Stress analysis of variable angle tow shells by high-order unified finite elements

STRESS ANALYSIS OF VARIABLE ANGLE TOW SHELLS BY HIGH-ORDER UNIFIED ELEMENTS

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ABSTRACT

Variable Angle Tow (VAT) composite components constitute a family of laminated structures in which the fibre orientation varies point-wise within the ply. VAT are commonly obtained by means of additive manufacturing techniques such as Automated Fibre Placement (AFP) [1], that allows to create curvilinear paths while placing pre-impregnated tows. In the recent VAT literature, this sort of composites have been studied for the case laminated plates and beams [2-3], being almost residual their application to shells, as far as the authors concern. In order to study the mechanical behaviour of such structures, Carrera Unified Formulation (CUF) [4] is utilised to obtain the numerical structural models. The potential of CUF resides in its capacity of choosing the degree of accuracy, i.e., the order of the structural model as input of each analysis. Particularly, CUF has been applied to a large spectre of mechanical problems, among which the reader can find some studies regarding shell [5] and VAT structures [6]. This work aims to address the influence of the parameters used to describe the variable fibre angle orientation on the through-the-thickness stress distribution of laminated composite shells.

Keywords: Variable Angle Tow, Composite Shells, Unified formulation

1 UNIFIED ELEMENTS FOR VAT SHELLS

1.1 Preliminary considerations

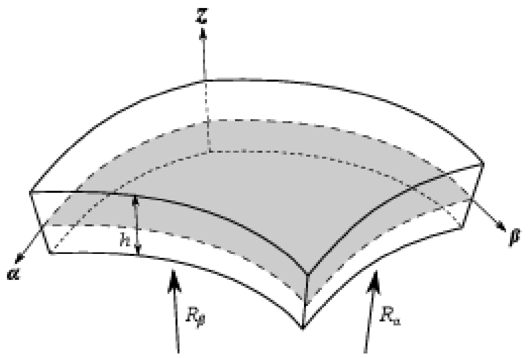


Figure 1. Graphical representation of a shell-like structure.

The analysis of VAT shell structures is carried out with refined shell models. A shell is a 2D structural element where the thickness is negligible in comparison with the other dimensions. In this work, an orthogonal curvilinear reference system (α, β, z) is employed (see Figure 1). The transposed displacement, strain and stress vectors for layer k of the multi-layered shell can be written as:

$$u^k = \{u_\alpha^k, u_\beta^k, u_z^k\}^T \tag{1}$$

$$\boldsymbol{\varepsilon} = \{\varepsilon_{\alpha\alpha}^k, \varepsilon_{\beta\beta}^k, \varepsilon_{zz}^k, \varepsilon_{\alpha z}^k, \varepsilon_{\beta z}^k, \varepsilon_{\alpha\beta}^k\}^T$$

$$\boldsymbol{\sigma} = \{\sigma_{\alpha\alpha}^k, \sigma_{\beta\beta}^k, \sigma_{zz}^k, \sigma_{\alpha z}^k, \sigma_{\beta z}^k, \sigma_{\alpha\beta}^k\}^T$$

Strains and displacements are related by means of the geometrical relationships which, for the sake of brevity, are not reported in this paper but can be found in the shell literature [7]. Then, using the constitutive equations, stresses can be calculated as:

$$\boldsymbol{\sigma}^k = C^k \boldsymbol{\varepsilon}^k \tag{2}$$

where C^k is the material elastic matrix. Note that, for the case of composite structures, matrix C^k has to be rotated to the global reference system through the rotation matrix T. This matrix changes pointwise for the case of VAT. That is, the material elastic matrix in the global reference frame is calculated as:

$$\tilde{C}^k = T(\alpha, \beta)^T C^k T(\alpha, \beta) \tag{3}$$

In this work, the local fibre angle orientation is supposed to vary linearly along direction α' as

$$\theta(\alpha') = \Phi + T_0 + \frac{T_1 - T_0}{d} |\alpha'| \tag{4}$$

in which T_0 is the initial fibre orientation, T_1 is the final fibre orientation, d is the length along which the fibre orientation varies, and $\alpha' = \alpha\cos(\Phi) + \beta\sin(\Phi)$. The reader is invited to read the work by [8] for further information.

1.2 Kinematic assumptions

In the framework of CUF, the refinement of the theory is considered as input of the analysis. Therefore, low to high-order models can be created in a hierarchic and unified manner. The 3D field of displacement can be expressed as an arbitrary through-the-thickness expansion of the in-plane variables as:

$$\boldsymbol{u}^k(\alpha, \beta, z) = F_\tau^k(z) \boldsymbol{u}_\tau^k(\alpha. \beta) \tag{5}$$

where $\tau = 1, \ldots, M$ represents the number of terms considered in the thickness expansion $F_\tau(z)$, and $\boldsymbol{u}_\tau^k$ is the generalized displacement vector that depends on the in-plane coordinates. In the literature, Equivalent Single Layer (ESL) and Layer-wise (LW) theories are typically employed for the characterisation of the stress state of composite shells. Due to paper constraints, these expansion families are not reported in this manuscript, but can be found in [4].

Then, the Finite Element Method (FEM) can be used to approximate the in-plane generalised displacement vector by means of the shape functions $N_i(\alpha, \beta)$ as:

$$\boldsymbol{u}_\tau^k(\alpha,\beta) = N_i(\alpha,\beta)\boldsymbol{q}_{\tau i} \tag{6}$$

in which $\boldsymbol{q}_{\tau i}$ denotes the unknown nodal variables and $i = 1, \dots, N_n$, where N_n is the number of nodes per element, and i indicates summation. Derivation of the governing equations is omitted for brevity, but can be found in [6].

2 NUMERICAL RESULTS

In this section, two numerical assessments are presented. First, a verification of the present modelling approach against commercial software Abaqus is conducted. Then, the influence of the parameters in Eq. 4 is addressed for the case of a curved VAT shell panel. The material properties of these two numerical examples are gathered in Table 1.

Case	E_{11} [GPa]	E_{22} [GPa]	ν_{12} [-]	ν_{23} [-]	G_{12} [GPa]	G_{23} [GPa]
Flat panel	143.17	9.64	0.252	0.349	6.09	3.12
Curved panel	3.30	1.10	0.25	0.25	0.66	0.66

Table 1. Elastic properties of the materials employed for the different numerical cases.

2.1 VAT flat panel

First, a VAT flat panel is analysed against commercial software. The plate is clamped on its four edges and the stacking sequence is $\theta = [0 < 90{,}45 > ,0 < 0{,}45 >]$. A uniform $P_z = 10$ kPa pressure is exerted on the top surface. The dimensions of the plate can be appreciated in Figure 2. Stress magnitudes are computed at point $Q \equiv (0.25{,}0.25)$ m.

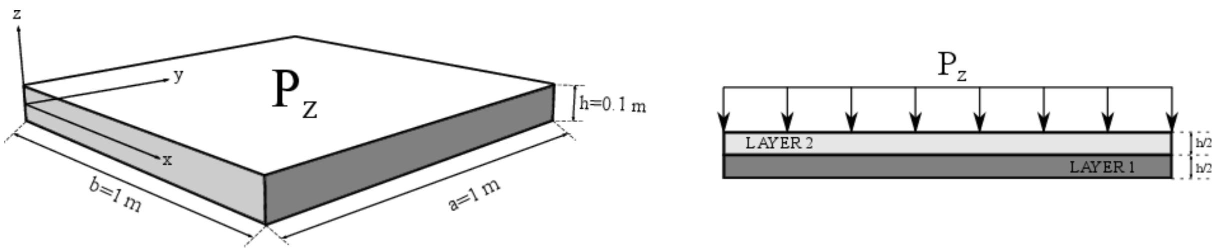
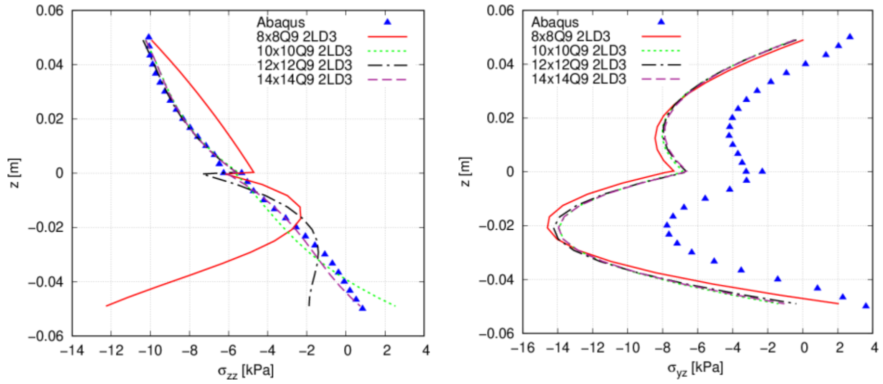


Figure 2. Illustration of the VAT flat panel.

A mesh convergence analysis was first conducted until the results provided by the current approach are in agreement with the ones provided by the commercial software. These results are available in Figure 3. According to this figure, a 14x14 mesh, conformed by quadratic Q9 elements, is necessary to retrieve an accurate evaluation of the stress state. However, in Figure 3.b, evident differences between both models appear. The commercial code is not able to provide a null value of the transverse shear at the bottom and top of the laminate, whilst the proposed method does. This might be due to the way in which fibre orientation is imposed in both models: in the commercial one, a constant fibre orientation per element is provided, while the current method varies at the integration points of the structure.



(a) (b)

Figure 3. Through-the-thickness representation of the stress components at point Q for the flat panel case. (a) σ_{zz}; (b) σ_{yz}.

2.2 VAT curved panel

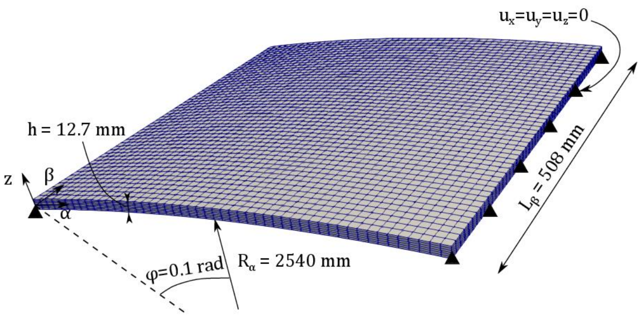
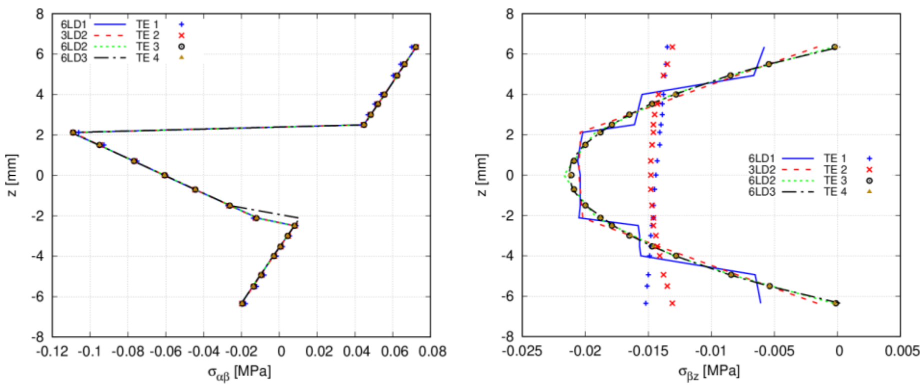


Figure 4. Representation of the VAT curved panel.

A VAT shell is studied in the following. The shell is hinged on its lateral edges and a concentrated load $F_z = 200$ N is applied at the centre point. The stacking sequence of the laminated structure is $\theta = [90 < 30,0 > ,0 < 30,0 > ,90 < 30,0 >]$. The shell dimensions are gathered in Figure 4. The stresses are evaluated at points $S \equiv (254,127)$ mm and $W \equiv (127,127)$ mm.



(a) (b)

Figure 5. Through-the-thickness representation of the stress components at point S for the curved VAT panel. (a) $\sigma_{\alpha\beta}$; (b) $\sigma_{\beta z}$.

Once the in-plane mesh convergence was carried out, yielding a 32x32 Q9 mesh needed for accurate results, the influence of the different expansion theories was addressed.

Both Taylor (TE) and Lagrange (LE) expansions were employed, which lead to ESL and LW modelling approaches, respectively. The differences between these two approaches is appreciated in Figure 5. The reader can see that both ESL and LW models agree in the prediction of the in-plane shear stress component (Figure 5.a), while they present some discrepancies in the evaluation of transverse shear. However, in the latter, it is appreciated that TE 3 and TE 4 are able to retrieve the same distribution as LE models.

3 CONCLUDING REMARKS

In this work, a CUF-based refined shell theory has been developed for the analysis of VAT panels. Particularly, a flat panel has been considered for verification purposes against commercial code Abaqus. The current approach presented a similar through-the-thickness stress distribution for most of the stress components. Nevertheless, some differences were found between the proposed approach and the commercial software. These might be due to the way in which the spatially varying fibre orientation is imposed in both numerical models: in the proposed one, a different fibre angle is computed at each integration point of the structure, whilst for the Abaqus model, a constant angle is imposed at each element.

The second numerical assessment comprised a hinged curved VAT panel. Different expansion theories were employed to characterise the stress distribution through the thickness, leading to the conclusion that LW models are necessary for an accurate retrieval of the stresses. Finally, the effect of T_0 and T_1 on the stresses was investigated. The different results suggest that this kind of structures can play a major role in the future of shell structures thanks to its tailoring capabilities, which allow the designers to extend the design space and find enhanced solutions.

4 ACKNOWELEDGEMENTS AND REFERENCES

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