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Free Vibration Analysis of Angle-Ply Laminated and Sandwich Plates Using Enhanced Refined Zigzag Theory

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Abstract. The paper presents a numerical assessment of the free vibration of angle-ply laminated and sandwich plates using the enhanced Refined Zigzag Theory (en-RZT). It has been observed that standard RZT cannot predict the coupling effect of in-plane displacements for anisotropic multilayered structures, such as angle-ply laminates. The recent enhancement introduces two more zigzag functions in the local displacement field that overcome this drawback. According to the partially enforcement of transverse shear stress continuity at the interfaces, the zigzag functions are characterized. The equations of motion and consistent boundary conditions are derived through the D'Alembert principle and specialized for the linear free vibration analysis. Furthermore, the Ritz method in conjunction with the Gram-Schmidt orthogonal polynomials is used to assess the model's performances when applied to the free-vibration analysis of plates with different types of boundary conditions. The numerical assessment investigates the influence of various design parameters, such as plate aspect ratios, boundary conditions and ply orientations.

INTRODUCTION

Multilayered composite and sandwich structures are widely used in the last decades in many engineering field (aerospace, marine, automotive,…) due to the their excellent properties, such as high tensile modulus and strength to weight ratios, damping characteristic and good fatigue behavior. The possibility to tailor them in order to optimize or control the structural response makes them very attractive for light-weight structures, typically of aerospace field. On the other hand, their through-the-thickness anisotropy and high transverse shear deformability require very accurate structural models in order to achieve a reliable design and to accurately describe their structural behavior during the operative life.

Many researchers have put their effort to provide mathematical models to describe the structural response of multilayered composite and sandwich structures. It is desirable using three-dimensional solutions for general laminates for any kind of load cases and boundary conditions, but this is possible only for very few cases, the most of all, difficulty to observe in real technical applications. Among the exact three-dimensional approaches available in literature framework, the analytical solution for angle-ply laminated structures have been investigated by Pagano [1], Noor and Burton [2], Savoia and Reddy [3], Chen and Lee [4] and Loredo [5].

Alternative approaches are those represented by the axiomatic theories. A detailed description has been done by Abrate and Di Sciuva [6,7]: the Equivalent Single Layer (ESL) theories are quite accurate for thin multilayered structures in predicting global quantities such as displacements and natural frequencies, but they have a poor description of through-the-thickness displacements and stresses. On the other hand the Layer-Wise (LW) theories with their independent displacement field assumed for each layer, can provide accurate results but only at high computational cost in structures with several layers. One of the best compromise is represented by the zigzag (ZZ) theories, in which the displacement field is based on the superposition of a global description (like ESL theories) and a finer contribute. Many theories have been presented in the last decades, but the Refined Zigzag Theory (RZT) has

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been shown remarkable accuracy in predicting the structural behaviour of beams [8], plates [9] and shells [10]. It is worthy to mention that the RZT requires only C0-continuity of the kinematic variables, thus the possibility for developing simple and efficient finite elements. Some examples in literature are represented by Oñate et al [11], Gherlone et al [12,13], Iurlaro [14] and Sorrenti et al [15]. On the other hand, the RZT has been demonstrated to be successfully implemented with other numerical methods such as the Higher Order Haar Wavelet Method [16] and the Isogeometric method [17].

Recently, the RZT has been further enhanced with two more zigzag functions that enable the possibility to include the in-plane coupling effect of displacements. Sorrenti and Di Sciuva [18] derived and applied the enhanced-RZT (en-RZT) to the static analysis of multilayered angle-ply composite plates assessing the great accuracy of this approach not only in predicting the maximum displacements but also the local through-the-thickness quantities.

Aim of this paper is to use the en-RZT to investigate the free vibration of general angle-ply multilayered and sandwich plates. The effect of boundary conditions, plate dimensions and thickness, number of layers and lamina orientations on the natural frequencies is evaluated.

FORMULATION

We consider a multilayered flat rectangular plate made of a finite number N of perfectly bonded layers. V is the volume of the plate, h the thickness; a1 and a2 are the length and width, respectively. The points of the plate are referred to an orthogonal Cartesian coordinate system defined by the vector $\mathbf{X}=\{x_i\}$ $(i=1,2,3)$. The vector $\mathbf{x}=\{x_\alpha\}$ $(\alpha=1,2)$ represents the set of in-plane coordinates on the reference plane, here chosen to be the middle plane of the plate, and x_3 being the coordinate normal to the reference plane, so that x_3 is defined in the range $x_3 \in \left[-\frac{h}{2},+\frac{h}{2}\right]$. The thickness of each layer, as well as of the whole plate, is assumed to be constant, and the material of each layer is assumed to be elastic orthotropic with a plane of elastic symmetry parallel to the reference surface and whose principal orthotropy directions are arbitrarily oriented with respect to the in-plane reference frame.

According to the en-RZT kinematic [18], the assumed displacement field is:

$$\begin{aligned} u_1^{(k)}(\mathbf{X},t) &= u_1^{(0)}(\mathbf{x},t)+x_3\theta_1(\mathbf{x},t)+\phi_{11}^{(k)}(x_3)\psi_1(\mathbf{x},t)+\phi_{12}^{(k)}(x_3)\psi_2(\mathbf{x},t) \\ u_2^{(k)}(\mathbf{X},t) &= u_2^{(0)}(\mathbf{x},t)+x_3\theta_2(\mathbf{x},t)+\phi_{21}^{(k)}(x_3)\psi_1(\mathbf{x},t)+\phi_{22}^{(k)}(x_3)\psi_2(\mathbf{x},t) \\ u_3^{(k)}(\mathbf{X},t) &= u_3^{(0)}(\mathbf{x},t) \end{aligned} \tag{1}$$

Where $u_i^{(0)}$ and θ_α are the global uniform displacement, and rotations of the normal, respectively, to the reference plane, ψ_α are the unknown spatial amplitudes of the $\boldsymbol{\phi}^{(k)}(x_3)$ zigzag functions (assumed continuous piecewise linear vanishing to top and bottom external surfaces). According to Ref. [18], the recursive expression for the generalized zigzag functions matrix $\boldsymbol{\phi}^{(k)}(x_3)$ of the kth layer is

$$\begin{aligned} \boldsymbol{\phi}^{(k)}(x_3) &= (x_3-z_{(\mathrm{B})})\left(\mathbf{S}_{\mathrm{t}}^{(k)}\mathbf{G}-\mathbf{I}\right)+\sum_{q=1}^{k}h^{(q)}\left(\mathbf{S}_{\mathrm{t}}^{(q)}-\mathbf{S}_{\mathrm{t}}^{(k)}\right)\mathbf{G} \\ &= (x_3-z_{(\mathrm{B})})\boldsymbol{\beta}^{(k)}+\sum_{q=1}^{k}h^{(q)}\left(\boldsymbol{\beta}^{(q)}-\boldsymbol{\beta}^{(k)}\right) \qquad (k=1,\ldots,N) \end{aligned} \tag{2}$$

where $\boldsymbol{\beta}^{(k)}=\boldsymbol{\phi}_{,3}^{(k)}=\mathbf{S}_{\mathrm{t}}^{(k)}\mathbf{G}-\mathbf{I}$; $\mathbf{G}=h\left(\sum_{k=1}^{N}h^{(k)}\mathbf{S}_{\mathrm{t}}^{(k)}\right)^{-1}$ and $\mathbf{S}_{\mathrm{t}}^{(k)}$ is the matrix of the transverse shear compliance coefficients.

The d'Alembert principle is used herein to derive the equations of motion and the variationally consistent boundary conditions. The virtual variation of the internal work given by stresses $\tilde{\boldsymbol{\sigma}}^{(k)}$ (δW_{int}) and the virtual variation of the inertial forces (δW_{in}) are given as follow

$$\delta W_{\mathrm{int}}=\int_\Omega\left\langle\tilde{\boldsymbol{\sigma}}^{(k)T}\delta\tilde{\boldsymbol{\varepsilon}}^{(k)}\right\rangle d\Omega \;\; ; \;\; \delta W_{in}=-\int_\Omega\left\langle\rho^{(k)}\ddot{\tilde{\mathbf{d}}}^T\delta\tilde{\mathbf{d}}\right\rangle d\Omega \tag{3}$$

where $\tilde{\boldsymbol{\sigma}}^{(k)T} = \left[\boldsymbol{\sigma}_{11} \quad \boldsymbol{\sigma}_{22} \quad \boldsymbol{\sigma}_{12} \quad \boldsymbol{\sigma}_{13} \quad \boldsymbol{\sigma}_{23}\right]^{(k)}$, $\tilde{\boldsymbol{\varepsilon}}^{(k)T} = \left[\boldsymbol{\varepsilon}_{11} \quad \boldsymbol{\varepsilon}_{22} \quad \boldsymbol{\gamma}_{12} \quad \boldsymbol{\gamma}_{13} \quad \boldsymbol{\gamma}_{23}\right]^{(k)}$, $\langle \bullet \rangle = \sum_{k=1}^{N} \int_{x_3^{(k-1)}}^{x_3^{(k)}} (\bullet) dx_3 = \sum_{k=1}^{N} \int_{z_{(\mathrm{B})}^{(k)}}^{z_{(\mathrm{T})}^{(k)}} (\bullet) dx_3$ and $\tilde{\mathbf{d}}^T = \left[u_1^{(0)} \quad u_2^{(0)} \quad u_2^{(0)} \quad \theta_1 \quad \theta_2 \quad \psi_1 \quad \psi_2\right]$. In Eq. (3), the overdot indicates differentiation with respect to the time. In this study, an approximate solution is obtained using the Ritz method, assuming the trial functions generated through the Gram-Schmidt orthogonal procedure. This method let us to investigate various combination of boundary conditions.

RESULTS

In this section natural frequencies of anti-symmetric angle ply multilayered composite and sandwich plates are evaluated. In the first example, see Table 1, the fundamental frequency of a ten layered-antisymmetric simply supported square plate is compared with the results coming from three dimensional elasticity, Noor and Burton [2].

TABLE 1. Non-dimensional fundamental frequencies $\bar{\omega} = \omega\sqrt{\rho h^2 / E_2}$ of simply-supported ten-layered anti-symmetric angle-ply square plate. (E_2/E_1=25, G_{13}/E_2=G_{12}/E_2=0.5, G_{23}/E_2=0.2, ν=0.25, ρ=1000 kg/m^3)

	θ=15°		θ=30°		θ=45°	
a_1/h	3D [2]	Present	3D [2]	Present	3D [2]	Present
4	0.4934	0.5192	0.5286	0.5629	0.5400	0.5773
5	0.3588	0.3748	0.3889	0.4105	0.3993	0.4231
10	0.1162	0.1184	0.1296	0.1328	0.1351	0.1388
100	0.001328	0.001328	0.001510	0.001511	0.001595	0.001595

Table 2 reports the effect of the number of layers and the span to thickness ratios on the fundamental frequencies for a simply supported anti-symmetric angle-ply squared plates. Results in Table 2 are also compared with those obtained by a refined higher order model developed by Swaminathan and Patil [19] as demonstration of the great accuracy of en-RZT.

TABLE 2. Non-dimensional fundamental frequencies $\bar{\omega} = \omega\sqrt{\rho h^2 / E_2}$ for simply supported angle-ply square plate. (E_2/E_1=40, G_{13}/E_2=G_{12}/E_2=0.6, G_{23}/E_2=0.5, ν=0.25, ρ=1000 kg/m^3)

	a_1/h	4	5	10	25	50	100
(45°/-45°)	Ref. [19]	8.8426	10.0350	12.9115	14.3500	14.6012	14.6668
	Present	8.7596	9.9353	12.8211	14.2899	14.5481	14.6150
$(45°/-45°)_2$	Ref. [19]	10.0731	11.9465	17.8773	22.2554	23.1949	23.4499
	Present	11.4573	13.4370	19.0657	22.5932	23.2830	23.4660
$(45°/-45°)_4$	Ref. [19]	10.7473	12.7523	19.1258	23.8713	24.8959	25.1741
	Present	11.6321	13.7316	19.9614	24.1221	24.9647	25.1900

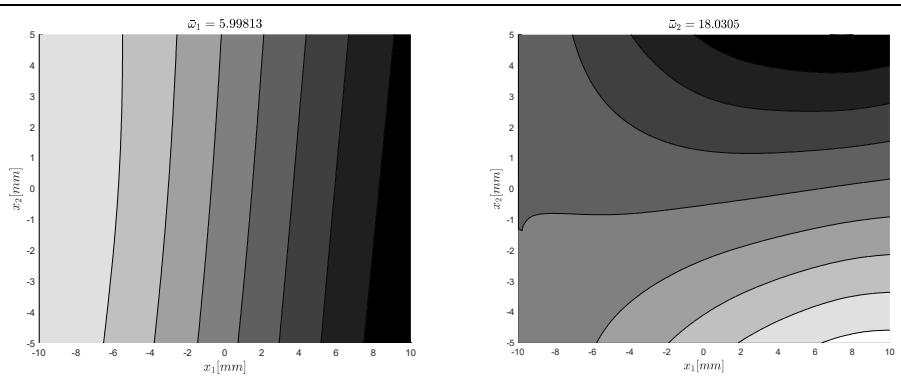

FIGURE 1. Modal shapes (flexural and torsional) for cantilever rectangular symmetric angle-ply sandwich plate (h_c/h_f=8).

Figure 1 shows the first flexional mode and the first torsional mode for a cantilever rectangular symmetric angle-ply sandwich plate. The clamped edge is the left edge, the lamination scheme from bottom to top layer is (45°/-45°/Core/-45°/45°). The facesheets are made of orthotropic material: E1=157.9 GPa, E2=E3=9.584 GPa, ν12= ν13=0.32, ν23=0.49, G12=G13=5.93 GPa, G23=3.227 GPa, ρ=1000 kg/m3. The core is isotropic with E=104 MPa, ν=0.3 and ρ=100 kg/m3. The core-thickness to facesheet-thickness ratio is 8. It is evident from Figure 1 the coupling effect of lamination scheme in the modal shape of the first two modes.

CONCLUSIONS

The numerical performances of the recently developed enhanced Refined Zigzag Theory (en-RZT) in conjunction of the Ritz method have been assessed for free vibration analysis of multilayered angle-ply laminated and sandwich rectangular plates. The numerical results show a remarkable accuracy of en-RZT in predicting fundamental frequencies for various angle-ply laminated plates. As expected, the frequencies are strongly affected the lamina orientation angle, their value increasing as the lamina orientation angle increases up to 45 degrees. Moreover, the effect of lamina orientation in sandwich plates has been investigated demonstrating the great potentialities of en-RZT for the dynamic structural analysis and in predicting the modal shape of these structures.

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