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Joint Identification in Bladed-disks using SEMM and VPT

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ABSTRACT

Bladed disks are fundamental bricks of the rotating parts of a turbomachine, which can include many of them. Although each blade-disk sector can be considered identical, the presence of imperfections or misalignments or inhomogeneity (so-called mistuning) induces high amplification of vibration response. The case of blade-root joints is analysed as a source of mistuning of the contact. The test-case is an academic bladed-disk comprising of 18 blade-root joints. The root has a typical dovetail configuration with two-sided interface per sector. Its relatively long interface (compared to the blade length) makes the joint identification challenging yet interesting. Therefore, some non-classic identification techniques are employed. The joint identification is done by using System Equivalent Model Mixing (SEMM) and Virtual Point Transformation (VPT). The measured dynamics of the internal degrees of freedom (DoFs) are expanded to the interface DoFs by SEMM at the substructure level i.e. the disk and the blade. The substructure SEMM models are transformed to one or more virtual points by interface displacement modes (IDMs). The virtual point IDMs are decoupled from the assembled structure (one blade coupled to the disk at a time) to identify joint parameters. Since each disk and blade interface is considered unique, therefore, the process can be extended to the second blade coupled to the disk (having the first one removed). Such joint characterization is aimed for all the remaining blade to disk interfaces.

Keywords: Bladed-disks, joint identification, SEMM, Virtual Point, Mistuning

INTRODUCTION

Mistuning in bladed-disks can result in high vibratory response amplification. The underlying assumption of symmetry breaks away to asymmetry which can arise due to various reasons, for example, geometric imperfections, manufacturing tolerances, contact conditions etc. Geometric or frequency mistuning have been vastly studied in the literature, but the mistuning arising from contacts has not been studied much. Since there are multiple contacts, it is assumed that each blade to disk contact interface is different and therefore, each contact one-by-one can be identified by keeping only one blade mounted at a time to the disk. The contact interfaces identification is done by decoupling in frequency domain. The classic experimental Lagrange Multiplier Frequency Based Substructuring (LM-FBS) has been shown to be quite challenging [1]. The main reasons remain i) measurement noise, ii) drive point FRFs, iii) rotational DoFs, iv) inaccessibility of the interface etc. Therefore, the techniques based on expansion System Equivalent Model Mixing (SEMM) [2] and filtering namely virtual point transformation [3] are employed for this purpose. The advantages are that i) inaccessible DoFs at the interfaces can be expanded based on the observed dynamics elsewhere, ii) unlike the modal domain expansion techniques the rank is full, iii) modal truncation error is avoided due to frequency domain formulation etc. among others.

BRIEF MATHEMATICAL OVERVIEW

Any two (or more) substructures can be coupled by the single line LM-FBS equation in the admittance form [1].

$$\mathbf{Y}^{AJB} = \mathbf{Y} - \mathbf{Y}\mathbf{B}^T(\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Y}, \quad \text{with} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^A & & \\ & \mathbf{Y}^J & \\ & & \mathbf{Y}^B \end{bmatrix} \tag{1}$$

where $\mathbf{B}$ is a Boolean matrix and $\mathbf{Y}$ is a block FRF matrix of the substructures to be coupled. $\mathbf{Y}^J$ is a joint model expressed in the admittance form to introduce a flexible coupling and needs to be identified in the following formulation. If it is not included, the result is a rigid coupling of the substructures at the interface DoFs. $\mathbf{Y}^J$ is obtained in a similar fashion to the equation above, in a process called decoupling:

$$\bar{\mathbf{Y}}^J = \mathbf{Y} - \mathbf{Y}\mathbf{B}^T(\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Y}, \quad \text{with} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{AJB} & & \\ & -\mathbf{Y}^A & \\ & & -\mathbf{Y}^B \end{bmatrix} \tag{2}$$

Note that here an extra step needs to be performed to receive the relevant DoF to truly obtain $\mathbf{Y}^J$. If a component has a finite element (FE) model as the numerical model providing a DoF structure and a model whose dynamics are important, for example from measurements, then a hybrid model can be created using System Equivalent Model Mixing (SEMM) [2] expressed in the following compact form:

$$\mathbf{H}_{gg}^{A,\text{SEMM}} = \mathbf{H}_{gg}^{A,par} - \mathbf{H}_{gg}^{A,par}(\mathbf{H}_{ig}^{A,par})^+ \left(\mathbf{H}_{ii}^{A,par} - \mathbf{H}_{ii}^{A,ov}\right)(\mathbf{H}_{gi}^{A,par})^+ \mathbf{H}_{gg}^{A,par} \tag{3}$$

where the subscript $_{gg}$ refers to the global DoFs and $_{ii}$ to the internal DoFs where the measurements are performed in the overlay model, and the superscripts ^{par} and ^{ov} refer to the parent and overlay models, respectively and $(\quad)^+$ is the pseudo inverse. The removed model $\mathbf{H}^{A,rem}$, as per SEMM convention, has been simply replaced with $\mathbf{H}_{ii}^{A,par}$. The resulting SEMM model mimics the overlay model exactly at the $_{ii}$ DoFs whereas for the other DoFs it is an expansion based on the observed dynamics in the overlay model. The expanded dynamics of the interface DoFs are then transformed to one or more virtual points (VP) by creating the interface displacement modes $\mathbf{T}_u$ and interface force modes $\mathbf{T}_f$ (cf. [3]):

$$\mathbf{Y}^A = \mathbf{T}_u\, \mathbf{H}^{A,\text{SEMM}}\, \mathbf{T}_f^T \tag{4}$$

Similarly the interface modes for the substructure B are created. Using the two interface modes of the SEMM models, one can make the parent model of the coupled structure by assuming a guessed joint [4] in the following equation.

$$\mathbf{Y}^{AJB,par} = \begin{bmatrix} Y_{ii}^{AA} & Y_{ib}^{AA} & Y_{ii}^{AB} & Y_{ib}^{AB} \\ Y_{bi}^{AA} & Y_{bb}^{AA} & Y_{bi}^{AB} & Y_{bb}^{AB} \\ Y_{ii}^{BA} & Y_{ib}^{BB} & Y_{ii}^{BB} & Y_{ib}^{BB} \\ Y_{bi}^{BA} & Y_{bb}^{BA} & Y_{bi}^{BB} & Y_{bb}^{BB} \end{bmatrix} \tag{5}$$

the double superscripts and subscripts imply coupling between the components A and B and between the internal i and boundary b DoFs, respectively. The overlay model of the coupled structure is obtained from the measurements on the same DoFs as in the uncoupled configuration which carries implicit joint dynamics. Applying the SEMM on the coupled structure as was done at the substructure level in Eq. 3 provides a hybrid model that has explicit joint dynamics and can be used to decouple the joint in Eq. 2. The process is repeated until the expansion error $||(\mathbf{Y}^{par} - \mathbf{Y}^{ov})||$ is minimized, see [4] for details.

THE EXPERIMENTAL SETUP

The test-case is a bladed-disk with 18 blades having as many blade-disk joints (blade root joints) and blade-blade joints (shrouds). Only the first blade is connected to its respective disk slot. The disk slot is narrow which makes it largely inaccessible. A recent study [5] on the same test-case explains its details and challenges. The experimental setup requires measurement of accelerance FRFs at internal DoFs of the blade (free-free) and the disk (fixed-free). Five triaxial accelerometers have been mounted on the blade and also on the disk. Impacts with a modal hammer are made on several locations on each substructure which are not collocated with accelerometers. The measured sensors and impact positions are preserved when the two substructures are coupled tightly. The setup is shown in Fig. 1.

RESULTS

The measurements are expanded to a set of boundary DoF. In this case the chosen boundary was one of 24 DoF. This also becomes the size of the joint model. The joint, identified by decoupling as shown in Figure 2b, is re-coupled

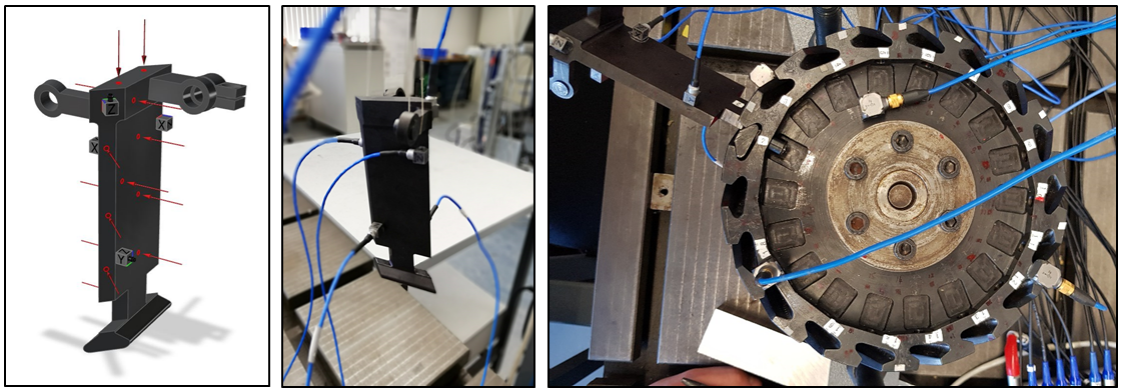
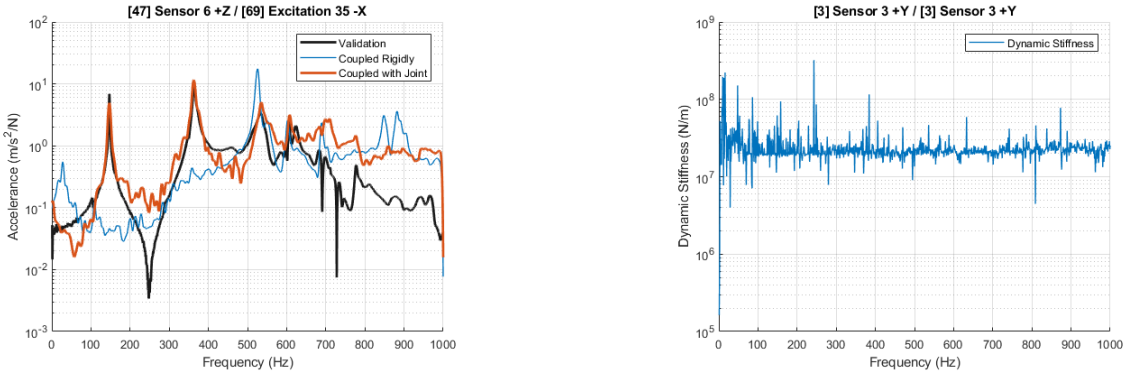


Fig. 1 The experimental setup: the position of sensors and impacts on the blade as designed in Dirac software (the left), the actual sensors configuration (the middle) and the coupled structure (the right). The sensor and impact positions on the disk alone are preserved after removing the blade.



(a) **(b)**

Fig. 2 (a)The addition of the unfitted joint to the substructures improves the results compared to a rigid connection. This FRF was not used for the identification of the joint, and is therefore a valid validation FRF. The results are smoothed. (b) A line of the unfitted joint stiffness for driving direction. Note the sensitivity to noise in the joint, here the results are not smoothed.

to the substructures to obtain the graphs in Figure 2a. The identified joint clearly heightens the coherence between the coupling results and the validation measurement. Figure 2b is the dynamic stiffness of one of the joint driving DoF. We can clearly see a stiffness line, albeit noisy.

CONCLUSIONS

The present work is aimed to show that the joint identification is possible with measurements expansion of the internal DoFs to the boundary DoFs. The method is sensitive to noise and bias errors included in the measurements so while the validated joint seems to improve the coupling, there exist artifacts from the structures in the joint model (e.g.measurement errors). The results show a good coherence with the identified joint model, but more work is required to obtain a valid and fitted model.

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