# Real Time Closed-Form Model for Nonlinearity Assessment of Fibre Optic Links with Lumped Loss

# Real Time Closed-Form Model for Nonlinearity Assessment of Fibre Optic Links with Lumped Loss

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**Abstract** *We upgraded the recently proposed closed-form model (CFM) for non-linear interference (NLI) estimation to account for lumped losses. Statistical validation analyses show a high level of accuracy with maximum errors below 1.1 dB in NLI which leads to GSNR estimation errors below 0.35 dB.*

## Introduction

In high-capacity wide-band optical systems, network control and operation must be optimized to avoid wasting resources. It has been demonstrated[1-2] that an effective approach is to bring physical layer awareness at the control plane level. A fundamental condition required to achieve this goal is the capability to get accurate estimation of non-linear interference (NLI) generated by Kerr effect during fiber propagation. In recent years, several models[3-8] have been proposed and validated. Most of them, despite their high level of accuracy, are not suitable for real-time use in network control and management because of the required computational effort. For this class of applications, fast closed-form models (CFMs) are needed, with the constraint of not compromising accuracy.

In this work, we consider the CFM proposed in[9] that has been thoroughly validated in a comprehensive set of conditions and we extend its validity to links affected by lumped losses. In real-world operation of optical line system installed in the field, the presence of lumped losses is unavoidable because of splices, connectors, and bends. Lumped losses characterization, both as to their position and attenuation, can be achieved through the analysis of OTDR traces, that are usually available to network operators. This information must be properly included in the NLI evaluation, especially when a lumped loss falls in the first portion of the span, as it has a strong impact on the signal power evolution and NLI generation.

We focus on a single span scenario and we propose two different approaches to upgrade the CFM proposed in[9]. In particular we select the most accurate version labelled CFM4. Through a statistical analysis of the NLI estimation error, we show the high level of effectiveness of the proposed corrections.

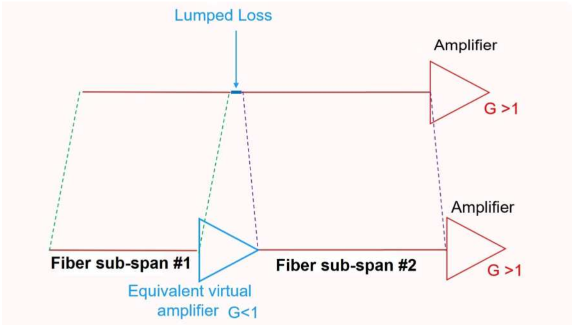


**Fig. 1:** Split-span model for lumped loss.

## The split-span model

The EGN model[5] can be used to accurately evaluate the NLI also in case of lumped losses by considering the split-span model, as shown in Fig. 1 for a single lumped loss. Every lumped loss is modeled with an *equivalent virtual amplifier* with a gain lower than 1, so that a single span is split in two sub-spans. This approach can be extended to more than one lumped loss. EGN can be used as a reference, but for real-time applications we must resort to a CFM, as proposed in[9]. First, we tested the accuracy of CFM with the split-span model. We focus on a single-span link, with 100 km length and an amplifier at the end. Fiber type is standard single mode (SMF) with parameters $\beta_2 = -21.68\, ps^2/km$, $\beta_3 = 0.12\ \ ps^3/km$, $\alpha = 0.21\, dB/km$ and $\gamma = 1.3\ (W.km)^{-1}$. The link is loaded with a comb of 45 Nyquist-WDM channels, each modulated at 69 Gbaud with PM-64QAM. Roll-off is set to 0.05 and channel spacing to 75 GHz. All comparisons are made by evaluating the normalized nonlinear coefficient ($\eta_{NLI}$), defined as the amount of NLI generated in the span divided by launch power cube.

In Fig. 2 we report the results for the case of a single lumped loss equal to 0.5 dB. We plot $\eta_{NLI}$ for the center channel as a function of loss position in the span.

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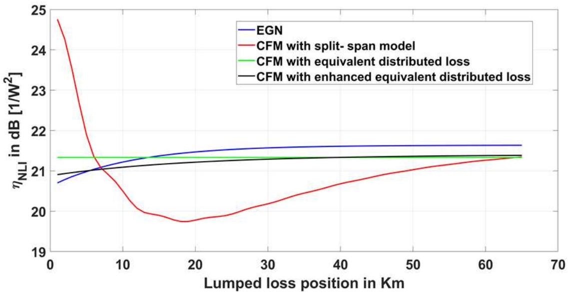


**Fig. 2:** Comparison of EGN, CFM and modified CFMs: $\eta_{NLI}$ as a function of the single lumped loss position.

Considering the EGN as a reference (blue line), we can see that the equivalent split-span model (red line) does not deliver a satisfactory NLI estimation with error extending beyond 4 dB.

The error reaches very high values because one key approximation applied in deriving CFM is that each span must have at least 8-10 dB of loss. Using the split-span model, we have now sub-spans with very short length and very low attenuation.

### Introducing the equivalent distributed loss

To improve the accuracy of CFM in the presence of lumped losses we must modify it to include such extra attenuation. Let us consider $\alpha$ as the distributed loss of the fiber span and we assume it to be frequency independent. Considering that we have $N^{LL}$ points of lumped loss, the total loss that is experienced by a signal in the span will be:

$$L = \exp(-2\alpha\, L_s) \times \prod_{j=1}^{N^{LL}} L'_j \qquad (1)$$

where $L_s$ is the span length and $L'_j$ is the amount of extra attenuation due to j-th lumped loss. As an alternative approach with respect to the split-span model, we propose to use in the CFM an equivalent distributed loss coefficient ($\alpha_{eq}$) in such a way that the total loss due to the span presented in (1) is exactly equal to an equivalent distributed loss as:

$$\alpha_{eq} = \alpha - \frac{\sum_{j=1}^{N^{LL}} ln\left(L'_j\right)}{2\times L_s} \qquad (2)$$

Results obtained with this method are reported in Fig. 2 (green line): we can observe an improved accuracy and the maximum error is reduced to 0.63 dB. Note that when the lumped loss position moves toward the end of the span, results converge to the split-span approach and they both levels at around 0.3 dB from the EGN, as expected from inherent error of CFM model[9]. To further validate this approach, we also statistically evaluated the error in the presence of two and three lumped losses in the span, each introducing a 0.5 dB of attenuation. We then generated 1000 random samples for loss position in both conditions. As shown in Fig. 3, the first, second and third lumped loss occur at $l_1$, $l_2$, and $l_3$ km, respectively. Lengths $l_1, l_2$ and $l_3$ are three independent random variables uniformly distributed between 0 and 10 km. In each sample the channel under test is selected randomly in the WDM comb and the $\eta_{NLI}$ are calculated both with EGN and CFM modified with the equivalent distributed loss. In Fig. 4 the probability density functions of:

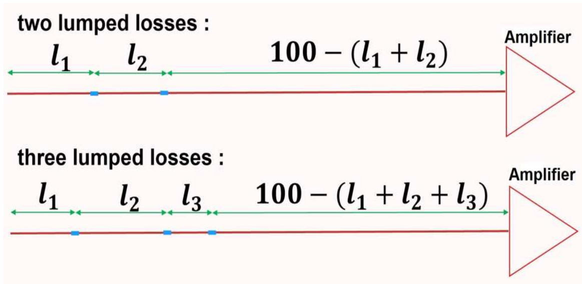


**Fig. 3:** Lumped losses positions in the statistical analysis.

$$\Delta\eta_{NLI} \triangleq 10 \cdot \log(\eta_{NLI,CFM}/\eta_{NLI,EGN}) \qquad (3)$$

for two and three lumped loss random cases are reported. NLI estimation errors are not as large as in the split-span model, but the maximum absolute error can still be as large as 1.55 dB and 2.29 dB, for the case of two and three lumped losses, respectively, not good enough for application in network optimization.

### Further improving the CFM

Looking for further improvements, we analysed in detail the CFM expressions to understand where it fails in NLI prediction. In the link function of the GN and EGN models a key element is the following integral[10]:

$$I(f_1, f_2, f) = \int_0^{L_s} \rho(z) \times \exp(j4\pi^2 \times (f_1 - f) \times (f_2 - f) \times [\beta_2 + \pi \times (f_1 + f_2) \times \beta_3] \times z)\, dz \qquad (4)$$

that plays an important role in the NLI evaluation. In this expression we can highlight the presence of the factor $\rho(z)$ that represents the normalized power evolution (with respect to input launched power) along z. From (4), it can be seen that the absolute value of $I(f_1, f_2, f)$ takes its maximum when $f_1 = f$ or $f_2 = f$ and is:

$$I_{max} \triangleq \max_{f_1,f_2,f}\{|I(f_1, f_2, f)|\} = \int_0^{L_s} \rho(z) dz \qquad (5)$$

If the span has only distributed loss without any lumped loss, we have $\rho(z) = \exp(-2\alpha\, z)$ and (5) becomes (assuming $\exp(-2\alpha\, L_s) \ll 1$):

$$I_{max} = \int_0^{L_s} \rho(z) dz = \int_0^{L_s} \exp(-2\alpha\, z)\, dz \cong \frac{1}{2\alpha} \qquad (6)$$

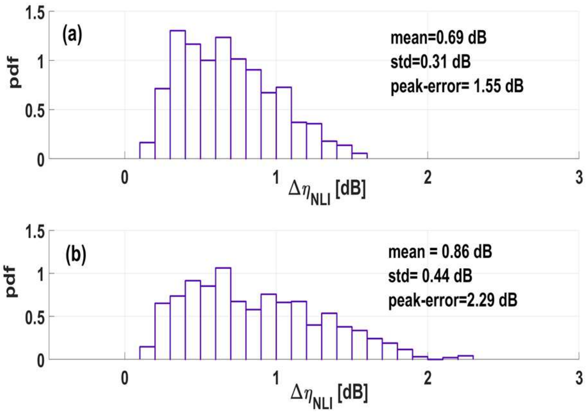


**Fig. 4:** Probability density functions of $\Delta\eta_{NLI}$ error obtained when applying the modified CFM with equivalent distributed loss: (a) 2 lumped losses and (b) 3 lumped losses.

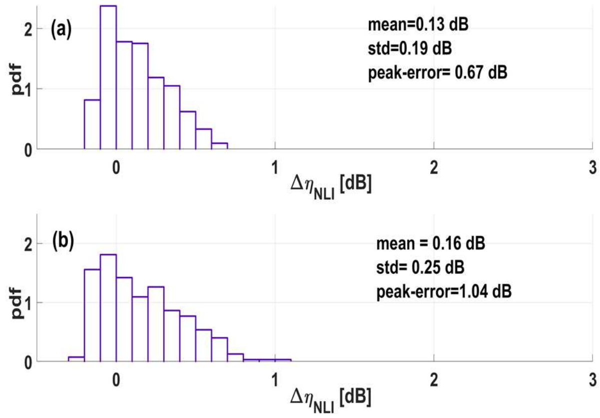


**Fig. 5:** Probability density functions of $\Delta\eta_{NLI}$ error obtained when applying the modified CFM with enhanced equivalent distributed loss: (a) 2 lumped losses and (b) 3 lumped losses.

If we have $N^{LL}$ lumped losses in the span, using the notation presented in previous section, the normalized power function can be expressed as:

$$\rho\,(z) = \exp(-2\alpha\, z) \times \sum_{r=1}^{(N^{LL}+1)} \left(\prod_{j=0}^{r-1} L'_j\right) \times \left(u(z - L^{LL}_{(r-1)}) - u(z - L^{LL}_r)\right) \quad (7)$$

$L^{LL}_r$ is defined as the location of r-th lumped loss in the span here assuming $L^{LL}_0 = 0$ and $L^{LL}_{(N^{LL}+1)} = L_s$ in (7). Also the function $u(x)$ in (7) is equal to 1 when $x \geq 0$ and is equal to 0 when $x < 0$.

To improve the CFM accuracy, we try to find a second equivalent loss parameter, called $\alpha'_{eq}$ which gives the same values of $I_{max}$ when equation (7) holds. Considering (7) and (8), we should have:

$$I_{max} = \int_0^{L_s} \rho\,(z)dz = \sum_{r=1}^{(N^{LL}+1)} \left(\prod_{j=0}^{r-1} L'_j\right) \times \int_{L^{LL}_{,(r-1)}}^{L^{LL}_r} \exp(-2\alpha\, z)dz = \frac{1}{2\alpha} \times \sum_{r=1}^{(N^{LL}+1)} \left(\prod_{j=0}^{r-1} L'_j\right) \times \left[exp\left(-2\alpha\, L^{LL}_{(r-1)}\right) - exp\left(-2\alpha\, L^{LL}_r\right)\right] = \frac{1}{2\alpha'_{eq}} \quad (8)$$

After simplification we have:

$$\alpha'_{eq} = \frac{\alpha}{\sum_{r=1}^{(N^{LL}+1)} \left(\prod_{j=0}^{r-1} L'_j\right) \times \left[exp\left(-2\alpha\, L^{LL}_{(r-1)}\right) - exp\left(-2\alpha\, L^{LL}_r\right)\right]} \quad (9)$$

In the derivation of CFM, $\alpha\,(f_{CUT})$ in (1),(2) in[9] are due to total span loss but $\alpha\,(f_{CUT})$ in (3),(4) in[9] are due to analytical approximation of the $I\,(f_1, f_2, f)$ presented in (4). Therefore in our final approach we replace $\alpha\,(f_{CUT})$ in (1),(2) in[9] with $\alpha_{eq}$ presented in (2) while $\alpha\,(f_{CUT})$ in (3),(4) in[9] will be replaced with $\alpha'_{eq}$ presented in (9).

In the following we refer to this approach as the enhanced equivalent distributed loss. We first test this new approach on the single lumped loss case, see results in Fig. 2 (black line): the maximum error is now 0.28 dB, considerably better than for the case of equivalent distributed loss.

To see if this improvement is maintained also for the cases of two and three lumped losses, we repeated the statistical analysis. In Fig. 5 we report the probability density function for NLI errors, same definition as for Fig. 4. Accuracy is improved and maximum absolute errors now are 0.67 dB and 1.04 dB, for the two and three lumped loss cases, respectively.

Considering that the NLI error impact in dB on the final GSNR, or maximum reach evaluation, is reduced by a factor of 1/3, see[4], the above figures reduce to 0.22 dB and 0.35 dB. We believe that this level of accuracy should be acceptable for most applications in real-time network optimization and control. Also notice that the model presented here can be used, as it is, in a multi-span link.

## Conclusions

We presented an extension of the closed-form model CFM4[9] to account for the presence of lumped losses in a single span link. In particular we have demonstrated that including the extra amount of loss as an equivalent distributed attenuation parameter is not sufficient to properly capture the reduced NLI generation. We propose a novel correction in the link function to obtain a CFM which allows much more precise NLI predictions. Through a statistical analysis of two classes of systems, with two and three losses in the span, we have quantified the level of accuracy with maximum errors on GSNR below 0.35 dB.

## Acknowledgements

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