Numerical Modelling of Glide Periodic Structures via Integral Equations

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***Abstract*—In this paper we propose to study the dispersion diagram of non-canonical glide-symmetric unit cells via integral equations with the use of the periodic Green's function and discretized via the method of moment. The proposed approach is validated with a fully metallic implementation of a Luneburg lens operating at Ka-band with potential use for 5G communications. The promising preliminary numerical results highlight the potentialities of this approach.**

***Index Terms*—Integral equation, periodic Green's function, periodic structures, glide symmetry**

I. Introduction

The study of properties of higher-symmetric metasurfaces require fast and accurate modelling tools to predict the behaviour of the structure and optimize a final prototype. By means of an accurate characterization of the behavior of these higher-symmetric metasurfaces, new radiating systems and guiding wave devices can be designed to achieve performance beyond the state-of-the-art in terms of beam-scanning capabilities, operational bandwidth, losses in different technologies such as slotted substrate-integrated waveguide (SIW), filters and resonators, open and closed gap waveguides, planar printed-circuit-board (PCB), 3-D printable materials for artificial lenses. These new materials are made of periodic cells invariant under higher symmetries, such as glide or twist symmetries. As an example, while a periodic structure is invariant under translations, a glide-symmetric structure is also invariant under a translation and a mirroring. These symmetries lead to marvelous uncommon properties: ultra wide bandwidth of operation, reduced losses, scanning capabilities, and enhanced stopband for electromagnetic (EM) bandgap materials. They have the potential to meet the expectation required by new communication standards.

The mode matching technique [1] has been commonly used to solve this kind of problems: the unit cell can be decomposed into sub-domains where Maxwell equations can be solved analytically. However, the application of this technique is limited to unit cells with canonical shapes. For more generic unit cells, formulations as the finite difference time domain (FDTD) [2] or the finite integration technique (FIT) [3] can be used due to their easy implementation from differential equations. However, it is well known that they are volumetric formulations strongly burdened by the discretization of the object and the surrounding space, where artificial termination boundary conditions has to be imposed. As a consequence, the accurate analysis of extended three-dimensional systems could require a high computational cost.

A different approach that has been extensively applied in the radio frequency (RF) and microwave regime, due to its high level of accuracy and versatility, is the well-known method of moments (MoM). This approach brings important advantages in problems including both perfect electric conductors and penetrable bodies, because they can be formulated in terms of surface integral equations (SIEs) defined over the material boundary surfaces and interfaces. This avoids the discretization of volumes and strongly reduces the required number of unknowns, which now scales linearly with the electrical size of the surfaces.

In this paper, we propose a novel tool, based on MoM-SIEs in conjunction with the free-space periodic Green's function (FSPGF) and a root-finding algorithm. Its aim is to correctly characterize higher-symmetric materials and to analyze and design non-canonical unit cells that are cases not suitable for the mode-matching approach.

II. Unit Cells Analysis via MoM-SIEs

Let us consider a perfect electric conductor (PEC) unit cell where we can formulate the electric field integral equations (EFIE). In the EFIE, the electric fields at the interface surfaces are written in terms of the equivalent surface electric (density) current $\mathbf{J}(\mathbf{r})$ as

$$-\mathbf{E}^S_{\tan}\left\{\mathbf{J}(\mathbf{r})\right\} = \mathbf{E}^i_{\tan}(\mathbf{r}) \tag{1}$$

where $\mathbf{E}^S_{\tan}$ is the tangential component on the PEC surface of the scattered field due to the PEC; $\mathbf{E}^i_{\tan}$ is a know incident field, also tangential to the PEC surface, and $\mathbf{r} = (x, y, z)$ is a point on the PEC surface. The scattered field $\mathbf{E}^S_{\tan}$ can be expressed as

$$\mathbf{E}^S_{\tan}\left\{\mathbf{J}(\mathbf{r})\right\} = -j\omega\mathbf{A} - \nabla\mathbf{\Phi} \tag{2}$$

where $\omega = 2\pi f$, being f the frequency. $\mathbf{A}$ is the vector potential and $\mathbf{\Phi}$ the scalar potential expressed as

$$\mathbf{A} = \mu \int_S \mathbf{J}(\mathbf{r})\, G(\mathbf{r},\mathbf{r}')\, \mathrm{d}S'$$
$$\nabla\mathbf{\Phi} = -\frac{1}{j\omega\varepsilon}\nabla \int_S \nabla' \cdot \mathbf{J}(\mathbf{r})\, G(\mathbf{r},\mathbf{r}')\, \mathrm{d}S' \quad (3)$$

where ε and μ are the constitutive parameters of vacuum, $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the problem and $\mathbf{r}' = (x', y', z')$ is a source point on the PEC surface. We are considering here a free-space periodic problem, since once the PEC surface is removed no other media (e.g., layered media) are present. Therefore, $G(\mathbf{r},\mathbf{r}')$ is the free-space periodic Green's function (FSPGF), defined as in [4]

$$G(\mathbf{r},\mathbf{r}',\mathbf{k}_{t00}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G_{m,n} \quad (4)$$

where the FEPGF is expressed as a sum of spatial terms $G_{m,n}$ with

$$G_{m,n} = e^{-j\mathbf{k}_{t00}\cdot\boldsymbol{\rho}_{mn}}\frac{e^{-jkR_{mn}}}{4\pi R_{mn}} \quad (5)$$

where $\mathbf{k}_{t00} = k_{x0}\hat{\mathbf{x}} + k_{y0}\hat{\mathbf{y}}$ is the transverse vector wavenumber that defines the inter-element phasing for the 2-D array in terms of the propagation angles of the first Floquet mode. $\boldsymbol{\rho}_{mn} = m\boldsymbol{s}_1 + n\boldsymbol{s}_2$, being $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ the lattice vectors, and $R_{mn} = \sqrt{(z-z')^2 + |\boldsymbol{\rho} - \boldsymbol{\rho}' - \boldsymbol{\rho}_{mn}|^2}$, with $\boldsymbol{\rho} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$ and $\boldsymbol{\rho}' = x'\hat{\boldsymbol{x}} + y'\hat{\boldsymbol{y}}$. In the same way the FEPGF can be written as a sum of spectral terms $\widetilde{G}_{p,q}$ as

$$G(\mathbf{r},\mathbf{r}',\mathbf{k}_{t00}) = \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \widetilde{G}_{p,q} \quad (6)$$

Evaluating (4) with the Ewald method, the FSPGF is then expressed as the sum of a "modified spectral" and a "modified spatial" series [5]

$$G(\mathbf{r},\mathbf{r}',\mathbf{k}_{t00}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G^E_{m,n} + \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \widetilde{G}^E_{p,q} \quad (7)$$

where $G^E_{m,n}$ and $\widetilde{G}^E_{p,q}$ are the terms of the spatial and spectral sums, respectively. The weight of the two sums can be tuned by modifying a real parameter, the E splitting parameter, present in the definition of the terms. E balances the the asymptotic rate of convergence of the spatial and spectral series, and consequently minimizes the overall number of terms needed to evaluate the FSPGF. The convergence rate is optimum when the "optimum" value of the Ewald splitting parameter is used

$$E_{opt} = \sqrt{\frac{\pi}{A}}, \quad (8)$$

where A is the area of the unit cell [6].

Once defined the FSPGF we solve the equation (1) using the MoM [7], in order to transform an integral equation into a system of linear equations, by discretizing the current $\mathbf{J}$ as

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N} I_n \mathbf{\Lambda}_n(\mathbf{r}), \quad (9)$$

where $\mathbf{\Lambda}_n(\mathbf{r})$ are known basis functions that discretize the surface of interest. The basis functions commonly used are the Rao-Wilton-Glisson (RWG) ones [8]. The discretization (9) allows us to write (1) as the matrix system

$$[Z(\mathbf{k}_{t00})][I] = [V], \quad (10)$$

where $[I]$ is the N-size vector collecting the unknown coefficients I_n that describe the current surface. $[Z]$ is the system $N \times N$ matrix with each element equal to

$$[Z]_{m,n}(\mathbf{k}_{t00}) = j\omega\mu \int_S \mathbf{\Lambda}_m(\mathbf{r}) \cdot \int_S \mathbf{\Lambda}_n(\mathbf{r}') G(\mathbf{r},\mathbf{r}',\mathbf{k}_{t00}) \mathrm{d}S'\mathrm{d}S$$
$$+\frac{1}{j\omega\varepsilon}\int_S \boldsymbol{\nabla}\cdot\mathbf{\Lambda}_m(\mathbf{r}) \int_S \boldsymbol{\nabla}\cdot\mathbf{\Lambda}_n(\mathbf{r}') G(\mathbf{r},\mathbf{r}',\mathbf{k}_{t00}) \mathrm{d}S'\mathrm{d}S \quad (11)$$

and $[V]$ is the N-size right hand side vector with each element equal to

$$[V]_m = \int_S \mathbf{\Lambda}_m(\mathbf{r}) \cdot \mathbf{E}^i_{\tan}(\mathbf{r})\, \mathrm{d}S. \quad (12)$$

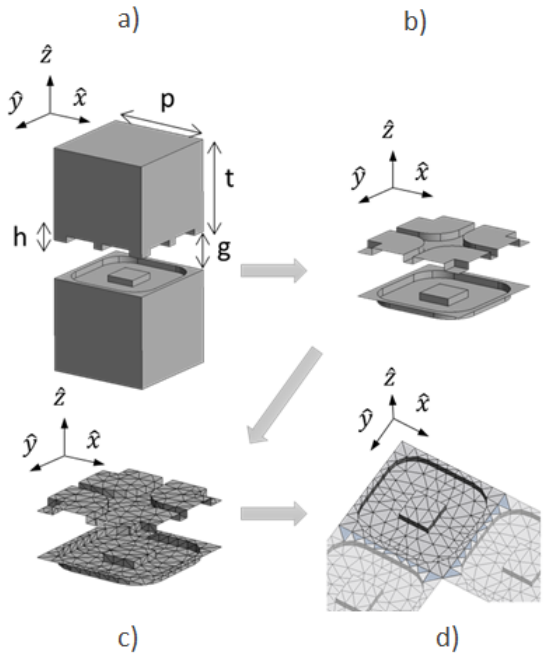

Fig. 1. Schematic description of the geometrical procedure.

To analyze the unit cell and draw its dispersion diagram, we have to find, for each frequency, the $\mathbf{k}_{t00}$ value for which the system matrix $[Z(\mathbf{k}_{t00})]$ is singular; in other words we are searching for the conditions of resonance. Hence, varying $\mathbf{k}_{t00}$, we estimate the reciprocal of the 1-norm condition number ("rcond") of $[Z(\mathbf{k}_{t00})]$ in order to find its minimum that

corresponds to badly conditioned $[Z(\mathbf{k}_{t00})]$. To search the minimum of the function "rcond", we will use the root finder algorithm developed in [9]. This algorithm needs three initial guesses for rcond$\left[Z(\mathbf{k}^i_{t00})\right]$, with $i = 1, 2, 3$; these three initial guesses can be chosen around the $\mathbf{k}_{t00}$ found for a previous frequency analasis. Each iteration n requires only one evaluation of rcond$[Z(\mathbf{k}^n_{t00})]$.

A schematic description of the described procedure is shown in Fig. 1. Figure 1.(a) reports the geometrical description of a fully metallic unit cell that is infinitely replicated along xy-plane. To analyze it with the MoM-SIEs procedure, we dismiss all the metallic surfaces orthogonal to the propagation (where there is zero field) and not present in the periodic structure, as shown in Fig. 1.(b). Then, the remaining surfaces are meshed with triangular cells with dimensions of at least one tenth of the working wavelength [see Fig. 1.(c)]. In order to guarantee the current continuity at the unit cell borders, during the meshing of the geometry, we have to ensure a conformal mesh at the unit cell borders and to add fictitious triangular cells in both axes of the periodicity, as shown in Fig. 1.(d). The RWG basis functions defined at the unit cell borders allow to properly model the current continuity.

III. Preliminary Numerical Results

In order to show the validity of the proposed method, we analyze the unit cell with a glide symmetry described in [10], shown here for completeness in Fig. 1.(a). The used design parameters are $g = 0.3\,\text{mm}$, $p = 3.2\,\text{mm}$, and $h = 0.8\,\text{mm}$; the considered frequency is 5 GHz and we assume a propagation along $\mathbf{k}_{t00} = k_x\hat{\mathbf{x}}$ only. In this work, we simplify the search for zeros and we run the simulations for different k_x in a range that include the expected value of k_x for which the system matrix $[Z(k_x)]$ is singular. In all cases the "rcond" is evaluated as shown in Fig. 2 for different values of n, where

$$n = \frac{k_x}{k_0}, \tag{13}$$

i.e., the equivalent refractive index. The reported "rcond" shows clearly a minimum close to 1.3 according to [10].

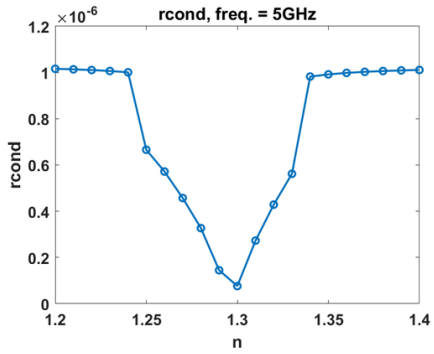

Fig. 2. Reciprocal of the 1-norm condition number of the system matrix for a range of n where $n = k_x/k_0$ at 5 GHz for the unit cell defined in [10].

IV. Conclusion and Perspectives

We presented a scheme based on the use of MoM-SIEs and periodic Green's function to analyze and design higher-symmetric metasurfaces with non-canonical unit cells. The method has been initially validated with a fully metallic implementation of a Luneburg lens [10].

The next steps are further validations of the results for a wide frequency range and the implementation of the tool for the search of zeros in the function "rcond". We will also study the acceleration of the evaluation of the MoM matrix using fast methods [11] and the interpolation of Ewald-accelerated periodic Green's function [5]. We also expect to extend the method to metallic and finite dielectric unit cell.

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