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Crosstalk analysis at near-end and far-end of the coupled transmission lines based on eigenvector decomposition

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Crosstalk Analysis at Near-End and Far-End of the Coupled Transmission Lines Based on Eigenvector Decomposition

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Abstract— In this paper, a new analytical method is proposed to accurately estimate the near-end and far-end crosstalk of a coupled Transmission Lines (TLs) based on eigenvector decomposition. For a non-homogenous two coupled lines, the related linear differential equations system (LDES) is derived for distributed voltage and current and then using matrix analysis, its four distinct eigenvalues and their associated eigenvectors are determined. It is shown that the two eigenvalues represent the self-propagation constant, while the other ones are linked to the mutual propagation constant of the coupled lines. In addition to, for these lines a closed form expression for near-end and far-end crosstalk is presented. In special case of homogenous coupled lines, the LDES is also determined and it is shown that they provide two couples of eigenvalues. Using the concept of generalized eigenvalues, the solution of these systems is derived and a closed form formula is derived for crosstalk. In order to verify the accuracy of the proposed method a few types of coupled lines, including homogeneous or non-homogeneous are investigated and the amount of crosstalk is estimated. The calculated crosstalk is presented and compared with those obtained by numerical investigation. It is shown that a good agreement is obtained between the calculated and measured results.

Key word: Crosstalk, Coupled Microstrip Line, Transmission Line (TL).

I. INTRODUCTION

In recent years, along with the developments of modern electronic and communication systems, the major tendency is toward the designing of compact, low consume power, high speed, low weight and high density circuits. The routing of a huge amount of wiring in each part of the entire structure is an important issue. In these compact device, the rout of signals is often grouped together with a closely distance due to limited available space. This phenomenon refers to the unplanned electromagnetic coupling between the adjacent lines, which are in close proximity. Crosstalk between wires in cables or between lands on PCBs is a near field coupling problem; that is, the source of the electromagnetic interference within the same system [1].

Today's, crosstalk has already become one of the dominant limiting factors in designing

process of Integrated Circuits (ICs). So, the problem of determining the electromagnetic coupling among coupled Transmission Lines (TLs) exists in many applications such as radio propagation, geophysical prospecting, Electromagnetic Compatibility (EMC), induction heating, radio remote sensing and radio direction finding [2-4].

In electromagnetic theory, the penetration of an unwanted electrical signal or electromagnetic wave from one transmission line to its adjacent one is called crosstalk. This phenomenon happens due to the mutual coupling because of mutual capacitance and inductance for any arbitrary two or more adjacent coupled lines. Multi-conductor Transmission Lines (MTLs) is one of the practical TLs in commercial systems, which are used in power and telephone lines and network cables over a wide area of applications. For this reason, the problem of calculating crosstalk between two lines located in close proximity is very important. This type of TL

consists of two or more shielded or unshielded conductors with a common conductor as a signal return path [5-8].

So far, various approaches including analytical and numerical methods have been used to calculate crosstalk for different applications [9]. Some of these methods are used at low frequencies and a few of them are suitable at high frequency applications. The observed discrepancy between these two methods is the electrical size. According to the above mentioned matter, an analytical method is required to be developed to obtain an expression for crosstalk of a coupled line. As a result, the closed form expression for crosstalk offers a clear relationship between the geometrical parameters of the coupled TL, which allows the designer to consider the initial results of the planned device.

With the aim of this paper, a new analytical method is introduced to quickly estimate crosstalk at near-end or far-end of a homogeneous or non-homogeneous coupled lines based on eigenvalue and eigenvector analysis. At first, a linear differential equations system is derived for distributed voltage and current along the lines. Using the matrix representation for the obtained equations, two types of eigenvalues are obtained and their related characteristic equations, eigenvectors and phase constants of the propagating waves along the lines are determined. It is shown that, the first type of eigenvalue shows the self-propagation constant of the propagated waves, while the other eigenvalues are related to the mutual propagation constant of the waves. Using the obtained results, a closed form expression is resulting to estimate the amount of crosstalk in the coupled line, which is valid for any type of coupled lines. In order to verify the accuracy of the proposed method, several coupled lines are investigated and the amount of crosstalk is calculated. The obtained results are compared with those of measurement and it is shown that the obtained results are in a good agreement.

II. CROSSTALK FORMULATION
a) General Case with Distinct Eigenvalues
Figure (1) shows a general form of a two coupled lines over a common ground plane. For simplicity, it is assumed that a weak coupling is occurred in the region of coupling. Let L_i, C_i, $i= 1,$

2 be self per unit length inductance and capacitance of the line respectively, which can be calculated, while the lines are isolated from each other [10]. It is known from electromagnetic theory that C_i is related to the capacitance to ground C_{ig} via $C_i=C_{ig}+C_m$. The corresponding propagation velocities and characteristic impedance are given by equations (1) and (2) [11].

$$\upsilon_i = \frac{1}{\sqrt{L_i C_i}}, \quad i = 1, 2 \tag{1}$$

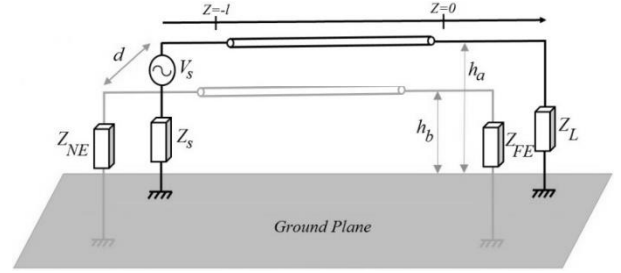$$Z_i = \sqrt{\frac{L_i}{C_i}}, \quad i = 1, 2 \tag{2}$$

Figure 1: The general structure of a coupled transmission line.

According to TL theory, the coupling between the lines is modeled by introducing two mutual per unit length parameters including L_m and C_m, which are mutual inductance and capacitance respectively. Then, the coupled versions of the transmission line equations are given by equations (3a) to (3d).

$$\frac{dV_1}{dz} = -(j\omega L_1 I_1 + j\omega L_m I_2) \tag{3a}$$

$$\frac{dV_2}{dz} = -(j\omega L_2 I_2 + j\omega L_m I_1) \tag{3b}$$

$$\frac{dI_1}{dz} = -j\omega C_1 V_1 + j\omega C_m V_2 \tag{3c}$$

$$\frac{dI_2}{dz} = -j\omega C_2 V_2 + j\omega C_m V_1 \tag{3d}$$

The above equations can be written by a 4×4 matrix form given by equation (4).

$$\frac{d}{dz}\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -j\omega L_1 & -j\omega L_m \\ 0 & 0 & -j\omega L_m & -j\omega L_2 \\ -j\omega C_1 & j\omega C_m & 0 & 0 \\ j\omega C_m & -j\omega C_2 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} \tag{4}$$

Equation (4) shows a first order linear coupled system of differential equations [12], which can be

written by a homogenous first order matrix differential equation given by (5).

$$\frac{d\mathbf{X}}{dz} = \mathbf{AX} \tag{5}$$

In equation (5), $\mathbf{A}$ is coefficient matrix of the equations; $\mathbf{X}$ is a vector whose components are unknown values in the system under investigation and $d\mathbf{X}/dz$ is the derivative of $\mathbf{X}$ with respect to z, the direction of propagation. It is well known from matrix algebra that the solution of the equation is given by equation (6) using distinct eigenvalue and eigenvectors.

$$\mathbf{X} = \sum_{i=1}^{4} d_i \mathbf{Y}_i e^{\zeta_i z} \tag{6}$$

In Which $\mathbf{Y}_i$ and ζ_i, $i=1, 2, 3, 4$ are eigenvectors and eigenvalues of matrix $\mathbf{A}$ respectively and d_i's are constant coefficient. The characteristic equation for the mentioned linear systems of differential equations in (5) is given by determinant of matrix $\mathbf{A}\text{-}\zeta\mathbf{I}$ given by equation (7), in which $\mathbf{I}$ is a 4-by-4 identity matrix.

$$|\mathbf{A} - \zeta\mathbf{I}| = 0 \tag{7}$$

By expanding equation (7), a polynomial power of 4 for ζ is obtained, while its roots are eigenvalues of the square matrix $\mathbf{A}$. It has at least one and at most four distinct eigenvalues [13]. In case of our coupled lines, for a 4×4 squared matrix $\mathbf{A}$, its characteristic equation is given by (8).

$$\zeta^4 + \omega^2 \left[|\mathbf{H}_{1m}| + |\mathbf{H}_{2m}|\right]\zeta^2 + \omega^4 |\mathbf{C}||\mathbf{L}| = 0 \tag{8}$$

In which $|\mathbf{C}|$, $|\mathbf{L}|$, $|\mathbf{H}_{1m}|$ and $|\mathbf{H}_{2m}|$ are the determinant of per unit length capacitance, inductance, hybrid capacitance-inductance type I and type II matrix respectively given by equations (9a) to (9d).

$$\mathbf{L} = \begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} \tag{9a}$$

$$\mathbf{C} = \begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix} \tag{9b}$$

$$\mathbf{H}_{1m} = \begin{bmatrix} C_m & C_1 \\ L_1 & L_m \end{bmatrix} \tag{9c}$$

$$\mathbf{H}_{2m} = \begin{bmatrix} C_m & C_2 \\ L_2 & L_m \end{bmatrix} \tag{9d}$$

Based on TL theory, in case of homogeneous medium, per-unit-length inductance and capacitance matrixes are directly linked by equation (10), in which υ represents phase velocity of the related electromagnetic wave.

$$\mathbf{CL} = \mathbf{LC} = \upsilon^{-2}\mathbf{I} \tag{10}$$

The eigenvalues of matrix $\mathbf{A}$ are determined by solving equation (8) given by (11).

$$\zeta_i = \pm\sqrt{\frac{-|\mathbf{H}_{1m}| - |\mathbf{H}_{2m}| \pm \sqrt{\left(|\mathbf{H}_{1m}| + |\mathbf{H}_{2m}|\right)^2 - 4|\mathbf{L}||\mathbf{C}|}}{2}} \tag{11}$$

$$i = 1, 2, 3, 4$$

According to the general solutions of equation (6) for voltage or current waves, it can be said that the eigenvalues are, in fact, phase constant of propagated waves in case of a single line. In this case, equation (11) is simplified as follows.

$$\zeta_i = \pm j\omega \begin{cases} \sqrt{L_1 C_1} \\ \sqrt{L_2 C_2} \end{cases} \Rightarrow \zeta_i = \pm j\omega \begin{cases} \sqrt{\mu_1 \varepsilon_1} = \pm j\beta_1 \\ \sqrt{\mu_2 \varepsilon_2} = \pm j\beta_2 \end{cases} \tag{12}$$

It is clear from (12) and Fig. 1 that in case of the uncoupled lines, $e^{-j\beta_i}$ and $e^{j\beta_i}$ $(i=1,2)$ show the propagating waves along $+z$ and $-z$ directions respectively for each line. For a specified TL, the per unit length capacitance and inductance can be computed from Method of Moment (MoM) with a good approximation [16], and therefore, the eigenvalues and eigenvectors of matrix $\mathbf{A}$ are known. Then, based on equation (6), four unknown values of d_i (i=1, 2, 3, 4) have to be calculated using boundary conditions at near-end and far-end of the coupled lines. These are given by equations (13a) to be (13d).

$$V_S = V_1(z = -l) + Z_S I_1(z = -l) \tag{13a}$$

$$V_1(z = 0) = Z_L I_1(z = 0) \tag{13b}$$

$$V_2(z = 0) = Z_F I_2(z = 0) \tag{13c}$$

$$V_2(z = -l) = -Z_N I_2(z = -l) \tag{13d}$$

The modal matrix $\mathbf{M}$ for matrix $\mathbf{A}$, which is a 4-by-4 matrix formed with the eigenvectors ($\mathbf{Y}_i$) of $\mathbf{A}$ given by equation (14) [14].

$$\mathbf{M} = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 & \mathbf{Y}_3 & \mathbf{Y}_4 \end{bmatrix} \tag{14}$$

By substituting equations (13) in (6) using (14), the following equations are obtained.

$$\sum_{i=1}^{4} d_i (\mathbf{M}_{1i} + Z_S \mathbf{M}_{3i}) e^{-\zeta_i l} = V_S \tag{15a}$$

$$\sum_{i=1}^{4} d_i (\mathbf{M}_{1i} - Z_L \mathbf{M}_{3i}) = 0 \tag{15b}$$

$$\sum_{i=1}^{4} d_i (\mathbf{M}_{2i} - Z_F \mathbf{M}_{4i}) = 0 \tag{15c}$$

$$\sum_{i=1}^{4} d_i (\mathbf{M}_{2i} + Z_N \mathbf{M}_{4i}) e^{-\zeta_i l} = 0 \tag{15d}$$

The new obtained linear system of equations (15) can be written in matrix form $\mathbf{Bd}=\mathbf{g}$, in which the coefficient matrix $\mathbf{B}$ is a 4×4 matrix, while $\mathbf{g}$ and $\mathbf{d}$ are column vectors. It is clear that $\mathbf{B}$ depends on the modal matrix elements $\mathbf{M}$, terminated impedances, lines length and the associated eigenvalues. Since vector $\mathbf{g}$ has one non-zero element, system is a non-homogeneous system of equations. Finally, unknown vector $\mathbf{d}$ is founded by $\mathbf{d}=\mathbf{B}^{-1}\mathbf{g}$ equation. Using the above procedure, the near-end and far-end crosstalk is defined by equations (16) and (17) [1].

$$CT_{\text{far end}} = \frac{V_2(z=0)}{V_S} = \sum_{i=1}^{4} d_i M_{2i} \tag{16}$$

$$CT_{\text{near end}} = \frac{V_2(z=-l)}{V_S} = \sum_{i=1}^{4} d_i M_{2i} e^{-\zeta_i l} \tag{17}$$

b) **Especial case with repeated eigenvalues**

The mentioned crosstalk estimation of the coupled lines based on eigenvalue analysis in previous section is quite general and is valid for asymmetrical transmission line with a non-homogeneous media. However, in case of practical applications, TLs are symmetrically arranged using a homogeneous material. In such a situations the per unit length self-capacitance and self-inductance are equal, $L_1=L_2$, $C_1=C_2$, and therefore, the system of linear equations provide repeated eigenvalues and special techniques should be considered to obtain the solution. In this case, the following equations is satisfied be.

$$|\mathbf{C}| = C_1^2 - C_m^2 \tag{18a}$$

$$|\mathbf{L}| = L_1^2 - L_m^2 \tag{18b}$$

$$|\mathbf{H}_{1m}| = |\mathbf{H}_{2m}| = C_m L_m - C_1 L_1 \tag{18c}$$

In this case, eigenvalues are given by (19), in which repeated eigenvalues exist.

$$\zeta_i = \pm j\omega \begin{cases} \sqrt{(C_1 + C_m)(L_1 - L_m)}, & i=1,2 \\ \sqrt{(C_1 - C_m)(L_1 - L_m)}, & i=3,4 \end{cases} \tag{19}$$

In case of a symmetrical or asymmetrical TL and using equation (10), the following equations are satisfied.

$$|\mathbf{L}| = \left(\upsilon^4 |\mathbf{C}|\right)^{-1} \tag{20a}$$

$$|\mathbf{H}_{1\mathbf{m}}| = |\mathbf{H}_{2\mathbf{m}}| = -\upsilon^{-2} \tag{20b}$$

By substituting equations (20) in (11), the repeated eigenvalues are obtained.

$$\zeta_1 = \zeta_2 = j\omega\sqrt{\mu\varepsilon} \tag{21a}$$

$$\zeta_3 = \zeta_4 = -j\omega\sqrt{\mu\varepsilon} \tag{21b}$$

In which μ and ε are permeability and permittivity of the material. It is well known from TL theory that the propagation constant of a symmetrical or asymmetrical homogeneous coupled TL is only dependent on the medium characteristics, but for symmetrical or asymmetrical lines made by a non-homogeneous medium, propagation constant is not only depending on the medium characteristics, but also it is reliant on the geometrical parameters of the lines.

To find two linearly independent eigenvector $\mathbf{Y_1}$, $\mathbf{Y_2}$ or $\mathbf{Y_3}$, $\mathbf{Y_4}$ in case of repeated eigenvalues, a set of generalized eigenvector have to be found to obtain the required solution. Based on linear algebra, the first and second eigenvectors $\mathbf{Y_1}$ or $\mathbf{Y_2}$ can be found using $\mathbf{AY}_1=\zeta_1\mathbf{Y}_1$ respectively. To obtain the other eigenvectors $\mathbf{Y_3}$ and $\mathbf{Y_4}$, a same process can be used. In other words, equation $\mathbf{AY}_3=\zeta_3\mathbf{Y}_3$ is used to determine $\mathbf{Y_3}$ and generalized eigenvector $\mathbf{Y_4}$. Therefore, in case of linearly independent eigenvectors $\mathbf{Y_1}$ and $\mathbf{Y_2}$ (or $\mathbf{Y_3}$ and $\mathbf{Y_4}$), equation (6) is still suitable to find the solution. But in case of linearly dependent $\mathbf{Y_1}$, $\mathbf{Y_2}$ or $\mathbf{Y_3}$ and $\mathbf{Y_4}$, the solution is the modified version of equation (6) given by equation (22) [14], in which by adding boundary conditions at $z=0$ and $z=-l$ the required coefficients d_is are determined.

$$\mathbf{X} = \left[d_1\mathbf{Y}_1 + d_2(z\mathbf{Y}_1 + \mathbf{Y}_2)\right]e^{\zeta_1 z} + \left[d_3\mathbf{Y}_3 + d_4(z\mathbf{Y}_3 + \mathbf{Y}_4)\right]e^{\zeta_2 z} \tag{22}$$

It should be considered that in case of practical applications using homogenous coupled lines, all eigenvectors $\mathbf{Y_i}$ ($i=1,2,3,4$) are always linearly independent due to validating (20a), and so far-end and near-end crosstalk is determined using equations (16) and (17) respectively.

To verify the validity of equation (11), an asymmetrical double conductor transmission line with a non-homogeneous material above a ground plane is considered. The coupled lines have circular cross section and one of the lines (culprit line) is coated with a lossless dielectric material having a relative permittivity of 2.1 with thickness of 1.5 mm. The lines have a conductor radius of 0.64 mm and 1.5 mm respectively and their length is about 100 mm. The lines are at a height 3.14 mm and 6.02 mm above the ground plane. The distance between the two lines is 9.17 mm.

The method of moment is used to determine the per unit length parameters [16], which gives per unit length parameters of C_1=38.93 pF/m, C_2=21.29 pF/m,

C_m=1.88 pF/m, L_1=4.23 nH/m, L_2=5.43 nH/m and L_m=0.20 nH/m. Due to non-homogeneous properties of this coupled lines, equation (10) cannot be used. Moreover, this asymmetrical TL provides two pairs of complex conjugates eigenvalues, which can be calculated by equation (11). In order to investigate the validity of equation (11), the mentioned structure is simulated using High Frequency Simulator Structure (HFSS) and the simulated propagation constant and calculated results are shown in Fig. 2a and Fig. 2b. It can be seen that the results are in a good agreement.

A symmetrical double conductor transmission line without dielectric insulation above the ground plane is also considered as the second example. The conductor radius and their length are 0.64 mm and 100 mm respectively. The lines are at a height of 5 mm above the ground plane and the distance between two lines is 11 mm. The self and mutual per unit length capacitance are 20.43 pF/m and 2.26 pF/m respectively. Also, from equation (10), L_1=L_2=0.55 µH/m, L_m=60.97 nH/m. According to equation (11), the imaginary part of all eigenvalues is equal. Comparison of the calculated propagation constant and the simulated results using HFSS in plotted in Fig. 2c versus frequency, which confirms the accuracy of the proposed method compared with that of the simulation.

III. RESULTS AND DISCUSSION

As an example of a wire-typed TLs, two wires with radius of r=0.4 mm are separated by distance of d=20 mm is considered to calculate its crosstalk [1]. The length of the lines is L=4.6 m and they are suspended at h=20 mm above the ground plane, which is the path for return current. No dielectric material is used as the media and so, the coupled lines are categorized as a homogeneous case study. The per unit length parameters were computed using the MATLAB code and MoM, which gives parameter values of C_1=C_2=20.29 pF/m, C_m=2.19 pF/m, L_1=L_2=0.91 µH/m, L_m=0.16 µH/m. All termination impedances are set to 100 Ω. Fig. 3 shows the predicted near-end crosstalk including the measured results versus frequency. Since the wires are widely separated, the crosstalk between the coupled lines is weak.

A double conductor transmission line above the ground plane is considered as a second example. The geometrical parameters of this line are as L=870 mm, d=90 mm, h=45 mm, r=0.4 mm. The per unit length parameters are as C_1=C_2=9.64 pF/m, C_m=0.66 pF/m, L_1=L_2=1.1 µH/m, L_m=63.3 nH/m [18]. In the designed experimental setup, the culprit line uses 290 Ω source impedance and an open circuit load at its end. The victim line uses a 1000 Ω near-end impedance and 1050 Ω far-end impedance. Fig. 4 shows the predicted and measured crosstalk for the mentioned line, which

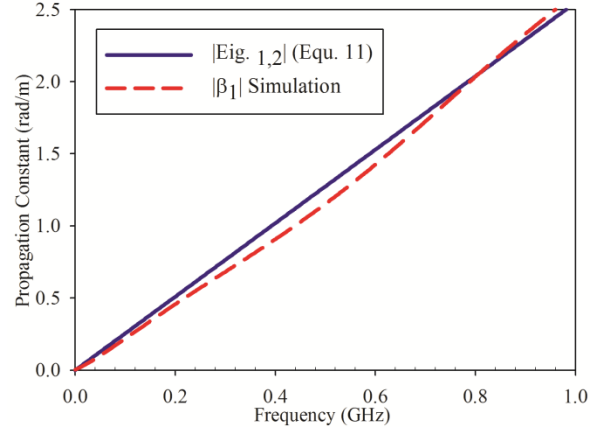confirms the accuracy of the proposed method of calculating.

Figure 2a: The simulated and calculated propagation constant of an asymmetrical TL, eigenvalues 1 and 2.
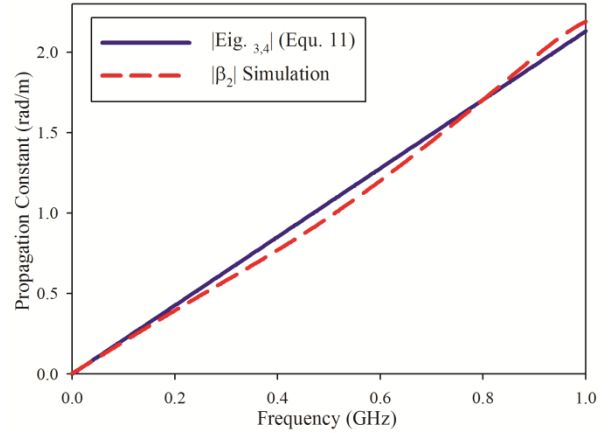

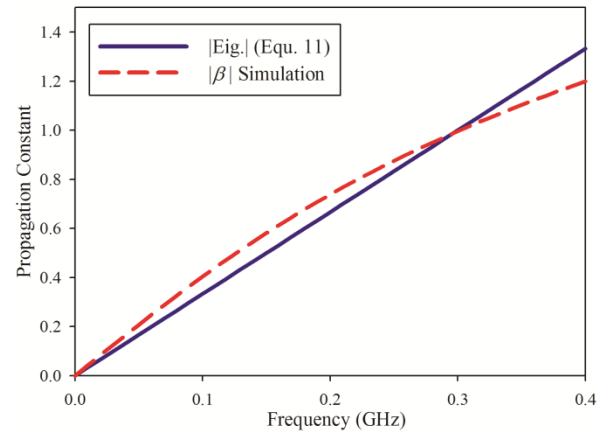**Figure 2b:** The simulated and calculated propagation constant of an asymmetrical TL, eigenvalues 3 and 4.

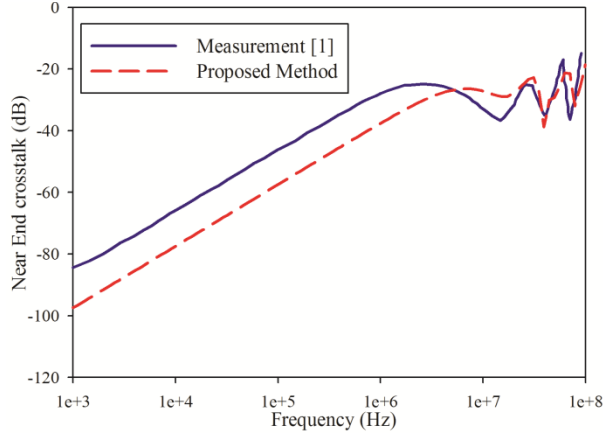**Figure 2c:** The simulated and calculated of propagation constant of a symmetrical TL.

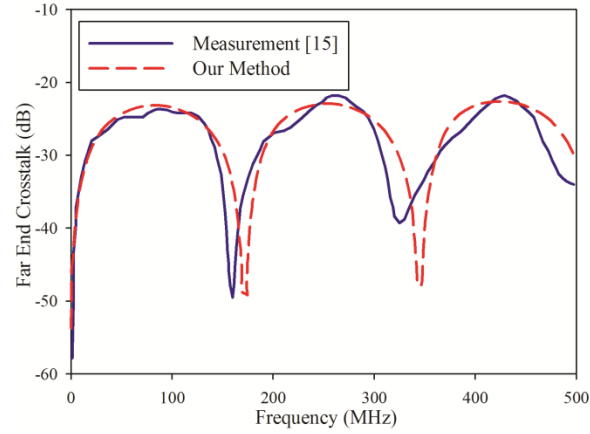**Figure 3:** The calculated and measured near-end crosstalk of a two wires TL.

Figure 4: Far-End Crosstalk of a double conductor TL.

Figure 5a: The calculated and measured near-end crosstalk of a PCB line.
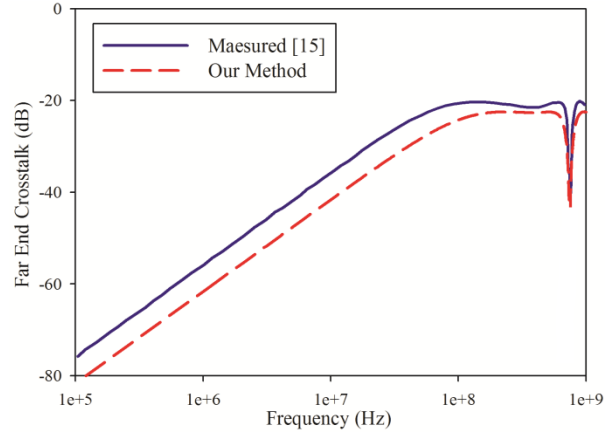
Figure 5b: The calculated and measured far-end crosstalk of a PCB line.

To determine the crosstalk between adjacent parallel lines on a printed circuit board (PCB), which is adapted for microstrip coupled line structures, two lines having same width along their length is considered in our investigation. It is assumed that quasi-TEM mode is propagating on the lines. Thus, TL theory can be used to determine the voltages and currents along the terminated lines. The applied substrate is glass-epoxy with relative permittivity of 4.7, while copper plane is on one side and two parallel coupled lines are etched on the other side with the lengths of 0.2 m. The width to height ration for both traces is about 0.4. All different load cases are considered as 50 Ω. The per unit length parameters are $C_1=C_2=24.4$ pF/m, $C_m=7.3$ pF/m, $L_1=L_2=0.5$ μH/m, $L_m=0.15$ μH/m. The computed near-end and far-end crosstalk coefficients are using equations (16) and (17) including the measurement results [17] are shown in Fig. 5a and Fig. 5b. It can be seen that a good agreement is obtained between the obtained simulation and measurement results up to 1 GHz, whereas the lines are in a non-homogeneous structure and an equivalent approximated homogeneous is used for the crosstalk calculation.
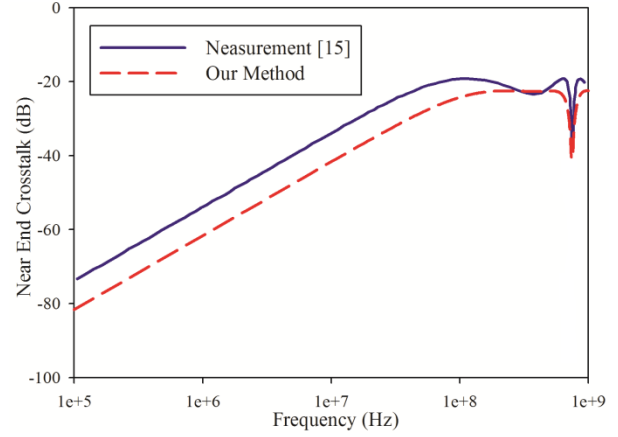
In order to verify the accuracy of the proposed method for estimating the crosstalk of a coupled microstrip line, two lines with same width of 4.8 mm and lengths of 55 mm is considered to adapt 50 Ω lines and the center to center distance between them is d. To carry out a parametric study, d is varied and the amount of crosstalk is studied. TLY062 substrate is used with relative permittivity and height of 2.2 and 1.56 mm respectively. Both strips are excited at one end and the other end is terminated to open circuit. The second strip is terminated to SMA connector to adapt with a 50 Ω load. The measurement setup is shown in Figure (6).

The measured and calculated crosstalk is plotted in Fig. 7 versus frequency for different values of d. The shape of the graphs is reasonably the same; however, the deviation between measured and computed results is due to fabrication imperfection and systematical errors in measurement. Moreover, it is believed that the non-homogeneous structure of this line may be another source of error. For this reason, effective dielectric constant is usually considered as the dielectric constant of an equivalently homogeneous medium, which

replaces air and dielectric regions of the coupled lines [9].

Finally, a coupled strip line is considered in our analysis. The culprit conductor is connected to a source at one end and to a 50 Ω resistive load at the other end, while the victim line is connected to 50 Ω loads at both ends. The lengths of the lines are 160 mm. The relative dielectric constant, ground plane spacing, line width and spacing between lines are 4.7 mm, 3.2 mm, 1 mm and 4 mm respectively, in which per unit length capacitances are $C_1=C_2=125.02$ pF/m, $C_m=2.5$ pF/m. Due to using two conductors in this structure in a homogeneous medium, it supports TEM wave and per unit length inductances are computed by equation (10). The near and far-end cross talks are plotted in Fig. 8 versus frequency. It can be seen that the obtained results are in a good agreement with those obtained by simulation using HFSS. In addition to, due to presence of the second ground plane on the top of the strips, the crosstalk coefficients are decreased slightly compared to that of the other coupled lines.

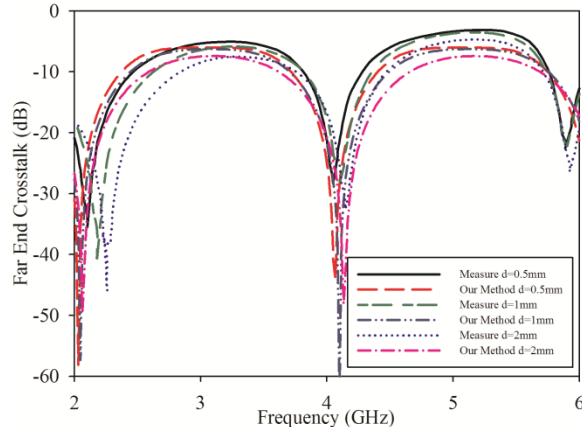**Figure 6:** The experimental setup of measurement of a microstrip coupled line.

Figure 7: The calculated and measured far end crosstalk of a microstrip line for different values of d.
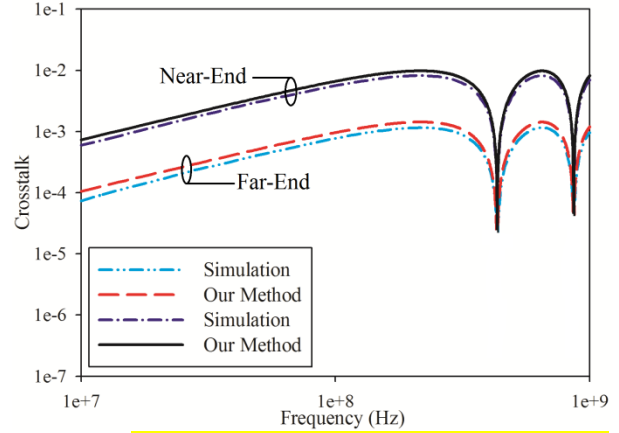

Figure 8: The calculated and simulated far-end and near-end crosstalk of a strip line.

VI. Conclusion

In this paper, a new analytical method based on eigenvector decomposition is introduced to quickly calculate the near-end and far-end crosstalk of coupled transmission lines. A linear differential equations system is derived for distributed voltage and current along the coupled lines. For this system of differential equations, using matrix formulation, generally four distinct eigenvalues are obtained for the two lines. It is shown that the propagated waves along the lines are related to these eigenvalues and their associated eigenvectors. Two eigenvalues show the self-propagation constant, while the other ones are associated to the mutual propagation constant of the coupled lines. By identifying the transmission line equations, a closed form formula for the near-end and far-end crosstalk is derived. In case of homogenous environment for the coupled lines, eigenvalues and eigenvalues are determined and it is shown for these lines provide repeated eigenvalues. Using generalized eigenvalues, the solution of the system of differential equation is presented by matrix computations and a closed form formula is derived for crosstalk. To verify the accuracy of the proposed method four types of the coupled lines are investigated and the amount of crosstalk is calculated and compared with those of experimental examination. It is shown that a very good agreement between the calculated and measured results is obtained.

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