Numerical nonlinear vibration-buckling investigation of variable stiffness composite plate and shell structures

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NUMERICAL NONLINEAR VIBRATION-BUCKLING INVESTIGATION OF VARIABLE STIFFNESS COMPOSITE PLATE AND SHELL STRUCTURES

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ABSTRACT

This work intends to present a novel numerical approach for carrying out virtual Vibration Correlation Technique (VCT) in variable angle tow (VAT) composite structures to predict the buckling load, to characterize the variation of the natural frequencies for progressively increasing loads, and to provide verification of the experimental VCT results. The study has been performed using the well-established Carrera Unified Formulation (CUF) able to describe several kinematic models for two-dimensional (2D) structures. All Green-Lagrange strain components are employed because far nonlinear regimes are investigated. Furthermore, the geometrical nonlinear equations are written in a total Lagrangian framework and solved with an opportune Newton-Raphson method along with a path-following approach based on the arc-length constraint. Different VAT composite structures have been investigated in order to validate the proposed approach and provide some benchmark solutions. The computed equilibrium paths are compared with results obtained using the commercial code ABAQUS. The results document the good accuracy and reliability of the proposed approach and show this numerical tool's potentialities.

Keywords: VAT; Natural frequencies; Buckling; Geometrical nonlinearity.

1 INTRODUCTION

One of the most important experimental methods used in aerospace industry for the assessment of the buckling is the Vibration Correlation Technique (VCT) [1,2]. This nondestructive experimental test allows to calculate the buckling load and the equivalent boundary conditions by interpolating the natural frequencies of the structures for progressively increasing applied loads without reaching instability. The first experimental VCT investigations were performed by Lurie [3], Meier [4] and Chu [5]. In view of its importance and potential, several experimental tests and studies were carried out for decades. Recently, Abramovich et al. [6] adopted the VCT to evaluate the buckling load of metallic and laminated structures. Jansen et al. [7] presented the capability of analysis tools for supporting and improving the accuracy of the VCT results obtained through semi-empirical methods. For the sake of brevity, readers are referred to [8,9] for further detailed investigations. The literature on VCT analyses of isotropic and classical composite two-dimensional (2D) structures is vast, while the one on variable angle tow (VAT) plates and shells is limited. Readers are referred to [10,11] for some interesting studies on the dynamic investigation of VAT structures.

The principal goal of this work is to provide an efficient methodology based on high-accuracy but efficient layerwise (LW) models to investigate the dynamic characteristics of VAT plates and shells under extreme loadings and a means for verifying experimental VCT results.

In this context, the VAT 2D structures are formulated in the Carrera Unified Formulation (CUF) [12] framework in order to obtain accurate results. The main advantage of the CUF is to be able to consider the structural model order as an input of the analysis. In this way, the refined generic models do not

need specific formulations. One of the advantages of the present formulation with respect to the others, often based on linear approaches, is to consider the geometrical nonlinearities that allow to guarantee a remarkable accuracy of the results. In fact, the nonlinear governing equations and the relative finite element (FE) arrays of the 2D theories are written in terms of Fundamental Nuclei (FNs). FNs represent the basic building blocks of the presented formulation.
The investigated structures are subjected to progressively higher applied loads, and for each state of equilibrium, on the deformed structure, the natural frequencies are calculated by solving a linearized eigenvalue problem, obtained from an analysis of the free vibration on the structure.

2 VIBRATION AROUND NONLINEAR EQUILBRIUM STATES

The application of the presented methodology to investigate the vibration around nonlinear equilibrium states can be described in the following steps: 1) first, the static geometrical nonlinear problem is solved using the Newton-Raphson method based on the arc-length approach. 2) Once the nonlinear equilibrium is computed, the tangent stiffness matrix $\boldsymbol{K}_T$ is obtained in each states of interest. 3) Then, since the modal behavior of a structure is not a property of the geometric and mechanical characteristics, but it is a property of the state of equilibrium, the free vibrations analysis is carried out around a linearized (non-trivial) equilibrium state along the nonlinear path. Namely, the linearization of the equation of motion is written as:

$$\delta(\delta L_{int} + \delta L_{ine}) = \delta \boldsymbol{q}_{sj}^T \boldsymbol{K}_T^{ij\tau s} \boldsymbol{q}_{\tau i} + \delta \boldsymbol{q}_{sj}^T \boldsymbol{M}^{ij\tau s} \delta \ddot{\boldsymbol{q}}_{\tau i} \tag{1}$$

where $\boldsymbol{M}^{ij\tau s}$ is the FN of the mass matrix and it is assumed to be linear, $\boldsymbol{K}_T^{ij\tau s}$ represents the FN of the tangent stiffness matrix and $\boldsymbol{q}_{\tau i}$ indicates the vector of the unknown nodal variables. 4) By assuming harmonic motion around non-trivial equilibrium states, the equation of motion is simplified into a linear eigenvalues problem, Eq. (2), from which it is possibile to evaluate natural frequencies and mode shapes.

$$\left(\boldsymbol{K}_T^{ij\tau s} - \omega^2 \boldsymbol{M}^{ij\tau s}\right) \delta \tilde{\boldsymbol{q}}_{\tau i} = 0 \tag{2}$$

in which ω indicates the natural frequencies and $\delta \tilde{\boldsymbol{q}}_{\tau i}$ is the eigenvector.
These vectors and matrices are formulated in the framework of the CUF. According to CUF, the three-dimensional (3D) displacement field in the dynamic case for a plate model, represented using a Cartesian system (x, y, z), is defined as a general expansion of the primary unknowns:

$$\boldsymbol{u}^k(x,y,z;t) = F_\tau^k(z) \boldsymbol{u}_\tau^k(x,y;t) \qquad \tau = 1,\dots,\mathrm{M} \tag{3}$$

in which $\boldsymbol{u}_\tau(x,y;t)$ is the generalized in-plane displacement vector, F_τ represent the expansion functions of the thickness coordinate z, M denotes the order of expansion in the thickness direction, k indicates the layer index in laminated composite structures and t stand for time.
In this research, Lagrange polynomials (LE) are adopted for the expansion functions. For brevity, readers are referred to [12] for a full explanation about the mathematical derivation of the 2D FE formulation in the domain of CUF.
The finite element method (FEM) is used to approximate the in-plane generalized displacement vector employing the shape function $N_i(x,y)$.

$$\boldsymbol{u}_\tau^k(x,y;t) = N_i(x,y) \boldsymbol{q}_{\tau i}^k(t) \qquad i = 1,\dots,\mathrm{N_n} \tag{4}$$

in which $\mathrm{N_n}$ stand for the number of nodes per element and i indicates summation.
In this study, employing the total Lagrangian formulation, the full Green-Lagrange nonlinear strain vector is adopted.

$$\boldsymbol{\varepsilon}^k = \boldsymbol{\varepsilon}_l^k + \boldsymbol{\varepsilon}_{nl}^k = (\boldsymbol{b}_l + \boldsymbol{b}_{nl}) \boldsymbol{u}^k \tag{5}$$

where $\boldsymbol{b}_l$ and $\boldsymbol{b}_{nl}$ represent the linear and nonlinear differential operators [13].

The stress vector is computed from the constitutive relation:

$$\boldsymbol{\sigma}^k = \boldsymbol{C}^k \boldsymbol{\varepsilon}^k \tag{6}$$

in which $\boldsymbol{C}$ is the material elastic matrix for orthotropic materials and it is defined in [14].
In a VAT structure, the fibre has a general orientation function of the space coordinates, i.e.; *θ(x,y)*. Thus, we have:

$$\boldsymbol{\sigma}^k = \widetilde{\boldsymbol{C}}^k \boldsymbol{\varepsilon}^k \tag{7}$$

with

$$\widetilde{\boldsymbol{C}}^k = \boldsymbol{T}^T \boldsymbol{C}^k \boldsymbol{T} \tag{8}$$

where $\boldsymbol{T}$ is the rotation matrix [15].
In this work, a linear fibre angle variation, as described by Gürdal [16], over the lamina is assumed. For the sake of brevity, the reader is refereed to [17] for a complete description of the VAT formulation.

3 NUMERICAL RESULTS

For the representative purpose, a 4-layer $[0\pm\langle 0/15\rangle]_2$ rectangular VAT laminate plate is considered. This structure has the following geometrical data: length (*a*) equal to 0.2 m, the width (*b*) is 0.05 m and a total thickness (*h*) of 0.002 m. This VAT plate is subjected to uniform axial compressive load in the *x*-direction, see Figure 1. All the plate edges are simply-supported. The lamina properties are given in Table 1.
The results in the following analyses are presented in the non-dimensional form using the equation:

$$\widetilde{\omega} = \boldsymbol{\omega}\left(\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}\right) \tag{9}$$

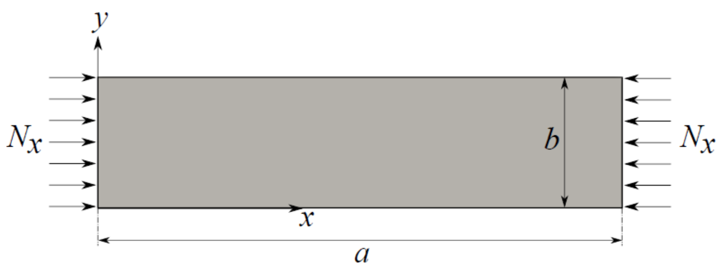

Figure 1: Geometry of the rectangular VAT plate subjected to uniform axial compressive load.

E_1 (GPa)	E_2=E_3 (GPa)	G_{12}=G_{13} (GPa)	G_{23} (GPa)	ν_{12}=ν_{13}	ν_{23}	ρ (kg/m^3)
250	6.25	5.125	3.25	0.24	0.49	1

Table 1: Material properties of the rectangular VAT plate under uniform axial compression.

For the following discussions, the convergent model is obtained by adopting at least 20x5Q9 for the in-plane mesh approximation and only one LE1 in each layer in the thickness direction. Figure 2 depicts the equilibrium curves of the rectangular VAT plate computed by the 2D CUF linear model, 2D CUF nonlinear model, and, for the sake of comparison, using the ABAQUS 3D solid nonlinear model.

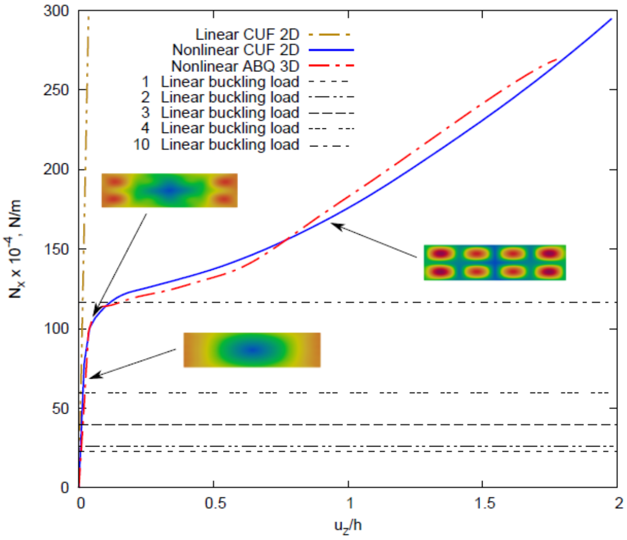

Figure 2: Comparison of equilibrium curves for the rectangular VAT plate under in-plane compressive loads obtained with the 2D Linear model, 2D CUF Nonlinear model and ABAQUS 3D Nonlinear model (60x15x4 C3D20R)

Figure 3 shows the comparison between the variation of the non-dimensional natural frequencies for progressively increasing loads via trivial linearized solution and via full nonlinear approach.

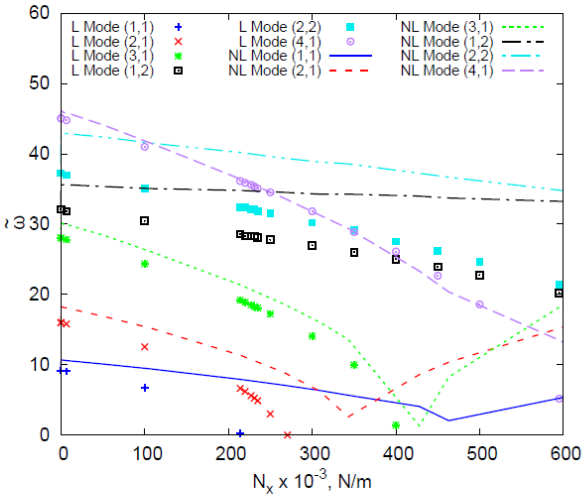

Figure 3: Comparison between the variation of the non-dimensional natural frequencies obtained via trivial linearized solution and full nonlinear approach for the rectangular VAT plate with simply-supported edge conditions.

The results suggest that a nonlinear approach is needed to perform accurate vibration analysis and, in particular, to have a reliable buckling prediction. In particular, this definite change in the slope of the frequency represents a criterion for the nonlinear buckling prediction. Moreover, it is clear that crossing and veering phenomena are completely unforeseen by the trivial linearized approach. Readers are invited to compare the modes reported in Figures 4; various crossing phenomena occur and mode aberration is evident.

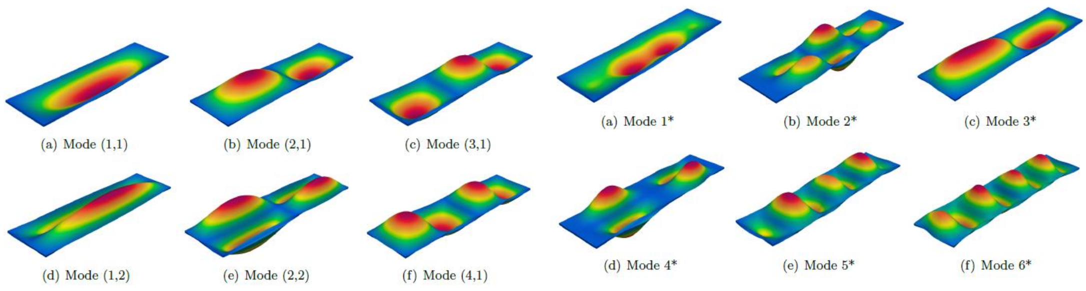

Figure 4: Characteristics first six vibration mode shapes for the rectangular VAT plate; (a) $N_x = 0\ N/m$; (b) $N_x = 1.83x10^6\ N/m$.

4 CONCLUDING REMARKS

The presented method allows to determine the buckling load of variable angle tow (VAT) structures, to evaluate the natural frequencies variation and to provide a means for verifying experimental VCT results with high reliability. The results demonstrated the potential of this approach and provide reasonable confidence for future applications in this topic. In particular, a full nonlinear approach is needed to perform accurate investigations. It was shown that eigenfrequencies and eigenmodes can suffer abrupt aberrations in deep nonlinear regimes. Moreover, mode aberration is evident compared to the modes calculated using the full nonlinear approach with those obtained in the trivial state.

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