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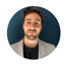

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***Abstract*—Energy efficiency and renewable energy generation is attracting increasing attention, due to the established understanding that a new sustainable approach to power human activities is compulsory, on the one hand, and thanks to technological advances, on the other hand. Energy harvesters, at different scales, are promising tools, but their performance and economic viability depend on advanced energy-maximisation control techniques. This paper borrows a control strategy from the vibration energy harvesting field, and implements it to a notional wave energy conversion application. A linear time variant damping coefficient of the power take-off unit is considered, oscillating at twice sinusoidal excitation force frequency, inducing parametric resonance in the system and significantly expanding the response bandwidth. Active and semi-active solutions are investigated, allowing or preventing bi-directional power flow, comparing results with traditional passive and impedance-matching control. Results show that, although both active and semi-active control successfully increase the mean power extraction, much larger than passive control, semi-active control ensures better power quality and lower control forces.**

***Keywords*—Energy harvesting, wave energy converter, linear time-variant system, energy-maximization optimal control, active/semi-active control, pseudospectral approach**

I. Introduction

Recent years are witnessing, more than ever, a clear and resolute worldwide momentum towards sustainability, based on the solid awareness that human impact on the planet must become at least neutral, if not positive. One of the major pathways towards sustainability is related to energy, which should be generated from renewable sources and should be used efficiently, avoiding waste. This is in line with the Sustainable Development Goals (SDG) [1], in particular SDG-11 (Sustainable cities and communities), SDG-7 (Affordable and clean energy), and SDG-13 (Climate action). Within this context, novel energy harvesting/scavenging technologies are proliferating, exploiting environmental energy to power electric systems. Energy harvesters are particularly common for low-power applications for sensors and the new Internet of Things (IoT) [2]–[5], with vibration energy harvesters (VEH) being among the most common.

The environment is full of sources of vibration, usually noisy or broadband. Trivial linear vibration energy harvesters are usually quite effective just at their own natural resonant frequency, whereas there is an abrupt degradation of response at any different frequency. Therefore, economical and technical feasibility can be achieved only through an structural improvement of the performance of the VEHs, with the objective to both magnify the power amplitude and, at the same time, broaden the frequency bandwidth. Prominent stratagems encompass including nonlinearities in the architecture of the system [6]: bistability [7], Duffing nonlinearity [8], stochastic oscillators [9] and parametric oscillators [10], among others. An alternative, less followed, route proposed to use linear time variant (LTV) systems instead, leading to either active or semi-active control strategies. *Active* control strategies normally guarantee the largest mean power, although the power flow is required to be a bidirectional between the transducer and the vibrator, hence often demanding for more expensive and complex machineries. Instances of active control strategies for VEH can be found in optimal stochastic control [11], including nonlinearities [12], in piezoelectric circuits [13], or considering positive/negative parametric resonance [14]. Considering the balance between economic and technical aspects, a question remains open, i.e. if the marginal cost and complexity increase is clearly justified by a greater gain in power delivered. *Semi-active* control strategies offer a hybrid solution, that refers to the introduction of an active alteration of a passive system parameter, so that no explicit actuator is required [15]. Mechanisms of this sort can be implemented, for example, via controllable fluid dampers, variable orifice dampers, or magneto-rheological dampers [16].

With a similar working principle to VEH, wave energy converters (WECs) produce electricity from an offshore renewable energy source. Despite the similarity, the order of magnitude of produced power is several orders of magnitude larger [17], [18]. Simple traditional strategies for WECs use passive control or impedance matching reactive control, while modern energy-maximising control strategies attempt to resolve the optimal control problem with model-predictive control (MPC) or MPC-like approaches [19].

This paper applies to WECs a control strategy mainly studied for VEH, *i.e.* to use a time-varying damping coefficient to induce parametric resonance into the system and increase power extraction. Such an objective is achieved via a convenient pseudo-spectral approximation framework used to readily compute the steady-state response of the associated linear time-varying (LTV) system. Results are provided considering active (bidirectional power flow) and semi-active (unidirectional power flow), also considering unconstrained and constrained displacement; results are also compared to standard passive and impedance-matching reactive control.

II. Problem definition

A. System model

A generic oscillating energy harvester, oscillating in the generic z direction, can be represented as a single-degree of freedom (SDOF) system, as schematically shown on the right-hand-side of Fig. 1, composed of a mass M, subject to an oscillating excitation force F_{ex}, connected to a fixed frame of reference by means of a spring with stiffness $K_h \in \mathbb{R}^+$, a damper with coefficient B representing internal dissipations, and a power take-off (PTO) unit for energy extraction. An assumption is that the force of the PTO is linearly proportional to the relative displacement (z) and velocity ($\dot{z}$), by means of K_{PTO} and B_{PTO}, respectively. The total PTO damping coefficient, useful for energy extraction, is defined in terms of a time-varying mapping $B_{\text{PTO}} : \mathbb{R}^+ \to \mathbb{R}$, $B_{\text{PTO}}(t) = b^0_{\text{PTO}} + b_{\text{PTO}}(t)$, where $b^0_{\text{PTO}} \in \mathbb{R}$ represents the mean PTO damping. In Fig. 1, it is already presented the equivalence between a heaving point absorber wave energy converter and a generic oscillating energy harvester, where the restoring force is provided by the hydrostatic stiffness (K_h), and the wave radiation force is decomposed in a term proportional to the acceleration via the added mass coefficient ($A(\omega)$), and a term proportional to the velocity via the radiation damping coefficient ($B(\omega)$), both frequency(ω)-dependent. Note that, for a fixed input frequency, the dependence on ω can be dropped accordingly, and $B(\omega) \equiv B$ and $A(\omega) \equiv A$. Additional physical phenomena, such as viscous damping [20], mooring stiffness [21], [22], mooring instability [23], are here neglected for simplicity. If necessary, an appropriate approximation may be defined [24].

The dynamic equation of the SDOF system follows directly from second law of dynamics [1], *i.e.*

$$M\ddot{z}+B\dot{z}+(b^0_{\text{PTO}}+b_{\text{PTO}}(t))\dot{z}+(K_h+K_{\text{PTO}})z = F_{ex}(t), \quad (1)$$

with z being the displacement of the mass and, taking a cosine (harmonic) monochromatic wave at angular frequency ω, $F_{ex}(t) = F^0_{ex}\cos(\omega t)$, $F^0_{ex} \in \mathbb{R}$. From now on, we refer to

[1]Hereafter, the dependence on t is omitted in the instances when it is obvious from the context.

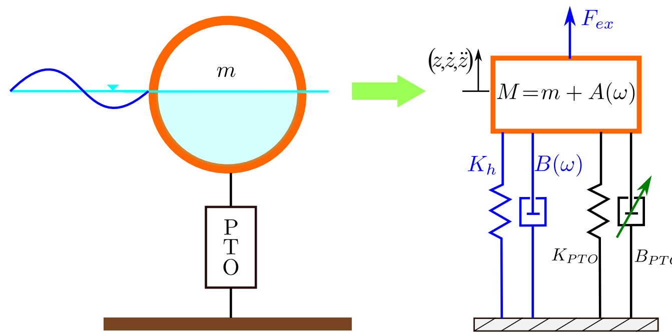

Fig. 1: Representation of the single-degree of freedom vibration energy harvester.

the input (excitation) frequency as the *fundamental frequency*, whereas its related period $T = 2\pi/\omega$ is named *fundamental period*.

B. Traditional control approaches

The traditional approach to energy extraction maximization, applicable for mono-chromatic excitation in linear-time invariant systems, is *impedance-matching* [25], where, for a fixed input frequency ω, the optimal stiffness and damping terms of the PTO system, as defined in Fig. 1, are:

$$\begin{cases} B_{\text{PTO}} = B, \\ K_{\text{PTO}} = M\omega^2 - K_h. \end{cases} \quad (2)$$

However, (2) eventually requires reactive power, due to a non-zero K_h, which may increase the complexity and cost of the PTO system; moreover, if K_h is negative, a bi-directional power flow is demanded, i.e. the PTO must provide power to the system during a portion of the oscillation period in order to extract more energy in the complementary part of the period. Therefore, a sub-optimal *passive control* can be alternatively defined as:

$$\begin{cases} B_{\text{PTO}} = \sqrt{B^2 + 1/\omega^2(M\omega^2 - K_h)^2}, \\ K_{\text{PTO}} = 0. \end{cases} \quad (3)$$

C. Optimal control problem

The objective considered in this work translated into the definition of a convenient design for the time-varying mapping B_{PTO} so that the absorbed energy by the WEC is *maximized*. Equivalently, such a design encompass an *energy-maximization* objective, where the criterion is to maximise the absorbed energy from the corresponding base excitation, over a finite time interval[2] $\Xi = [0, T] \subset \mathbb{R}^+$. Being more specific, the excitation input is converted into useful energy by the PTO system, which can be calculated in a straightforward way as the time integral of converted (instantaneous) power: Let the *control force* be defined as $u = (b^0_{\text{PTO}} + b_{\text{PTO}}(t))\dot{z}$ (*i.e.* PTO input); It follows that such an energy-maximizing design

[2]We highlight no loss of generality in assuming the starting time to 0.

approach is translated into an optimal control problem (OCP), with *objective function* $\mathcal{J}$ defined as

$$\mathcal{J}(B_{\text{PTO}}) = \frac{1}{T}\int_{\Xi}(b^0_{\text{PTO}} + b_{\text{PTO}}(t))\dot{z}^2(\tau)d\tau, \tag{4}$$

Using the control objective function as declared in (4) (*i.e.* the mapping $\mathcal{J}$), and the WEC dynamics as in (1), the energy-maximizing OCP is properly expressed as:

Problem 1 (Energy-maximizing OCP)**.** Look for an optimal control damping $B^{\text{opt}}_{\text{PTO}} : \Xi \to \mathbb{R}$ so that

$$\begin{aligned} B^{\text{opt}}_{\text{PTO}} = \arg\max_{B_{\text{PTO}}}\ & \mathcal{J}(B_{\text{PTO}}), \\ \text{subject to:}\ & \\ \text{Wave energy converter dynamics (1).} & \end{aligned} \tag{5}$$

D. On the selection of the time-varying B_{PTO}

Since the external excitation is harmonic, it is reasonable to also let B_{PTO} assume a corresponding harmonic form. It's worth noting that, although not a strict requirement nor condition for optimality, such an assumption is in line with previous work found in [16] and [14]. Therefore, hereafter, we let the time-varying damping b_{PTO} to be defined as

$$B_{\text{PTO}}(t) = b^0_{\text{PTO}} + \sum_{p=1}^{N}\alpha^b_p\cos(p\omega t) + \beta^b_p\sin(p\omega t), \tag{6}$$

with $N \in \mathbb{N}$, and where $\{\alpha^b_p, \beta^b_p\}^N_{p=1} \subset \mathbb{R}$. Furthermore, based on previous work, such as [14], [16], [26], it is found that time-variation simply at twice the fundamental frequency is already highly-beneficial, thanks to exploiting parametric resonance; therefore, we simplify the expression in (6) as

$$B_{\text{PTO}}(t) = b^0_{\text{PTO}} + \alpha^b_2\cos(2\omega t) + \beta^b_2\sin(2\omega t), \tag{7}$$

i.e. $\alpha^b_p = \beta^b_p = 0$ for $p = 1$ and $\forall\, p \geq 3$ in (6). Therefore, three variables are optimized, namely the mean damping coefficient (b^0_{PTO}) and the two terms corresponding with the time-varying damping coefficient at twice the excitation frequency (α^b_2 and β^b_2).

Note that, if B_{PTO} becomes negative in portions of the time period, bidirectional power flow is required at the PTO, hence realizing an *active* control strategy. Since additional complexity and cost may arise from bidirectional PTOs, a *semi-active* control strategy may be preferred; this is realized by imposing an additional constraint to the control problem in (1), namely requiring the instantaneous power to never change sign.

E. A pseudospectral-based approach

The OPC, defined in (5), using the parameterization of the damping coefficient in (6), can be formulated using a pseudo-spectral representation, in turns enabling a computationally efficient mapping for the algebraic approximation of the steady state response. While the method is briefly illustrated here in Figure 2, we refer the reader to [27] for an extensive discussion on the theoretical foundations of this strategy. The major advantages and properties are listed hereafter:

- The steady-state response of the associated LTV system, *i.e.* the WEC driven by a time-varying damping (feedback) force, can be written in terms of a linear algebraic equation which always has a *unique* solution.
- Such steady-state equation can be utilized to transcribe the infinite-dimensional OCP (5) to a finite-dimensional nonlinear program (NP).
- The resulting NP can be solved using state-of-the-art optimization routines, leading to a well-posed computation of the associated time-varying optimal damping force.
- Constraints on both motion variables, and final damping control force, can be added to the NP straightforwardly via collocation.

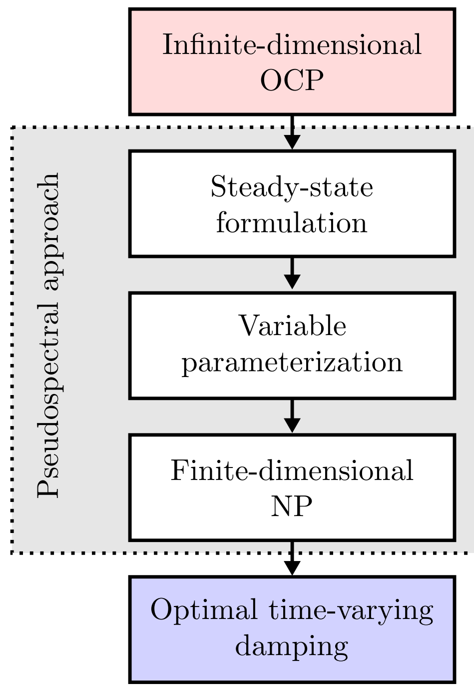

Fig. 2: Schematic representation of the pseudospectral based approach.

III. Case study: a wave energy converter

As a case study, a single-degree of freedom wave energy converter (WEC) is considered, inspired at the Wavestar device. The WEC is a piercing spherical heaving point absorber, schematically shown in Fig. 1, whose geometrical and inertial properties are shown in Table I, and hydrodynamic curves are shown in Fig. 3 for different wave periods (T_w).

TABLE I: Properties of the wave energy converter.

Parameter	Symbol	Quantity	Units
Radius	R	2.5	m
Draft	d	2.5	m
Mass	m	33543	kg
Hydrostatic stiffness	K_h	1.9743	N/m
Natural period	T_n	3.1	s

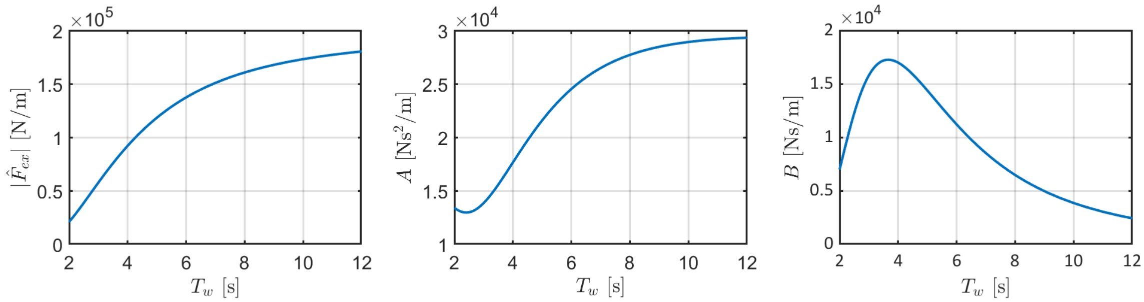

Fig. 3: Excitation force magnitude per unit of wave amplitude ($|\hat{F}_{ex}|$), radiation added mass (A) and radiation damping (B).

IV. Results

The two time-varying PTO damping coefficient strategies (active and semi-active) have been compared to the constant damping strategy, shown in (3), and the reactive impedance-matching (IM), shown in (2). Results are discussed in Sect. IV-A. However, it becomes evident that it is necessary to include a constraint on the maximum absolute displacement, leading to results presented in Sect. IV-B.

Regular waves are considered, with wave period (T_w) ranging from 2s to 12s, and wave height (H_w) equal to 3m.

A. Unconstrained optimization

Figure 4 shows a representative time trace of the steady-state response to an incoming wave of T_w equal to 6s, showing the velocity ($\dot{z}$), the total PTO damping coefficient (B_{PTO}), the resulting PTO control force (F_{PTO}) and instantaneous power (P), both active (left) and semi-active (right) control strategies.

It can be verified that the power flow is bidirectional with the active strategy, while always positive in the semi-active control; furthermore, peaks are two orders of magnitude higher in the active control, leading to scarce power quality. Similarly, the PTO force is one order of magnitude higher for the active control, requiring a more costly PTO. The uni- or bi-directionality of the power flow is due to the total PTO damping, which becomes negative in the active strategy, while always positive in the semi-active control.

The resulting mean power ($\overline{P}$), for all wave periods, is shown in Fig. 5, also highlighting the ratio with respect to the worst-benchmark case of constant damping control ($\overline{P}_r$). It is readily evident that, for long-period waves, the active control strategy is able to extract several times more energy than both the passive and the semi-active controls, even though no reactive term is explicitly included (as K_{PTO} in the reactive IM). However, such results are merely a mathematical aberration due to the simplifying assumptions of the linear model. In fact, as clearly shown in Fig. 6, the resulting displacement according to the reactive IM and the active control, for long-period waves, would easily exceed several tens of meters. This is not realistic, since the draft of the buoy is just 2.5m, equal to the radius. Therefore, a constrained optimization is carried out, whose results are discussed in Sect. IV-B.

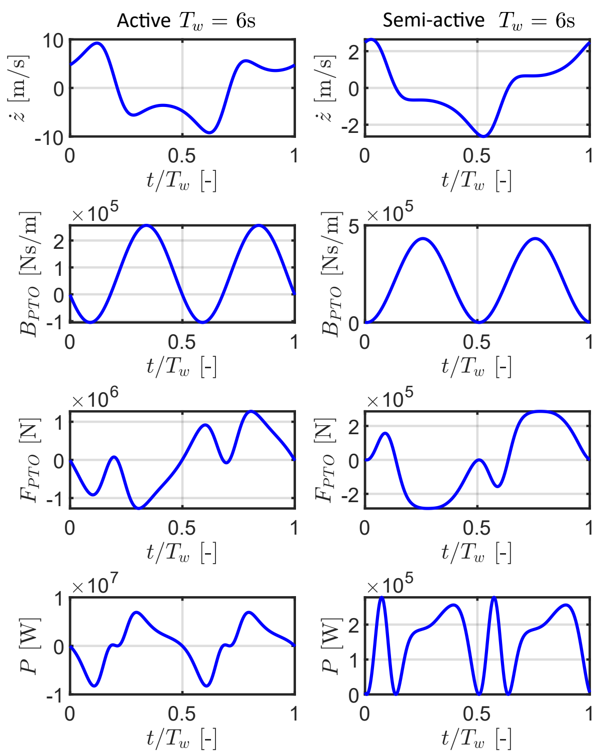

Fig. 4: Steady state response to an incoming wave of T_w of 6s, H_w of 3m, without maximum displacement constraint.

B. Constrained optimization

A constrained optimization algorithm is implemented in order to withstand the requirement of the maximum absolute displacement to remain below 2.5m, as shown in Fig. 7. The reactive IM is excluded in this section, since no optimization is performed, and the displacement does not comply with the constraint for most of wave periods. Conversely, the constant damping naturally complies with the constraint. Similarly, the semi-active control remains below the constraint for all wave periods, although achieving a larger displacement than the constant damping.

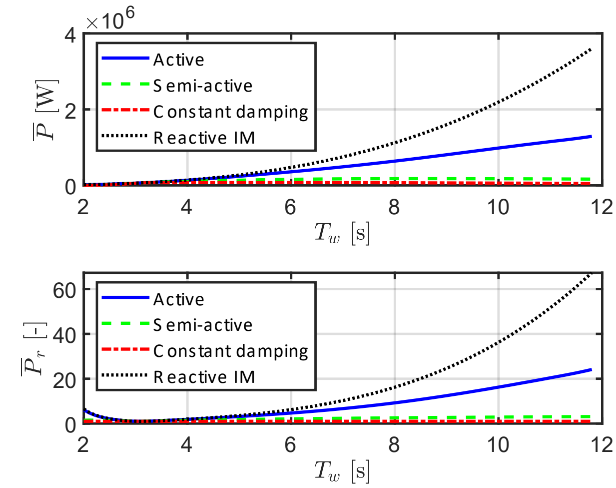

Fig. 5: Mean ($\overline{P}$) and relative ($\overline{P}_r$) power extracted, without maximum displacement constraint.

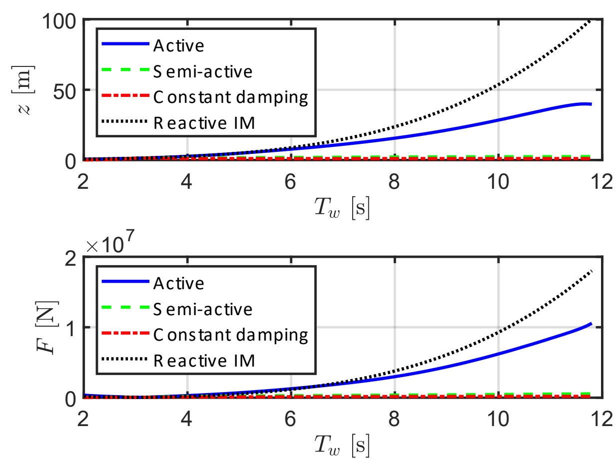

Fig. 6: Maximum displacement and PTO force, without maximum displacement constraint.

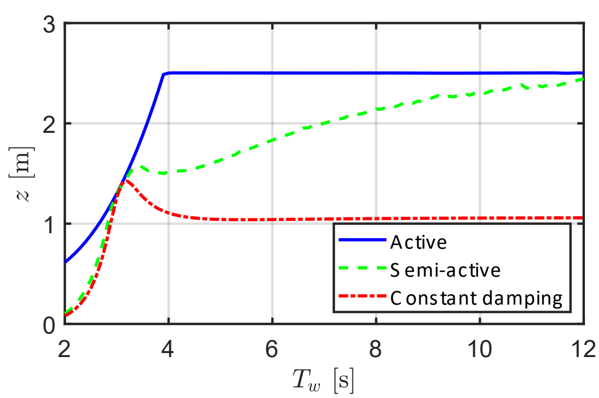

Fig. 7: Maximum displacement and PTO force, constraining the maximum displacement to 2.5m.

A representative time trace of the steady state response is shown in Fig. 8, for a wave of T_w of 6s, H_w of 3m, similarly to Fig. 4. It can be noted that the PTO force have the same order of magnitude with active and semi-active strategies; conversely, the instantaneous power with the active control has high peaks, much larger than the mean value, realizing low power quality. Conversely, in the semi-active strategy, the peak-to-mean ratio is significantly lower.

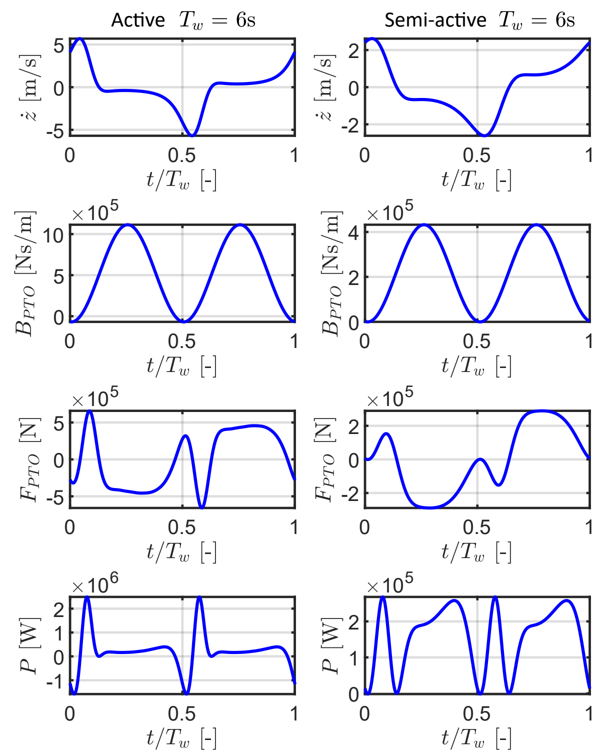

Fig. 8: Steady state response to an incoming wave of T_w of 6s, H_w of 3m, constraining the maximum displacement to 2.5m.

Finally, Fig. 9 shows the mean and relative power extraction for different wave periods. It is worth noting that, at the natural frequency ($T_w = T_n = 3.1$s), all controllers behave equivalently; in fact, at resonance, no action is required by the controller, since the system is already naturally responding at its best. Overall, it can be noticed that, with the displacement constraint keeping the predicted response of the device realistic, the performance of the semi-active and active control strategies are similar, several-fold higher than the constant damping strategy.

V. Conclusions

This paper proposes the application of an energy-maximisation control strategy for vibration energy harvesters/scavengers to wave energy converters, exploiting similar working principles at different power generation size. The

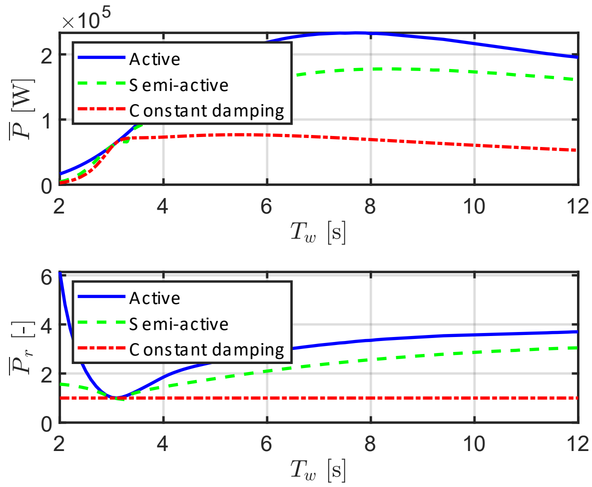

Fig. 9: Mean ($\overline{P}$) and relative ($\overline{P}_r$) power extracted, constraining the maximum displacement to 2.5m.

control strategy, based on time-varying power take-off damping parameter at twice the excitation frequency, is shown effective since it leverages on parametric resonance. Active (bi-directional power flow) and semi-active (uni-directional power flow) are both effective, although the semi-active strategy seems to require less complex generator machinery, lower loads, and ensures higher power quality.

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