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Abstract: We propose an accurate nonlinearity closed-form model (CFM) for forward Raman-amplified WDM links which enables fast system optimization. We show a detailed study of a single-span link SNR maximization and flattening.

1. Introduction

Optical systems with Distributed Raman Amplification (DRA) increasingly play a key role in modern high capacity data transmission networks. DRA drastically improves performance in multi-span systems [1,2]. It also enables *single-span* long-distance links in many critical scenarios, at substantially lower cost than multi-span solutions, often by means of *both* backward and forward-pumped DRA [3,4].

A key issue in the design of such Raman-amplified systems is the optimization of pump frequency and power, as well as channel launch power. To perform such optimization, models for fiber non-linear effects (or NLI, Non-Linear-Interference) are needed, that are accurate and *fast computable*. While several NLI models are available (see [5] for overview), none of them is real-time in their original form, as they all include numerical integrals. Recently, though, Closed-Form Models (CFMs) that closely approximate the GN/EGN models have been proposed, capable of assessing whole links in fractions of a second [6,7]. However, a CFM accurately supporting Forward-pumped DRA (F-DRA) has not been available so far, making it computationally difficult to address the optimization of long and ultra-long single-span links, or of multi-span links where one or more spans use F-DRA.

In this paper we describe a CFM that accurately approximates the EGN model and takes into account both F-DRA and Inter-Channel Stimulated Raman Scattering (ISRS). We then focus on a specific 235-km single-span link that uses both F-DRA and Backward-pumped DRA (B-DRA), whose layout and parameters closely match those of an existing submarine system. We analyze and carefully optimize the link, under the constraint of obtaining the highest and most flat SNR profile across the whole C-band. The optimization results found with the CFM are then extensively compared to two high-accuracy independent benchmarks: full C-band split-step simulations and the full numerically-integrated EGN model. In both cases we find a very good match.

2. Modeling power evolution and channel SNR

To obtain a CFM supporting forward-pumped Raman amplification, a simple model for the power evolution of each channel is needed. The model that we use, proposed in [8], assumes that *loss* for the m-th WDM channel in the presence of F-DRA can be written vs. distance z as:

$$\alpha_m(z) = \alpha_{0,m} + \alpha_{1,m} \cdot \exp(-\sigma_m \cdot z) \tag{1}$$

Note that $\alpha_{0,m}$, $\alpha_{1,m}$ and σ_m can be all different, channel by channel. The model is inspired by an approximate physical picture where $\alpha_{0,m}$ is fiber loss in the absence of F-DRA; $\alpha_{1,m}$ is the F-DRA *gain* experienced at the start of the fiber (hence is negative); σ_m regulates how fast the F-DRA gain extinguishes along the fiber and loosely relates to pump loss. However, instead of trying to use empirical laws to assign the above parameters based on such physical interpretation, we decided to fit them so that they best approach the actual power evolution of each channel. Such evolution is found by numerically integrating the forward-Raman differential equations (Eq. (1) in [9]), which can be done with high accuracy and low calculation time. An example of such fitting is Fig. 1(g), which shows the high accuracy of the model over the high-power section of the span, where NLI is generated.

Remarkably, Eq. (1) *permits to derive a NLI model in closed-form*, which is by far the most critical aspect for enabling overall fast computation. Performance estimation hinges on the total SNR, which is defined as:

$$SNR = \left[(SNR^{\mathrm{NLI}})^{-1} + (SNR^{\mathrm{ASE}})^{-1}\right]^{-1} \qquad \text{where :} \qquad SNR^{\mathrm{NLI}} = P_{\mathrm{ch}}/P^{\mathrm{NLI}} \tag{2}$$

where P_{ch} and P^{NLI} are signal power and NLI power in the channel under test (CUT), at the receiver input. SNR^{ASE} can be found very accurately with well-known formulas and relatively low complexity.

Based on Eq. (1), SNR^{NLI} for the n-th WDM channel, assumed to be the 'channel under test' or CUT, at the end of a single-span link with F-DRA, can be derived in closed form as:

$$SNR_n^{\mathrm{NLI}} = \left[\frac{16}{27}\gamma^2 \sum_{\substack{1\le m\le N_c\\ 0\le k\le M,\, 0\le q\le M\\ 0\le j\le 1}} \frac{P_{0,m}^2 \cdot \rho_m \cdot (2-\delta_{m,n}) \cdot e^{-4\alpha_{1,m}/\sigma_m}\cdot(-1)^j}{2\pi R_m^2 \cdot k!q!\cdot \bar{\beta}_{2,m}\cdot(4\alpha_{0,m}+(k+q)\sigma_m)} \cdot \left(\frac{2\alpha_{1,m}}{\sigma_m}\right)^{k+q} \psi_{m,n,j,k}\right]^{-1} \tag{3}$$

where $P_{0,m}$, R_m, $\bar{\beta}_{2,m}$, ρ_m are for the m-th channel, respectively, the input launch power, the symbol rate, the effective dispersion (see Eq. (5) in [7]), and the 'correction term' (see Eq. (14) and Table IV in [7]) evaluated at the peak power location for that channel. γ is the nonlinear coefficient of the fiber. $\delta m,n$ is 1 if $m=n$, and 0 otherwise. N_c is the number of channels. Furthermore:

$$\psi_{m,n,j,k} = \operatorname{asinh}\left(\left\{\pi^2 \bar{\beta}_{2,m} R_n \cdot \left(f_m - f_n + \left[(-1)^j \cdot R_m/2\right]\right)\right\} / \{2\alpha_{0,m} + k\sigma_m\}\right)$$

where f_i represents the center frequency of the i-th channel. Also, $M = \max\{\lfloor 10\cdot|2\alpha_{1,m}/\sigma_m|\rfloor + 1\}$ where the maximum is calculated across all channels ($1 \le m \le N_c$) in the WDM comb. M represents the order of a series expansion, whereby more terms are called into play as the strength of F-DRA goes up. This series expansion is *one of the key modeling novelties* that we propose. Without it, F-DRA *cannot* be modeled accurately. The details of the derivation of Eq. (3) can be found in [10], but all that is needed to implement and run the model is included in this paper. Note that, besides F-DRA, Eq. (3) takes ISRS into account too. It does *not* take into account B-DRA. The reason is that the power evolution model Eq. (1) cannot structurally account for B-DRA. However, in most deployed systems, and in particular in those using F-DRA, the NLI that is produced at the end of the link is negligible vs. the NLI produced at the start of the link. In this paper we make such assumption. Otherwise, B-DRA can be accounted for as shown for instance in [11]. However, we carefully take into account B-DRA, channel by channel, as for its impact on linear loss/gain and SNR^{ASE}.

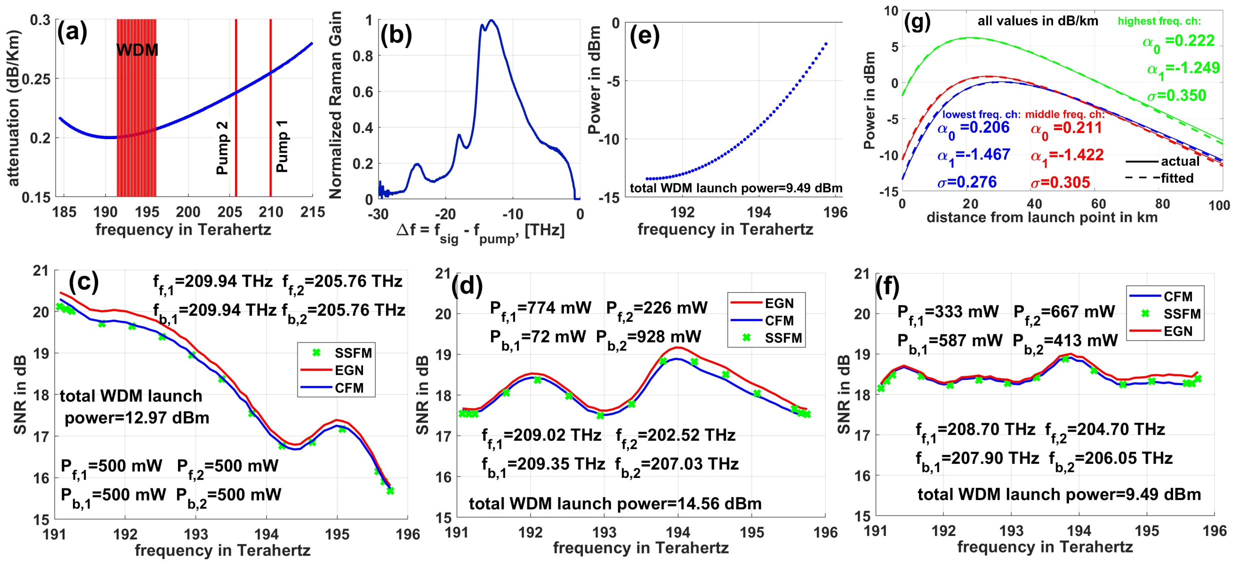

Fig. 1. **(a):** fiber attenuation profile versus frequency. **(b):** fiber normalized Raman gain. **(c):** system SNR with uniform optimized input power and fixed nominal wavelengths and powers for Raman pumps. **(d):** system SNR with uniform optimized input power and optimized wavelengths and powers for Raman pumps. **(e):** optimized quadratic channel launch power profile. **(f):** system SNR with the optimized channel launch power profile (e) and optimized wavelengths and powers for Raman pumps. **(g):** spatial power evolution up to 100 km for the system whose SNR is shown in (f), from Raman equation integration (solid) or bestfit using Eq. (1) (dashed).

3. Test link description and power optimization results

A 235 km single-span SMF link is considered, whose parameters mimic an actual operational undersea link. The fiber loss and normalized Raman gain profile of the fiber are depicted in Fig. 1 (a) and (b) respectively. The max Raman gain is $0.392\ (\mathrm{W\cdot km})^{-1}$. The nonlinearity coefficient, second and third order dispersion parameters of the fiber are $\gamma = 1.2\ (\mathrm{W\cdot km})^{-1}$, $\beta_2 = -21.68\ \mathrm{ps^2/km}$ and $\beta_3 = 0.1445\ \mathrm{ps^3/km}$, respectively. We consider a full C-band comb in the range 191.4 to 196.1 THz (1528.8-1566.3 nm) which contains 56 channels at 64 GBaud, roll-off 0.2 and channel spacing $\Delta f = 85$ GHz, transmitting PM-64QAM. For *forward Raman amplification*, two pumps are placed at 210 and 205.75 THz (to be optimized), see Fig. 1 (a). The total power into the fiber of the two pumps is limited to 1 Watt. Another two pumps are injected at the Rx side for backward Raman amplification, with same nominal frequencies and maximum total power. At the Rx end there is also an EDFA with noise figure

5 dB. Our optimization goal is to *maximize the minimum-SNR* across all channels at the Rx. This procedure also tends to lead to a flatter SNR profile. We use standard Matlab optimization software based on the Nelder-Mead simplex algorithm to achieve it. For each algorithm iteration, the Raman differential equations (Eq. (1) in [9]) are first solved for the two backward pumps without any signal. Then, we solve them for both the WDM signal and the forward pumps. Note that for B-DRA we assume undepleted pumps (also verified a posteriori), while for F-DRA we take pump depletion fully into account. Next, using Eq. (3) in [9], the ASE due to spontaneous Raman Scattering is calculated for both F-DRA and B-DRA and is added to the Rx EDFA ASE noise to obtain SNR_n^{ASE}. Then, for each channel we best-fit the three parameters $\alpha_{0,m}$, $\alpha_{1,m}$ and σ_m and use them in Eq. (3) to find SNR_n^{NLI}. Finally, total SNR is calculated according to Eq. (2) for each channel and passed to the optimization algorithm, which computes the new values of all parameters for the following iteration.

We applied the above procedure to the optimization of the described links, as follows. In a first run, we kept all pumps at their nominal frequencies and at 0.5 W launch power each. Equal launch power P_0 was assumed for all WDM channels, and only P_0 was optimized. The resulting SNR is shown in Fig. 1(c). The minimum SNR across all channels was 15.75 dB and the ΔSNR between min and max was a very large 4.56 dB.

Next, besides P_0, the frequencies and launch powers of all four pumps were subject to optimization, keeping the total pump power constraints indicated above. The optimized values are reported in Fig. 1(d). The minimum SNR was a much higher 17.5 dB and ΔSNR was down to 1.38 dB. The last optimization removed channel launch power uniformity: we considered a quadratic law which contains three unknown parameters A, B, C, i.e.: $P_{0,m} = A + B \times m + C \times m^2$ (in dBm). The resulting optimized channel launch power profile is shown in Fig. 1(e). The optimized values of pump frequencies and powers are written in Fig. 1(f). The minimum SNR was now up to 18.24 dB and ΔSNR was down to a remarkably low 0.66 dB. This optimization, which involved 11 free parameters overall, took about 30 minutes on a modern desktop, without using any GPUs. If we had used the numerically-integrated EGN model for the optimization, we estimate that about 5 days would have been needed.

After optimization, the three systems SNR was also assessed using (i) the full-band split-step simulations and (ii) the full numerically-integrated EGN model. Note that both (i) and (ii) used the exact power evolution delivered by numerical integration of the Raman differential equations, and not Eq. (1). These two independent validations agree very well with the CFM results, as shown in Fig. 1(c),(d),(f). This suggests that the new CFM which we propose in this paper retains the very good accuracy of the CFMs presented in [7], while fully supporting both ISRS and F-DRA. It also suggests that the power evolution model Eq. (1), with channel-by-channel best-fitted parameters, is quite effective, as also shown by Fig. 1(g), and does not degrade accuracy. Finally, note that while tested here on a single-span system for simplicity, the CFM Eq. (3) is capable of modeling multi-spans systems as well, using the same procedure as in [7].

4. Comments and conclusion

A closed-form-model (CFM) of fiber nonlinearity in forward-pumped Raman amplified systems is presented. A single-span link was optimized based on the max-min-SNR strategy and the obtained results were verified through the EGN model and split step simulations. Accuracy and low computation time are the two main advantages which make the proposed CFM a suitable tool for the optimization of WDM coherent systems with Raman amplification.
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