MISSIONS FOR ASTEROID INSERTION INTO EARTH-MARS CYCLER

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This paper analyzes the feasibility of insertion of a 1000-metric ton asteroid into an Earth-Mars cycler. An electric propulsion space tug comparable to the one selected for the now-canceled Asteroid Robotic Redirect Mission is considered. An indirect optimization method is used to compute the low-thrust trajectories. Resonant and non-resonant trajectories that allow for multiple Earth encounters are explored to define the trajectory structure in order to limit the propellant consumption to feasible values. Two examples for the insertion of asteroid 2010 UY7 into Earth-Mars S1L1 cyclers are presented.

I. INTRODUCTION

The NASA and ESA programs indicate Mars as the main objective for human exploration beyond the Earth-Moon system. The goal is challenging and key technologies are under development to accomplish this task, in terms of propulsion system, power system, and deep space habitat. One of the showstopper is the protection of the astronauts from radiation when they are not subject to the Earth magnetic field for a long period of time. A round trip towards Mars can take 500 days and the dose of radiation absorbed by a human being without a viable shielding technology could be lethal. The state of the art considers the use of materials rich of carbon and hydrogen as the best strategy for the shielding from Galactic Cosmic Ray and Solar Particle Events. The basic concept is to have a sufficient amount of mass in order to intercept the charged particles and the particles deriving from their scattering. In order to stop most of the dangerous radiation, the shields need to be thick and massive.[1] All this mass has traditionally to be launched from Earth to LEO and then moved towards Mars. A more innovative strategy[2] has been suggested in the past to accomplish the shielding task; the protection of a Deep Space Habitat (DSH), which is inserted into a Cyclic Earth-Mars trajectory,[3] can be done by harvesting shielding material from an asteroid and putting it on the cycler with the DSH. This strategy obviously avoids the high-cost transportation from Earth. A method for preliminary selection of the possible target asteroid, based on the Tisserand criterion, has been presented in the past.[4]

The aim of this research is the optimization of a mission aimed at the transfer of a large asteroid into an Earth-Mars cycler, in order to actually assess its feasibility and envisaged performance. The cycler trajectories taken into account are Two-Synodic-Period Earth-Mars Cycler.[5] These cycler trajectories can be used for human transportation because they can be built up to have a short leg from Earth to Mars (outbound cycler) or a short leg from Mars to Earth (inbound). The short leg means that the time of flight is lower than 180 days. The cycler will have a flyby of the Earth every two synodic periods, meaning that the phasing angle between Earth and Mars is the same (at least in the simplified circular coplanar model): the cycler can use the Earth to change the argument of periapsis and to encounter again Mars with relative low fuel consumption. The two-synodic period cycler is interesting because has a low V_∞ (from 3 up to 7.8 km/s at the Earth flyby) with respect to the Aldrin Cycler. However, in order to have a continuous occupation of Mars, four cyclers are needed. The cycler can be divided in two legs: leg1 starts from and arrives to the Earth, but the trajectory can encounter Mars; during leg 2, the trajectory will remain on the same orbit plane of the Earth to re-encounter our planet.

For shielding purpose, a properly selected asteroid (o part of it) is moved to the cycler orbit by means of a space tug with low thrust maneuvers and Earth flyby(s). The large mass involved creates issues both in terms of required propellant and available acceleration, which can make the mission unfeasible. For this reason, the mission must exploit multiple gravity assists from the Earth to limit the propulsive requirements. First, a strategy to define the mission structure (i.e., number and location of Earth encounters) is developed. Then, actual trajectories for a space tug from the Earth to the selected asteroid and from the asteroid to the cycler are computed by means of an indirect optimization procedure. The propulsion system has Hall thrusters with high Isp based on the ARM design.

II. TRAJECTORY ANALYSIS AND OPTIMIZATION

The patched conic approximation is typically employed for preliminary mission analysis and is adopted in the present research. The dimension of the Earth sphere of influence can be neglected. The same obviously hold for the asteroid's one. Only the heliocentric leg can therefore be considered. Flybys are treated as instantaneous discontinuities of the spacecraft velocity.

Variables are made non-dimensional by using the radius of the Earth's orbit, the corresponding circular velocity and the spacecraft initial mass as the reference values. The equations of motion in the heliocentric reference frame are

$$\mathrm{d}\boldsymbol{r}/\mathrm{d}t = \boldsymbol{v} \tag{1}$$
$$\mathrm{d}\boldsymbol{v}/\mathrm{d}t = \boldsymbol{g} + \boldsymbol{T}/m \tag{2}$$
$$\mathrm{d}m/\mathrm{d}t = -q \tag{3}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are the spacecraft position and velocity vectors, $\boldsymbol{g}$ is the gravitational acceleration (an inverse-square gravity field is assumed here), $\boldsymbol{T}$ is the engine thrust, and q is the propellant mass flow rate. The effective exhaust velocity is $c = T/q = g_0 I_{sp}$ and is proportional to the specific impulse.

The space tug is assumed to have characteristics similar to the now-cancelled Asteroid Robotic Redirect Mission.[6,7] The available power of a solar electric propulsion system is in an inverse-square relation with the distance from the sun (the degradation of the solar arrays is here neglected). The solar arrays produce 47 kW of power at 1 AU but 5 kW are reserved for operation of on-board systems not related to propulsion. The characteristic of the three Hall effect thrusters on board, that is the values of thrust and mass flow rate as a function of input power (between 7 and 13.95 kW) are described by third-degree polynomials[8] and shown in Fig. 1. A 90% duty cycle is introduced to account for windows of non-available thrust.

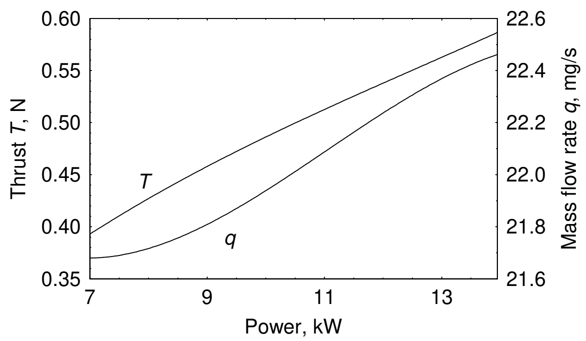

Fig. 1: Thruster performance as a function of input power.

An indirect optimization procedure[9,10] is used to maximize the payload. An adjoint variable is associated with each state equation and the Hamiltonian

$$H = \boldsymbol{\lambda}_r \boldsymbol{v} + \boldsymbol{\lambda}_v(\boldsymbol{g} + \boldsymbol{T}/m) - \lambda_m/c \tag{4}$$

is defined. The Euler-Lagrange equations for the adjoint variables are easily derived. According to Pontryagin's Maximum Principle (PMP), the optimal controls maximize H. Therefore, the thrust must be parallel to $\boldsymbol{\lambda}_v$, i.e., the primer vector.[11] The Hamiltonian becomes

$$H = \boldsymbol{\lambda}_r \boldsymbol{v} + \boldsymbol{\lambda}_v \boldsymbol{g} + \bar{H} \tag{5}$$

where the thrust dependent terms are collected in $\bar{H}$.

Three equal thrusters ($N = 3$) are considered here and

$$\bar{H} = \sum_{i=1}^{N} T_i - K q_i \tag{6}$$

There is a single parameters that determines (with the availabe power) the optimal power repartition between the engines: $K = m\lambda_m/\lambda_V$, which is varying along the trajectory. At any given trajectory point, K is known and the power repartition that maximizes $\bar{H}$ can be determined rather easily. Details of this procedure can be found in Ref. 8.

Boundary conditions on the state variables define the trajectory. In the present paper, the outbound leg (from Earth to the asteroid) will be treated separately from the inbound leg (from the asteroid to a specified Earth flyby for insertion into the cycler).

At departure of the outbound leg, spacecraft and Earth (subscript E) have the same position. The initial mass (10^4 kg) and the magnitude of the hyperbolic excess velocity $V_{\infty 0}$ (1.3 km/s) are fixed; these values are consistent with a Delta IV Heavy launch and a lunar gravity assisted escape.[7,12] At asteroid (subscript A) rendezvous, position and velocity match the asteroid values and the final mass is maximized. Initial and final time are in general left free. One has

$$\boldsymbol{r}_0 = \boldsymbol{r}_E(t_0) \tag{7}$$
$$[\boldsymbol{V}_0 - \boldsymbol{V}_E(t_0)]^2 = V_{\infty 0}^2 \tag{8}$$
$$m_0 = 1 \tag{9}$$
$$\boldsymbol{r}_1 = \boldsymbol{r}_A(t_1) \tag{10}$$
$$\boldsymbol{V}_1 = \boldsymbol{V}_A(t_1) \tag{11}$$

where subscripts 0, 1 refer to Earth departure and asteroid arrival, respectively. A constraint on t_1 may be added to evaluate the influence of an early arrival; in this case, $t_1 = k_1$ is added to the set of boundary conditions.

At departure of the return leg (subscript 2) position and velocity again match the asteroid values. Performance improves for earlier departures; time is here fixed to allow for sufficient stay time at the asteroid, as explained in the following. At the first Earth flyby (subscript 3) the spacecraft position coincide with the Earth and the magnitude of the hyperbolic excess velocity is continuous across the flyby. No constraint is imposed on the flyby height as resonant trajectories can be used to split the required V_∞ rotation, when it exceeds the maximum allowed value. At insertion into the cycler (subscript f) spacecraft and Earth position are again the same; date and V_∞ magnitude are imposed by the required cycler. The available propellant is fixed (here, in the reference case, at 4000 kg) and either the initial or final mass is the performance index to be maximized. Therefore:

$$t_2 = k_2 \tag{12}$$
$$\boldsymbol{r}_2 = \boldsymbol{r}_A(t_2) \tag{13}$$
$$\boldsymbol{V}_2 = \boldsymbol{V}_A(t_2) \tag{14}$$
$$\boldsymbol{r}_{3-} = \boldsymbol{r}_{3+} = \boldsymbol{r}_E(t_3) \tag{15}$$
$$[\boldsymbol{V}_{3-} - \boldsymbol{V}_E(t_3)]^2 = [\boldsymbol{V}_{3+} - \boldsymbol{V}_E(t_3)]^2 \tag{16}$$
$$\boldsymbol{r}_f = \boldsymbol{r}_E(t_f) \tag{17}$$
$$t_f = k_f \tag{18}$$
$$\left[\boldsymbol{V}_f - \boldsymbol{V}_E(t_f)\right]^2 = V_{\infty f}^2 \tag{19}$$
$$m_f - m_2 = 0.4 \tag{20}$$

The theory of optimal control[9,10] provides the lacking boundary conditions, which are omitted for the sake of conciseness. The indirect method transforms the optimization problem into a multi-point boundary value problem, which is solved by a procedure based on Newton's method.[13]

III. PRELIMINARY ANALYSIS

The most demanding part of the mission, in terms of propellant consumption, is the transfer of the asteroid from its initial orbit to the cycler orbit. The mass that must be accelerated in this phase is extremely large and even small ΔVs require large amount of propellant. In addition, the large asteroid mass translates into low acceleration that can be obtained from the propulsion system, also limiting the available ΔV for reasonable thrusting times. As a consequence, in order to maximize the mass of the asteroid to be moved to the cycler, a massive use of Earth Gravity assist (flyby) is needed.

The preliminary selection of the asteroid[4] must take these aspects into account and can be made with considerations concerning Tisserand's criterion. Only asteroids that have a natural close approach with the Earth can be redirected without excessive cost, and the first intercept of the Earth must occur in the proximity of the close encounter. An "a priori" estimation of encounter date (that is, position of the Earth along its orbit) and V_∞ at first encounter is therefore available.[4] In the most optimistic case the asteroid will encounter the Earth with exactly the right V_∞ and date required for insertion into the cycler orbit. As a matter of fact, only day and month are relevant, as resonant flybys can be used to get another encounter after an arbitrary integer number of years at the same position along the orbit. It is easy to compare the close encounter V_∞ and date[4] to the values required by the cycler Earth encounters[5] (in a given time-window of interest) and determine, for each pair, the difference in terms of V_∞ and the amount of time Δt required to move the encounter date, e.g., in terms of number of days between the two dates (modulus 365). Obviously, the most favorable situation (0 and 0) does not occur in practice. In the general case, the required date and V_∞ will be different, and thrust and Earth flybys will be required to achieve insertion into the cycler.

After the first encounter with the Earth at time t_3, if the space tug trajectory lays on the Earth orbit plane, both resonant and non-resonant trajectories with the Earth are allowed. Two concepts can be exploited to change the encounter V_∞ and date while maintaining the required propellant consumption to feasible values. First, V_∞-leveraging maneuvers[15–18] can be used to alter the hyperbolic excess velocity (however, they also perturb the encounter date and necessarily require some propellant usage). Second, non-resonant trajectories that encounter again the Earth at a different point along its orbit can be used to change day and month of the encounter. The V_∞ at the new encounter would not be modified for circular Earth orbit; the difference between the values at the two encounters amounts at most to few tens of m/s when Earth's eccentricity is taken into account.

Due to the propellant cost of leveraging, small values of V_∞-change must be preferred. The most favorable opportunities occur when this change is below 50 m/s. For these cases, non-resonant orbits can be explored to correct the encounter date by the prescribed amount Δt.

In a resonant trajectory, the spacecraft repeatedly encounters the Earth on the same day/month after an integer number of revolutions and years. However, an elliptic coplanar orbit intercepts the Earth's orbit also at another position, which is symmetric with respect to the line of apsides if Earth's eccentricity is neglected. The Earth passes through this point on a different day/month but note that the spacecraft on a resonant orbit arrives there at a different time, due to its eccentric orbit. To move the encounter to a given position (i.e., day/month) a non-resonant orbit must have proper values of period (i.e., semimajor axis a) and eccentricity e. On the other hand, these values are not independent when the V_∞ is assigned, as in the present case.

A procedure to find non-resonant trajectories is described in Ref. 14. A non-resonant trajectory is specified by number of integer revolutions made by the Earth (N), the one made by the spacecraft (M), and by the features of the additional part of spacecraft trajectory that completes the transfer after its M integer revolutions; this additional part either comprises the perihelion (inbound-outbound,

i = 1) or the aphelion (outbound-inbound, i = 2).

For each combination of N, M (between 1 and 6) and $i = 1, 2$ the unknowns a and e are found, and the change of the encounter date Δt can be computed

$$\Delta t = 2\nu = 2\left|\cos^{-1}((a - ae^2 - 1)/e))\right| \quad (21)$$

A favorable opportunity for the cycler injection arises when it is close to the required encounter date change. In fact, in this case, only minor orbit corrections will be required and a very large asteroid mass can be moved to the cycler. It is worth noting that, if sufficient time is available, two or more non-resonant trajectories can be used to achieve the correct Δt value.

When a larger change of V_∞ is required and/or a suitable non-resonant trajectory cannot be found for the required Δt, a V_∞-leveraging maneuver is necessary. However, due to the limited thrust authority and available propellant, only limited V_∞ changes (few 100 m/s) and date (few days) can be obtained, and the trajectory must necessarily remain quite similar to the closer non-resonant trajectory. The latter can be used as a tentative guess for the indirect optimization.

IV. RESULTS

The solution of the boundary value problem that comes from the indirect optimization of the trajectory, may be numerically hard, because of the very low acceleration value. A good estimation of the trajectory must be available to define a tentative solution that allows for convergence. The concepts outlined during the preliminary analysis allow for the definition of a suitable initial guess.

The case of asteroid 2010 UY7 is chosen as an example. The cyclers described in Ref. 5 are targeted. The asteroid orbital elements at Epoch 2458200.5 (2018-Mar-23.0) provided by JPL[19] are shown in Table 1. Its size is estimated between 4 and 19 meters. The mass estimation ranges between 77 and 6900 tons.

element	value
semimajor axis a, AU	0.89672453264552
eccentricity e	.1502422213661351
inclination i, deg	0.4570171865581963
RAAN Ω, deg	40.05077132239148
arg. of periapsis ω, deg	210.3824263817305
mean anomaly M, deg	28.19464888827728

Table 1: Orbital elements of asteroid 2010 UY7.

Departure at either node is advisable for the outbound (Earth-asteroid) leg to exploit the escape hyperbolic excess velocity and change the orbit plane (even though the inclination is rather small). Departure close to the aphelion where the asteroid orbit has the minimum orbit intersection distance with the Earth, which is close to the ascending node) is also advisable. The optimal maneuver tends to look like a two-burn rendezvous, where the spacecraft is placed on an elliptic orbit with proper period to assure favorable phasing after integer revolutions. Several local maxima of final mass are easily found by exploring Earth' passages at the node as departure date, and varying the time length. An optimal length transfer with duration close to 15 months, Earth escape in November 2022 and arrival in February 2024 is selected and shown in Figs. 2 and 3; the propellant consumption is 686 kg, with 4314 kg remaining; 4000 kg will be used during the inbound leg leaving 314 kg as reserve.

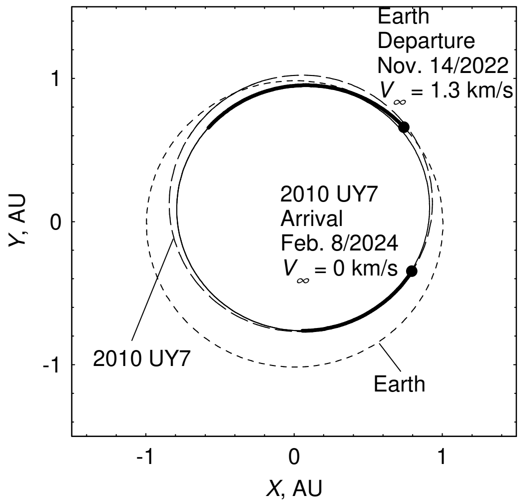

Fig. 2: Outbound trajectory (thrust arcs are in bold).

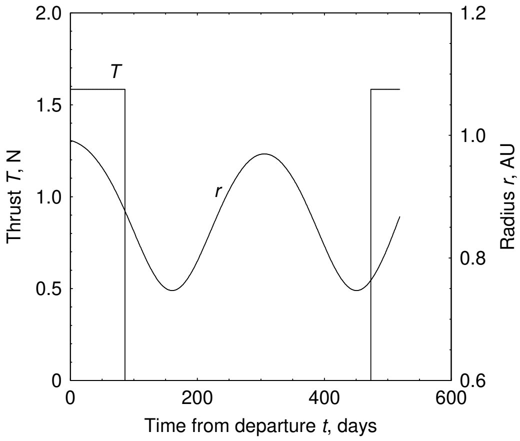

Fig. 3: Outbound leg thrust and radius histories.

The Earth close approach occurs on Oct. 25, 2027 with $V_\infty \approx 4$ km/s. The trajectory from the asteroid to the first Earth encounter (from t_2 to t_3) is initially treated separately, assuming 2000 kg of available propellant, which correspond to about 500 days of thrusting. With the addition of coast arcs, a three year time of flight is expected. Assuming a stay-time on the asteroid of about 5 months, the departure date of the return leg is fixed a July 5, 2024 (the influence of this date will be analyzed in the following). The optimization of this part of the trajectory does not show convergence problem and the returned mass is around 750 metric tons. This solutions is used to build a tentative guess for the optimization of the complete return leg.

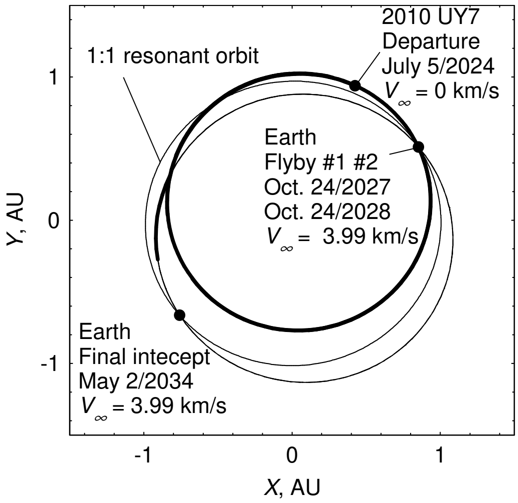

Fig. 4: Inbound trajectory to vehicle 3 cycler on May 2 (thrust arcs are in bold).

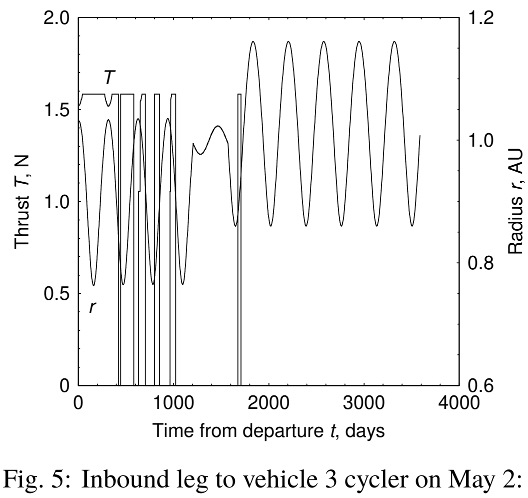

Fig. 5: Inbound leg to vehicle 3 cycler on May 2: thrust and radius histories.

The analysis of cycler insertion V_∞ and dates[5] shows that the Earth-22 encounter for vehicle 3 (May 2, 2037) has a 3.99 km/s V_∞, that almost exactly matches the first encounter value. Analysis of non-resonant orbits shows that the $N = M = 5$ inbound-outbound trajectory has $\Delta t \approx 189$ days, which is only 1 day shy of the number of days from Oct. 24 to May 2. A tentative solution is thus easily built by joining the first leg tentative solution and this non-resonant orbit and the complete return leg is optimized.

The final date is imposed to be May 2, 2033 with $V_\infty = 3.99$ km/s. The maximized asteroid mass that can be retrieved is 1256 metric tons. About 3840 kg of propellant are required to first intercept the Earth (ΔV around 80 m/s), whereas only 160 kg are needed to adjust date and V_∞ (ΔV around 10 m/s). The required V_∞ rotation at flyby is too large and a 1:1 inclined resonant orbit is introduced to split the rotation between two flybys. An infinite number of transfers are available, depending on inclination; the trajectory that correspond to equal V_∞ rotation is selected. Flyby heights result to be about 4900 km above Earth's surface. As a consequence, the final encounter has a delay of one year to May 2, 2034. Insertion into a $3 : M$ resonant orbit (whatever M is selected) generates the required encounter on May 2, 2037 for cycler injection. The characteristics of the trajectory are shown in Figs. 4 and 5. The leg to the first encounter is almost completely propelled, as a relatively large ΔV is sought. The 1:1 resonant orbit seems to pass through the final intercept point, but this is only a projection effect. Just a relatively short burn is required after the first encounter, as date and V_∞ adjustments are very small. The burn moves the intersection point and arrival date. The following non-propelled revolutions allow for the correct phasing.

Insertion into vehicle-4 cycler at Earth-18 flyby on Sep. 22, 2033[5] is another interesting case. V_∞ is 3.86 km/s, so some sort of leveraging is required, and the necessary Δt is 333 days. It has been observed that the change of the encounter date of a non-resonant orbit modifies V_∞ in a regular way. In particular, an inbound-outbound trajectory experiences an increased V_∞ when either the semimajor axis is lower than 1 (i.e., $N < M$) and the encounter date is advanced, or the semimajor axis is larger than 1 (i.e., $N > M$) and the encounter date is delayed. The change is usually below 100 m/s per day, with low values for low-eccentricity orbits (usually, when N is close to M) and long transfers. The V_∞ modification changes sign for any switch in trajectory type (e.g., from inbound-outbound to outbound-inbound), date modification (e.g., from advanced to delayed encounter), or orbit energy (e.g., from $a < 1$ to $a > 1$). In the light of these observation, the $N = 4$ $M = 5$ outbound-inbound trajectory, which has $\Delta t \approx 335$ days, should reduce V_∞ in the range 3.8-3.9 km/s (the required value is 3.86 km/s) when Δt is reduced to the required 333 days. The non-resonant trajectory with a 2-day advance of the encounter (thus moved to Sep. 22) is first optimized separately and this partial solution (which has an encounter $V_\infty = 3.82$ km/s) is matched to the first leg to build a tentative solution for the optimization of the complete return trajectory.

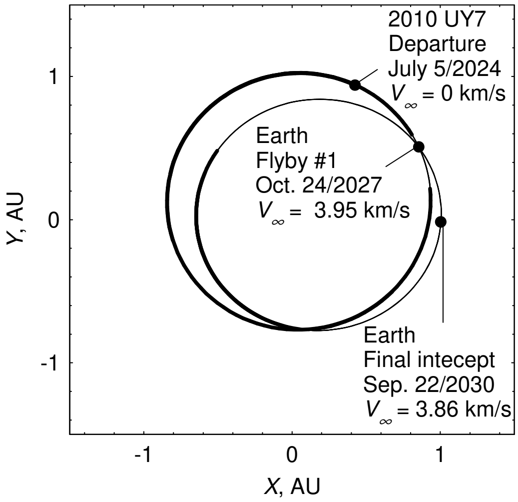

Fig. 6: Inbound trajectory to vehicle 4 cycler on Sep. 22 (thrust arcs are in bold).

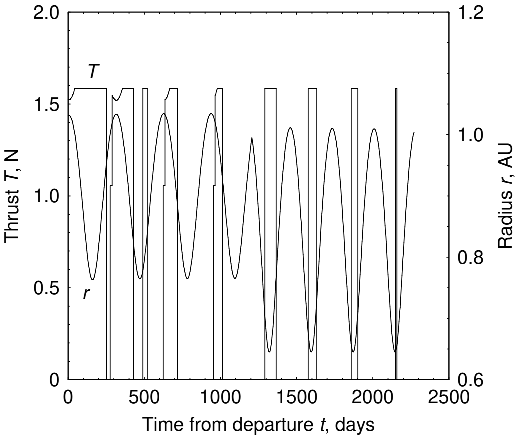

Fig. 7: Inbound leg to vehicle 4 cycler on Sep. 22: thrust and radius histories.

The optimal trajectory is shown in Figs. 4 and 5. The retrieved asteroid mass is 1056 metric tons. The propellant used before the first Earth encounter is slightly more that 3000 kg, with almost 1000 kg for the leveraging maneuver, mainly to correct V_∞ from 3.82 to 3.86 km/s (ΔVs are about 80 and 25 km/s, respectively). The flyby height is about 2700 km above the Earth's surface, meaning that the required rotation can be obtained with a single flyby. Use of thrust after the first encounter is different from the previous solution, as a more intense leveraging is required, and burn arcs appear during each revolution in correspondence of the perihelion passages. Final intercept is on Sep. 22, 2030; insertion into a $3 : M$ resonant orbit (whatever M is selected) moves the encounter to the correct year (2033).

A sensitivity analysis has been carried out to evaluate the influence on performance of the assumptions made for the reference cases. The departure of the return leg has a major influence. A 30-day delay, for instance to increase the stay time, would reduce the retrieved mass by 20-25 tons. It is instead more convenient to advance the arrival at the asteroid, which costs about 80 kg of propellant per month; in fact, a reduction of 80 kg of available propellant for the return leg decreases the retrieved mass by (only) 15 tons. It is worth noting that these changes are however below 2% of the retrieved mass. In the present analysis 4686 kg of propellant are used and 314 kg remain of the nominal 5000 kg for reserve. A 500 kg increase to 814 kg of reserve has been considered, reducing the propellant available for the return leg to 3500 kg. In this case, the retrieved mass shows a decrease of 100-120 tons (10 %). The penalty of a lower propellant mass is partially offset by a larger available acceleration, as the mass is now lower.

V. CONCLUSIONS

Indirect Optimization has been effectively used for finding trajectory to move an asteroid from its orbit to a Earth-Mars Cycler without imposing an a priori thrust/coast structure. The difficulties to find the initial guess for the indirect optimization have been overcome using resonant and non-resonant transfers and V_∞-leveraging maneuvers as building blocks.

The feasibility of the trajectory for a space tug similar to the ARRM envisaged spacecraft is demonstrated and the retrievable mass is interesting (more than 1000 metric tons), and suitable for the shielding purpose. A large number of additional aspects (e..g., docking and anchoring, attitude control, asteroid characterization) must however be considered before this concept can be actually employed, but feasibility, at least from the propulsive point of view, seems guaranteed.

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