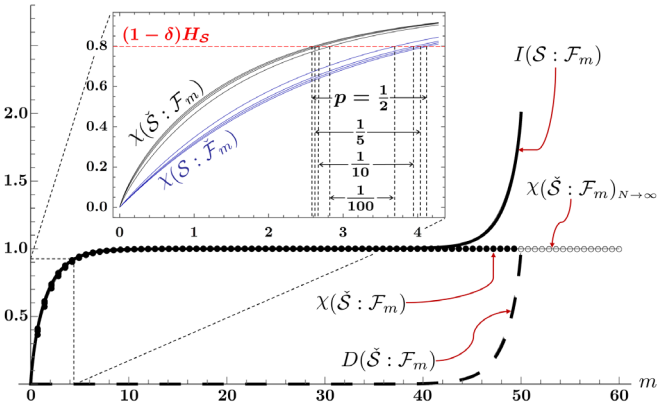


FIG. 1. Approximate universality of mutual information: Symmetric $I(\mathcal{S}:\mathcal{F}_m)$ and Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ coincide until the fragment $\mathcal{F}_m$ becomes almost as large as $\mathcal{E}$. Renormalized $I(\mathcal{S}:\mathcal{F})/H_{\mathcal{S}}$ and $\chi(\check{\mathcal{S}}:\mathcal{F})/H_{\mathcal{S}}$ depend only weakly on the probabilities of the outcomes (see inset). Their difference—quantum discord $D(\check{\mathcal{S}}:\mathcal{F})$—vanishes until $\mathcal{F}_m$ begins to encompass almost all of $\mathcal{E}$, $m \sim N/R_\delta$. The inset also compares the ratios $\chi(\check{\mathcal{S}}:\mathcal{F})/H_{\mathcal{S}}$ and $\chi(\mathcal{S}:\check{\mathcal{F}})/H_{\mathcal{S}}$ computed for several probabilities p of the pointer state $|0_{\mathcal{S}}\rangle$ in Eq. (3). Note that the fragment sizes m_δ that supply ~80% of information about $\mathcal{S}$ are only modestly affected by p and quite similar for these two different information measures.

since $H_{\mathcal{S}} = H_{\mathcal{SF}_m}$ cancel one another in Eq. (8). Furthermore, when the environment fragments are typical [50] (and in our model all fragments of the same size are identical—hence, each is typical) the plot of $I(\mathcal{S}:\mathcal{F}_m)$ is antisymmetric around $I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{S}}$ and $m = N/2$ [42].

We will see that the behavior of $I(\mathcal{S}:\mathcal{F}_m)$ is approximately universal. This means that after suitable rescaling its functional form is nearly independent of the size of the environment N, of the quality of the C-MAYBE gate $U_{\oslash}$, and almost independent of the initial state of $\mathcal{S}$.

Agents generally do not insist on knowing the state of $\mathcal{S}$ completely, but tolerate a finite information deficit δ. When $I(\mathcal{S}:\mathcal{F}_{m_\delta}) \geq (1-\delta)H_{\mathcal{S}}$ is attained already for a fragment with $m_\delta \ll N$ subsystems, a fraction $f_\delta = m_\delta/N$ of the environment, then there are many $(1/f_\delta)$ such fragments. We define redundancy of the information about $\mathcal{S}$ in $\mathcal{E}$ via

$$\mathcal{R}_\delta \equiv N/m_\delta \quad \text{with} \quad I(\mathcal{S}:\mathcal{F}_{m_\delta}) = (1-\delta)H_{\mathcal{S}}. \quad (12)$$

Redundancy $\mathcal{R}_\delta$ is the length of the classical plateau in the units set by m_δ, see Fig. 1. The beginning of the plateau is determined by the smallest m_δ such that $I(\mathcal{S}:\mathcal{F}_{m_\delta}) \geq (1-\delta)H_{\mathcal{S}}$.

In realistic models $I(\mathcal{S}:\mathcal{F}) = H_{\mathcal{S}}$ only when $f = 1/2$ (see Ref. [42]). Thus, significant redundancy appears only when the requirement of completeness of the information about $\mathcal{S}$ that can be extracted from $\mathcal{F}$ is relaxed. Moreover, Eq. (12) is an overestimate since $I(\mathcal{S}:\mathcal{F}_{m_\delta})$ is only an upper bound of what can be found out about $\mathcal{S}$ from $\mathcal{F}$ [51].

We will now consider better estimates: Inset in Fig. 1 compares $I(\mathcal{S}:\mathcal{F}_{m_\delta})$ with the two Holevo—like χ's we are about to discuss and illustrates resulting fragment sizes (hence, redundancies) they imply.

Asymmetric mutual information is defined using conditional entropy. We mark the system whose states are used for such conditioning by an inverted "hat," so when it is $\check{\mathcal{S}}$ we consider the asymmetric mutual information:

$$J(\check{\mathcal{S}}:\mathcal{F}_m)_{\{|s_k\rangle\}} = H_{\mathcal{F}} - H_{\mathcal{F}|\check{\mathcal{S}}_{\{|s_k\rangle\}}}. \tag{13}$$

Above, $H_{\mathcal{F}_m|\check{\mathcal{S}}_{\{|s_k\rangle\}}}$ is the conditional entropy [31] that quantifies the missing information about $\mathcal{F}$ remaining after the observable with the eigenstates $\{|s_k\rangle\}$ was measured. Accordingly, the joint entropy in Eq. (8) is replaced by

$$H_{\mathcal{F}_m,\check{\mathcal{S}}_{\{|s_k\rangle\}}} = H_{\mathcal{F}_m|\check{\mathcal{S}}_{\{|s_k\rangle\}}} + H_{\check{\mathcal{S}}_{\{|s_k\rangle\}}}. \tag{14}$$

The asymmetric joint entropy depends on whether $\mathcal{S}$ or $\mathcal{F}$ are measured and on the measurements that are used. The entropy increase associated with the wave packet reduction means that the asymmetric entropy (14) is typically larger than the symmetric version $H_{\mathcal{S},\mathcal{F}_m}$ in Eq. (8): Local measurements cannot extract information encoded in the quantum correlations between $\mathcal{S}$ and $\mathcal{F}_m$, which is why the asymmetric $J(\check{\mathcal{S}}:\mathcal{F}_m)$ is needed, [31]; see also Ref. [52].

For optimal measurements the asymmetric $J(\check{\mathcal{S}}:\mathcal{F}_m)$ defines the Holevo bound [30],

$$J(\check{\mathcal{S}}:\mathcal{F}_m) = \max_{\{|s_k\rangle\}} J(\check{\mathcal{S}}:\mathcal{F}_m)_{\{|s_k\rangle\}} \equiv \chi(\check{\mathcal{S}}:\mathcal{F}_m). \tag{15}$$

In our model, measurement of the pointer observable of $\mathcal{S}$ is optimal [46]. Indeed, Eq. (3) shows that in the pointer basis $\{|0_{\mathcal{S}}\rangle, |1_{\mathcal{S}}\rangle\}$ the conditional entropy disappears, $H_{\mathcal{F}_m|\check{\mathcal{S}}} = 0$, as states of $\mathcal{F}_m$ correlated with pointer states of $\mathcal{S}$ are pure.

The limit of large $\mathcal{E}$ ($N \geq N - m \gg m$) reflects the typical situation of agents (who do not even know the size of $\mathcal{E}$, and only access "their $\mathcal{F}_m$," with $m \ll N$). This is good decoherence, $s^N \leq s^{N-m} \ll s^m$, and equations simplify: Using $H_{\mathcal{S},\mathcal{F}_m} = H_{\mathcal{S}}$ and Eq. (11) we can thus write,

$$I(\mathcal{S}:\mathcal{F}_m) \approx H_{\mathcal{F}_m} = h(\lambda^+_{m,p}) = \chi(\check{\mathcal{S}}:\mathcal{F}_m). \tag{16}$$

An immediate important consequence is that $H_{\mathcal{F}_m}$ determines both the symmetric $I(\mathcal{S}:\mathcal{F}_m)$ (except for the final rise) as well as the asymmetric (optimal) $J(\check{\mathcal{S}}:\mathcal{F}_m) = \chi(\check{\mathcal{S}}:\mathcal{F}_m)$. We have

$$\chi(\check{\mathcal{S}}:\mathcal{F}_m) = -\frac{1}{2}\log_2[pq(1-s^{2m})] - \sqrt{1-4pq(1-s^{2m})} \times \mathrm{Arctanh}_2\left[\sqrt{1-4pq(1-s^{2m})}\right], \tag{17}$$

where "$\mathrm{Arctanh}_2$" denotes $\mathrm{Arctanh}/\ln(2)$.

Figure 1 compares $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ with $I(\mathcal{S}:\mathcal{F}_m)$ for finite and infinite N and for different values of s and p. As it shows, Eq. (17) matches $I(\mathcal{S}:\mathcal{F}_m)$ until the far end ($N - m \ll m$) of the classical plateau. This is a consequence of two scalings: (i) "vertically," the plateau appears at $H_{\mathcal{S}} = -p\log_2(p) - q\log_2(q)$, and it is easy to see that for $s^N \ll s^m \ll 1$ we have $\chi(\check{\mathcal{S}}:\mathcal{F}_m) = H_{\mathcal{S}}$ in Eq. (17); (ii) "horizontally", $H_{\mathcal{F}_m}$ depends on s^m, so weakly entangling gates can be compensated by using more of them—larger m. What is surprising is how insensitive are these plots to p, the probability of the outcome.

This remarkably universal behavior is a consequence of *good decoherence* [46]. Both, $\rho_{\mathcal{S}}$ and $\rho_{\mathcal{S}\mathcal{F}_m}$, Eqs. (5) and (7), become diagonal in the pointer basis. Moreover, the quality of $U_{\oslash}$ (set by c and s) determines the "information flow rate" from $\mathcal{S}$ to $\mathcal{F}$. Thus, when (at a fixed p) one demands the same $H_{\mathcal{F}_m}$, this translates into identical $\rho_{\mathcal{F}_m}$ when $s_1^{m_1} \simeq s_2^{m_2}$. Therefore, less efficiently entangling gates can be compensated by relying on more of them—on a suitably enlarged $\mathcal{F}$, with $m_2 = m_1\log(s_1)/\log(s_2)$.

Environment as a communication channel.—While the mutual information $I(\mathcal{S}:\mathcal{F}_m)$ is easier to compute and a safe upper bound on the accessible information in $\mathcal{F}_m$, it is important to verify it is also a reasonable estimate of that accessible information (as generally assumed in much of the Quantum Darwinism literature). The asymmetric mutual information extracted by optimal measurements on the environment fragment $\mathcal{F}_m$ is

$$J(\mathcal{S}:\check{\mathcal{F}}_m) = H_{\mathcal{S}} - H_{\mathcal{S}|\check{\mathcal{F}}_m} = \chi(\mathcal{S}:\check{\mathcal{F}}_m). \tag{18}$$

The joint entropy given in terms of the conditional entropy $H_{\mathcal{S}|\check{\mathcal{F}}_m}$ now becomes

$$H_{\mathcal{S},\check{\mathcal{F}}_m} = H_{\mathcal{S}|\check{\mathcal{F}}_m} + H_{\check{\mathcal{F}}_m}. \tag{19}$$

As in Eq. (14) above, all terms in Eq. (19) depend on how $\mathcal{F}$ is measured. However, while measuring $\mathcal{S}$ in the pointer basis simplified the analysis (since, e.g., $H_{\mathcal{F}|\check{\mathcal{S}}_{\{|s_k\rangle\}}} = 0$) this is no longer the case when $\mathcal{F}_m$ is measured.

To compute $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ we rely on the Koashi-Winter monogamy relation [53]. Details of that calculation are relegated to the Supplemental Material [54].

We focus again on the limit of large $\mathcal{E}$ ($N \geq N - m \gg m$): Agents only access "their $\mathcal{F}_m$," a small fraction of $\mathcal{E}$ with $m \ll N$. Assuming good decoherence we obtain

$$\chi(\mathcal{S}:\check{\mathcal{F}}_m) = H_{\mathcal{S}} + \frac{1}{2}\log_2(pqs^{2m}) + \sqrt{1-4pqs^{2m}} \times \mathrm{Arctanh}_2\left(\sqrt{1-4pqs^{2m}}\right). \tag{20}$$

Equation (20) constitutes our main result. We have decomposed the Holevo-like quantity $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ into the plateau

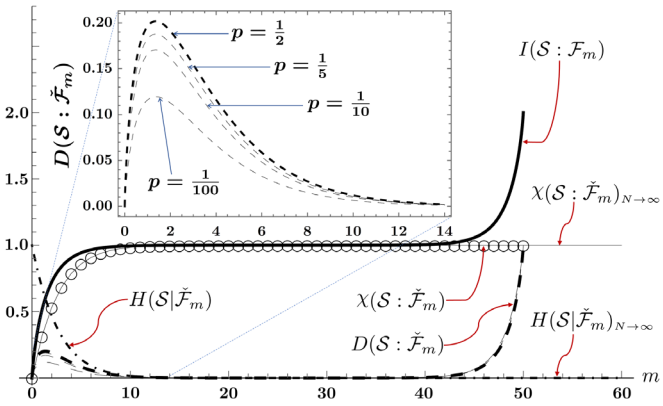


FIG. 2. Accessible information in the environment fragment. Fragment $\mathcal{F}_m$ carries at most $\chi(\mathcal{S}:\check{\mathcal{F}})$ of classical information about the system it helped decohere. As seen above, this Holevo-like quantity is less than the symmetric mutual information $I(\mathcal{S}:\mathcal{F})$ or the Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$. Their difference [quantum discord $D(\mathcal{S}:\check{\mathcal{F}})$] is significant already early on [in contrast to $D(\check{\mathcal{S}}:\mathcal{F})$], but disappears as the plateau is reached. It reappears again [as did $D(\check{\mathcal{S}}:\mathcal{F})$] when $\mathcal{F}_m$ begins to encompass almost all of $\mathcal{E}$.

entropy $H_{\mathcal{S}}$ and $H_{\mathcal{S}|\check{\mathcal{F}}_m}$—the ignorance about $\mathcal{S}$ remaining in spite of the optimal measurement on $\mathcal{F}_m$ [59]. Rather remarkably, $H_{\mathcal{S}|\check{\mathcal{F}}_m}=H_{\mathcal{S}}-\chi(\mathcal{S}:\check{\mathcal{F}}_m)$—the conditional entropy—scales *exactly* with pqs^{2m}. What remains to do is to quantify the differences of $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ with $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$. In Fig. 2 we compare it with these other, easier to compute, quantities.

Redundancy of the information about $\mathcal{S}$ in the channel $\mathcal{F}_m$ can be now computed using $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$, Eq. (20), and compared with the estimates based on $I(\mathcal{S}:\mathcal{F}_m)$. The fragment $\mathcal{F}_{m_\delta}$ can carry all but the deficit δ of the classical information about the pointer state of $\mathcal{S}$ when $\chi(\mathcal{S}:\check{\mathcal{F}}_{m_\delta})\geq(1-\delta)H_{\mathcal{S}}$. This leads to a transcendental equation for m_δ that we solve numerically: $R_\delta=N/m_\delta$, where N is the number of subsystems in $\mathcal{E}$.

The inset in Fig. 1 shows that—while m_δ deduced using $I(\mathcal{S}:\mathcal{F}_m)\approx\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ do not coincide with those obtained using $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$—the difference is modest, unlikely to materially affect conclusions about the emergence of objective classical reality. Indeed, in the Supplemental Material [54] we estimate that the redundancy estimates based on $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ differ at most by ~37% for $\delta\leq 0.2$, and by much less in the regime where $\delta\to 0$.

In situations relevant for observers who rely on photons, $R_{\delta=0.1}\simeq 10^8$ is amassed when sunlight illuminates a 1 μm dust grain in a superposition with a 1 μm spatial separation for 1 μs [48,49]. It may seem like we are stretching the applicability of our C-MAYBE model too far, but the equations for $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ derived for photon scattering *coincide* with our Eq. (17), see Supplemental Material [54]. Thus, it appears that the information transfer from $\mathcal{S}$ to $\mathcal{E}$ leading to the buildup of redundancy has universal features captured by our model.

Quantum discord is the difference between symmetric (9) and asymmetric quantum mutual information [32–36,38–41]. The *systemic discord* is defined as

$$D(\check{\mathcal{S}}:\mathcal{F}_m)=I(\mathcal{S}:\mathcal{F}_m)-\chi(\check{\mathcal{S}}:\mathcal{F}_m). \tag{21}$$

The measurements on pointer observables of $\mathcal{S}$ are optimal.

Mutual information for pure decoherence induced by noninteracting subsystems of $\mathcal{E}$ can be written as [46,60]

$$I(\mathcal{S}:\mathcal{F})=\overset{\text{local/classical}}{[H_{\mathcal{F}}-H_{\mathcal{F}}(0)]}+\overset{\text{global/quantum}}{(H_{\mathcal{S}d\mathcal{E}}-H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}})}. \tag{22}$$

As $\mathcal{SE}$ is a pure product state, the initial entropy of $\mathcal{F}$ is zero, $H_{\mathcal{F}}(0)=0$. Assuming good decoherence and conditioning on the pointer basis [hence, $\chi(\check{\mathcal{S}}:\mathcal{F}_m)=H_{\mathcal{F}_m}$], Eq. (16) we have

$$I(\mathcal{S}:\mathcal{F}_m)-J(\check{\mathcal{S}}:\mathcal{F}_m)=H_{\mathcal{S}d\mathcal{E}}-H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}_m}}, \tag{23}$$

where $H_{\mathcal{S}d\mathcal{E}}$ ($H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}}$) is the entropy of the system *decohered* by $\mathcal{E}$ (or just by $\mathcal{E}_{\setminus\mathcal{F}}$—i.e., $\mathcal{E}$ less the fragment $\mathcal{F}$).

The global or quantum term represents quantum discord in the pointer basis of $\mathcal{S}$ [46]. Good decoherence implies $H_{\mathcal{S}d\mathcal{E}}\approx H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}}$, so $D(\check{\mathcal{S}}:\mathcal{F}_m)\approx 0$. As long as $\mathcal{E}_{\setminus\mathcal{F}}$ is large enough to induce good decoherence, Eq. (16) holds, and, hence, the systemic discord (21) vanishes [61].

Systemic quantum discord can become large again when $\mathcal{F}_m$ encompasses almost all $\mathcal{E}$, as in this case $H_{\mathcal{SF}_m}$ approaches $H_{\mathcal{SE}}=0$ (given our assumption of a pure $\mathcal{SE}$). In this (unphysical) limit $I(\mathcal{S}:\mathcal{F}_m)$ climbs to $H_{\mathcal{F}_m}+H_{\mathcal{S}}=2H_{\mathcal{S}}$, while $\chi(\check{\mathcal{S}}:\mathcal{F}_m)\leq H_{\mathcal{F}_m}$. As good decoherence implies $\chi(\check{\mathcal{S}}:\mathcal{F}_m)\approx H_{\mathcal{F}_m}$, $D(\check{\mathcal{S}}:\mathcal{F}_m)$ can reach $H_{\mathcal{F}_m}$. Indeed, when $\mathcal{SE}$ is pure, $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ and $D(\check{\mathcal{S}}:\mathcal{F}_m)$—classical and quantum content of the correlation—are complementary [36], see Fig. 1.

The *fragmentary discord* is the difference between the mutual information $I(\mathcal{S}:\mathcal{F})$ and what can be extracted from $\mathcal{SE}$ by measuring only the fragment $\mathcal{F}$:

$$I(\mathcal{S}:\mathcal{F})-\chi(\mathcal{S}:\check{\mathcal{F}})\approx\chi(\check{\mathcal{S}}:\mathcal{F})-\chi(\mathcal{S}:\check{\mathcal{F}}). \tag{24}$$

It can be evaluated:

$$\begin{aligned}D(\mathcal{S}:\check{\mathcal{F}}_m)&\approx H_{\check{\mathcal{F}}_m}-(H_{\mathcal{S}}-H_{\mathcal{S}|\check{\mathcal{F}}_m}),\\&=(H_{\mathcal{S}|\check{\mathcal{F}}_m}+H_{\check{\mathcal{F}}_m})-H_{\mathcal{S}}.\end{aligned} \tag{25}$$

The bracketed terms in the last two expressions represent different quantities. The difference between the symmetric

and asymmetric mutual information $H_{\check{\mathcal{F}}_m} - (H_{\mathcal{S}} - H_{\mathcal{S}|\check{\mathcal{F}}_m})$ is the original definition of discord.

Note that initially decoherence does not suppress fragmentary discord $D(\mathcal{S}:\check{\mathcal{F}}_m)$. This is because the states of $\mathcal{F}_m$ that are correlated with the pointer states of $\mathcal{S}$ are not orthogonal: The scalar product of the branch fragments $\mathcal{F}_m$ corresponding to $|0_{\mathcal{S}}\rangle$ and $|1_{\mathcal{S}}\rangle$ is s^m. Thus, while the symmetric mutual information increases with m, orthogonality is approached gradually, also as m increases. Perfect distinguishability, i.e., orthogonality of record states of $\mathcal{F}$ is needed to pass on all the information about $\mathcal{S}$ [63–65]. See again Fig. 2 for an illustration of these findings.

Concluding remarks.—We found that in the pre-plateau regime relevant for emergence of objective reality [where $I(\mathcal{S}:\mathcal{F})$ increases with the size of $\mathcal{F}$] the mutual information as well as the Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F})$ coincide and exhibit universal scaling behaviors independent of the size of $\mathcal{E}$, of how imperfect are the C-MAYBE's, and only weakly dependent on the probabilities of pointer states. The corresponding Holevo $\chi(\check{\mathcal{S}}:\mathcal{F})$ and $I(\mathcal{S}:\mathcal{F})$ coincide until $\mathcal{F}$ encompasses almost all of $\mathcal{E}$.

However, the accessible information $\chi(\mathcal{S}:\check{\mathcal{F}})$ in the environment fragments $\mathcal{F}$ differs somewhat from $I(\mathcal{S}:\mathcal{F})$ in the pre-plateau region. This difference tends to be small compared to, e.g., the level of the plateau, and disappears as the plateau is reached for larger fragments. This behavior—generic when many copies of the information about $\mathcal{S}$ are deposited in the environment—facilitates estimates of the redundancy of the information about the system in the environment, as the differences between $I(\mathcal{S}:\mathcal{F}) \approx \chi(\check{\mathcal{S}}:\mathcal{F})$ or $\chi(\mathcal{S}:\check{\mathcal{F}})$ are noticeable but inconsequential.

To sum up, sensible measures of information flow lead to compatible conclusions about R_δ. The differences in the estimates of redundancy based on these quantities are insignificant for the emergence of objective classical reality—the overarching goal of Quantum Darwinism. The functional dependence of the symmetric mutual information in the photon scattering model [48,49] is the same as in our model. Thus, the universality we noted in scaling with s and p [approximate for $I(\mathcal{S}:\mathcal{F}) = \chi(\check{\mathcal{S}}:\mathcal{F}_m)$, exact for $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$] may be a common attribute of the information that reaches us human observers.

We acknowledge several discussions with Michael Zwolak, who has provided us with extensive and perceptive comments that have greatly improved presentation of our results. This research was supported by Grants No. FQXiRFP-1808 and No. FQXiRFP-2020-224322 from the Foundational Questions Institute and Fetzer Franklin Fund, a donor advised fund of Silicon Valley Community Foundation (S. D. and W. H. Z., respectively), as well as by the Department of Energy under the LDRD program in Los Alamos. A. T., B. Y., and W. H. Z. also acknowledge support from U.S. Department of Energy, Office of Science, Basic Energy Sciences, Materials Sciences and Engineering Division, Condensed Matter Theory Program, and the Center for Nonlinear Studies. D. G. acknowledges financial support from the Italian Ministry of Research and Education (MIUR), Grant No. 54_AI20GD01.

*akramt1@umbc.edu

[1] W. H. Zurek, Einselection and decoherence from an information theory perspective, Ann. Phys. (N.Y.) **9**, 855 (2000).
[2] W. H. Zurek, Decoherence, einselection, and the quantum origins of the classical, Rev. Mod. Phys. **75**, 715 (2003).
[3] H. Ollivier, D. Poulin, and W. H. Zurek, Objective Properties from Subjective Quantum States: Environment as a Witness, Phys. Rev. Lett. **93**, 220401 (2004).
[4] H. Ollivier, D. Poulin, and W. H. Zurek, Environment as a witness: Selective proliferation of information and emergence of objectivity in a quantum universe, Phys. Rev. A **72**, 042113 (2005).
[5] W. H. Zurek, Quantum Darwinism, Nat. Phys. **5**, 181 (2009).
[6] G. L. Giorgi, F. Galve, and R. Zambrini, Quantum Darwinism and non-markovian dissipative dynamics from quantum phases of the spin-1/2xx model, Phys. Rev. A **92**, 022105 (2015).
[7] N. Balaneskoviċ, Random unitary evolution model of Quantum Darwinism with pure decoherence, Eur. Phys. J. D **69**, 232 (2015).
[8] N. Balaneskovic and M. Mendler, Dissipation, dephasing and Quantum Darwinism in qubit systems with random unitary interactions, Eur. Phys. J. D **70**, 177 (2016).
[9] P. A. Knott, T. Tufarelli, M. Piani, and G. Adesso, Generic Emergence of Objectivity of Observables in Infinite Dimensions, Phys. Rev. Lett. **121**, 160401 (2018).
[10] N. Milazzo, S. Lorenzo, M. Paternostro, and G. M. Palma, Role of information backflow in the emergence of Quantum Darwinism, Phys. Rev. A **100**, 012101 (2019).
[11] S. Campbell, B. Çakmak, O. E. Müstecaplıoğlu, M. Paternostro, and B. Vacchini, Collisional unfolding of Quantum Darwinism, Phys. Rev. A **99**, 042103 (2019).
[12] E. Ryan, M. Paternostro, and S. Campbell, Quantum Darwinism in a structured spin environment, arXiv:2011.13385.
[13] G. García-Pérez, D. A. Chisholm, M. A. C. Rossi, G. M. Palma, and S. Maniscalco, Decoherence without entanglement and Quantum Darwinism, Phys. Rev. Research **2**, 012061(R) (2020).
[14] S. Lorenzo, M. Paternostro, and G. M. Palma, Anti-zeno-based dynamical control of the unfolding of Quantum Darwinism, Phys. Rev. Research **2**, 013164 (2020).
[15] M. Kiciński and J. K. Korbicz, Decoherence and objectivity in higher spin environments, Phys. Rev. A **104**, 042216 (2021).
[16] J. Korbicz, Roads to objectivity: Quantum Darwinism, spectrum broadcast structures, and strong Quantum Darwinism, Quantum **5**, 571 (2021).
[17] R. Blume-Kohout and W. H. Zurek, Quantum Darwinism in Quantum Brownian Motion, Phys. Rev. Lett. **101**, 240405 (2008).

[18] M. Zwolak, C. J. Riedel, and W. H. Zurek, Amplification, Redundancy, and Quantum Chernoff Information, Phys. Rev. Lett. **112**, 140406 (2014).

[19] M. Zwolak, H. T. Quan, and W. H. Zurek, Quantum Darwinism in a Mixed Environment, Phys. Rev. Lett. **103**, 110402 (2009).

[20] J. P. Paz and A. J. Roncaglia, Redundancy of classical and quantum correlations during decoherence, Phys. Rev. A **80**, 042111 (2009).

[21] C. J. Riedel, W. H. Zurek, and M. Zwolak, Objective past of a quantum universe: Redundant records of consistent histories, Phys. Rev. A **93**, 032126 (2016).

[22] C. J. Riedel, Classical Branch Structure from Spatial Redundancy in a Many-Body Wave Function, Phys. Rev. Lett. **118**, 120402 (2017).

[23] F. G. Brandão, M. Piani, and P. Horodecki, Generic emergence of classical features in Quantum Darwinism, Nat. Commun. **6**, 7908 (2015).

[24] X.-L. Qi and D. Ranard, Emergent classicality in general multipartite states and channels, Quantum **5**, 555 (2021).

[25] H.-F. Fu, Uniqueness of the observable leaving redundant imprints in the environment in the context of Quantum Darwinism, Phys. Rev. A **103**, 042210 (2021).

[26] M. A. Ciampini, G. Pinna, P. Mataloni, and M. Paternostro, Experimental signature of Quantum Darwinism in photonic cluster states, Phys. Rev. A **98**, 020101(R) (2018).

[27] M.-C. Chen, H.-S. Zhong, Y. Li, D. Wu, X.-L. Wang, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Emergence of classical objectivity of Quantum Darwinism in a photonic quantum simulator, Sci. Bull. **64**, 580 (2019).

[28] T. K. Unden, D. Louzon, M. Zwolak, W. H. Zurek, and F. Jelezko, Revealing the Emergence of Classicality Using Nitrogen-Vacancy Centers, Phys. Rev. Lett. **123**, 140402 (2019).

[29] G. García-Pérez, M. A. C. Rossi, and S. Maniscalco, IBM Q Experience as a versatile experimental testbed for simulating open quantum systems, npj Quantum Inf. **6**, 1 (2020).

[30] A. S. Holevo, Bounds for the quantity of information transmitted by a quantum communication channel, Prob. Peredachi Inf. **9**, 3 (1973).

[31] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2002).

[32] H. Ollivier and W. H. Zurek, Quantum Discord: A Measure of the Quantumness of Correlations, Phys. Rev. Lett. **88**, 017901 (2001).

[33] L. Henderson and V. Vedral, Classical, quantum and total correlations, J. Phys. A **34**, 6899 (2001).

[34] P. Giorda and M. G. A. Paris, Gaussian Quantum Discord, Phys. Rev. Lett. **105**, 020503 (2010).

[35] M. Shi, W. Yang, F. Jiang, and J. Du, Quantum discord of two-qubit rank-2 states, J. Phys. A **44**, 415304 (2011).

[36] M. Zwolak and W. H. Zurek, Complementarity of quantum discord and classically accessible information, Sci. Rep. **3**, 1729 (2013).

[37] M. Zwolak, C. J. Riedel, and W. H. Zurek, Amplification, decoherence and the acquisition of information by spin environments, Sci. Rep. **6**, 1 (2016).

[38] A. Brodutch, A. Gilchrist, D. R. Terno, and C. J. Wood, Quantum discord in quantum computation, J. Phys. **306**, 012030 (2011).

[39] G. Adesso, M. Cianciaruso, and T. R. Bromley, An introduction to quantum discord and non-classical correlations beyond entanglement, arXiv:1611.01959.

[40] A. Bera, T. Das, D. Sadhukhan, S. S. Roy, A. Sen(De), and U. Sen, Quantum discord and its allies: A review of recent progress, Rep. Prog. Phys. **81**, 024001 (2017).

[41] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, The classical-quantum boundary for correlations: Discord and related measures, Rev. Mod. Phys. **84**, 1655 (2012).

[42] R. Blume-Kohout and W. H. Zurek, A simple example of Quantum Darwinism: Redundant information storage in many-spin environments, Found. Phys. **35**, 1857 (2005).

[43] W. H. Zurek, Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?, Phys. Rev. D **24**, 1516 (1981).

[44] W. H. Zurek, Environment-induced superselection rules, Phys. Rev. D **26**, 1862 (1982).

[45] Deducing the reduced quantum states $\rho_{\mathcal{S}}$ and $\rho_{\mathcal{S}\mathcal{F}_m}$ in Eqs. (5) and (7) is straightforward, whereas $\rho_{\mathcal{F}_m}$ is slightly more involved, see e.g. [46].

[46] M. Zwolak, H. T. Quan, and W. H. Zurek, Redundant imprinting of information in nonideal environments: Objective reality via a noisy channel, Phys. Rev. A **81**, 062110 (2010).

[47] R. Blume-Kohout and W. H. Zurek, Quantum Darwinism: Entanglement, branches, and the emergent classicality of redundantly stored quantum information, Phys. Rev. A **73**, 062310 (2006).

[48] C. J. Riedel and W. H. Zurek, Quantum Darwinism in an Everyday Environment: Huge Redundancy in Scattered Photons, Phys. Rev. Lett. **105**, 020404 (2010).

[49] C. J. Riedel and W. H. Zurek, Redundant information from thermal illumination: Quantum Darwinism in scattered photons, New J. Phys. **13**, 073038 (2011).

[50] T. M. Cover and J. A. Thomas, *Elements of Information Theory* (John Wiley & Sons, New York, 1999).

[51] Note that $H_{\mathcal{S}}$—in the circumstances of interest to us—is not the thermodynamic entropy of $\mathcal{S}$. Rather, it is the missing information about the few *relevant* degrees of freedom of $\mathcal{S}$. The thermodynamic entropy of a cat, for instance, will vastly exceed the information an observer is most interested in—e.g., the one crucial bit in the 'diabolical contraption' envisaged by Schrödinger.

[52] A. Touil, K. Weber, and S. Deffner, Quantum euler relation for local measurements, Entropy **23**, 889 (2021).

[53] M. Koashi and A. Winter, Monogamy of quantum entanglement and other correlations, Phys. Rev. A **69**, 022309 (2004).

[54] See Supplemental Material http://link.aps.org/supplemental/10.1103/PhysRevLett.128.010401, which contains the technical details leading to the analytic expression of the information, about $\mathcal{S}$, accessible in environmental fragments, which includes Refs. [54–57].

[55] W. K. Wootters, Entanglement of formation and concurrence, Quantum Inf. Comput. **1**, 27 (2001).

[56] S. Hill and W. K. Wootters, Entanglement of a Pair of Quantum Bits, Phys. Rev. Lett. **78**, 5022 (1997).

[57] W. K. Wootters, Entanglement of Formation of an Arbitrary State of Two Qubits, Phys. Rev. Lett. **80**, 2245 (1998).

[58] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Quantifying Entanglement, Phys. Rev. Lett. **78**, 2275 (1997).

[59] Strictly speaking, Holevo bound χ on the channel capacity presumes conditioning on a predetermined classical basis (such as the pointer basis of $\mathcal{S}$ in $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$). In general, $\mathcal{F}_m$ in $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ does not possess such a predetermined classical basis, hence the "Holevo-like" designation.

[60] W. H. Zurek, Relative states and the environment: Einselection, envariance, Quantum Darwinism, and the existential interpretation, arXiv:0707.2832.

[61] Some have advocated "strong Quantum Darwinism" [62] where the Holevo information $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ based on the measurement of $\mathcal{S}$ (rather than $I(\mathcal{S}:\mathcal{F}_m)$) would play a key role. At least in case of good decoherence, and in view of Eq. (16) which implies $\chi(\check{\mathcal{S}}:\mathcal{F}_m) \approx I(\mathcal{S}:\mathcal{F}_m)$ when $m \ll N - m < N$, such distinctions appear unnecessary.

[62] T. P. Le and A. Olaya-Castro, Strong Quantum Darwinism and Strong Independence Are Equivalent to Spectrum Broadcast Structure, Phys. Rev. Lett. **122**, 010403 (2019).

[63] W. H. Zurek, Quantum origin of quantum jumps: Breaking of unitary symmetry induced by information transfer in the transition from quantum to classical, Phys. Rev. A **76**, 052110 (2007).

[64] W. H. Zurek, Wave-packet collapse and the core quantum postulates: Discreteness of quantum jumps from unitarity, repeatability, and actionable information, Phys. Rev. A **87**, 052111 (2013).

[65] B. Gardas, S. Deffner, and A. Saxena, Repeatability of measurements: Non-hermitian observables and quantum Coriolis force, Phys. Rev. A **94**, 022121 (2016).