# CUF-BASED FINITE ELEMENTS FOR THE FREE-EDGE STRESS CONCENTRATION ANALYSIS OF COMPOSITE COUPONS AND STRUCTURES

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*Summary* Based on the Carrera Unified Formulation (CUF), the present study proposes refined finite elements for the free-edge stress concentration analysis of composite structures. CUF makes use of a hierarchical and scalar procedure to approximate the 3D elasticity problem with 1D or 2D theories. In detail, the displacements are expressed as an arbitrary expansions of the generalized unknowns by using generic expansion functions, which determines the theory approximation. In this manner, the theory accuracy can be refined with ease close to critical regions, e.g. at the free-edge, which remains an unresolved problem and for which analytical solutions are limited to a few cases. We demonstrate the high versatility of this approach, which can be indistinctly used at the coupon scale up to the size of a final composite assembly, such as a composite wing or a tape spring for space applications.

## INTRODUCTION

Free-edge effects arise in laminates at the interfaces between dissimilar layers in the vicinity of discontinuities in the structure. This phenomenon has been demonstrated to be a crucial actor in damage initiation in composite structures. Nevertheless, current numerical methodologies for the analysis of composites lack in accuracy and efficiency, and the evaluation of stress fields close to free edges may be prohibitive in industrial practise. As a matter of fact, classical laminate theories cannot provide any useful information of the free-edge effects and active interest is still focused nowadays in the development of accurate tools for their evaluation in real composite applications [1].

This work introduces a numerical approach based on higher-order models for the free edge analysis of composite laminates. The Carrera Unified Formulation (CUF) [2] is employed to generate a theory of structure which makes use of displacements as only variables and Lagrangian polynomials as cross-sectional assumptions, named as Lagrange expansion (LE). This theory of structures was first introduced by Carrera and Petrolo [3] and has proven to be a powerful tool for the accurate stress analysis of composite structures in many works, see [4, 5, 6]. The use of a distribution of mathematical expansion domains to assume the deformation of the cross-section of the laminated beam enables the model to capture 3D-like stress distributions at the ply level. Moreover, it also allows the user to refine the cross-section domain in the zones of interest, such as the free-edges. The accuracy and robustness of the proposed method to represent the free-edge effects in generic composite beams is presented in the present work and particular emphasis is given to the scalable nature of CUF, which allows to study indistinctly composite coupon samples under traction/compression and complex assemblies under general loadings and boundary conditions.

## LAYERWISE LAMINATED THEORIES BASED ON CUF

Many displacement-based theories of structure have been introduced in the past decades for the study of the mechanical response of laminated structures. Among the others, the layerwise (LW) models make use of independent assumptions for each layer and provide more information of the meso-scale effects by accounting for the deformation of each ply independently. Furthermore, by taking displacement assumptions at the ply-level, LW models are able to capture the zig-zag effect of the displacements in the thickness direction, which is strongly related to the complex distribution of transverse stresses in composite laminates.

In the framework of CUF for beam theories, the LW displacement field of the composite beam is written as:

$$\mathbf{u}(x, y, z) = F_\tau(x, z)\mathbf{u}_\tau^k(y) \quad \tau = 1, 2, ..., M \tag{1}$$

where $\mathbf{u}(x, y, z)$ is the three-dimensional displacement field, $\mathbf{u}_\tau(y)$ is the vector of generalized displacements that depends on the longitudinal coordinate $y$ and $F_\tau(x, z)$ are the expansion functions of the cross-sectional domain. The class and number of expansion functions is arbitrary, being $M$ the maximum number of expansions, which is a user defined parameter. Repeating indexes denote summation.

The LE beam theory, introduced by Carrera and Petrolo [3], is based on the use of interpolating Lagrange polynomials as expansion functions $F_\tau$ of the cross-sectional coordinates. In this manner, the cross-section of the composite beam can be discretized with an arbitrary number of Lagrangian domains, which are used to represent the surfaces of each layer. Since LE beam models make use of displacement unknowns as degrees of freedom, they represent a useful tool for the efficient stress analysis of beam-like structural components. Moreover, LE are in particular interesting to generate LW models since the displacement compatibility at the interfaces between plies is automatically satisfied with no need of numerical artifacts.

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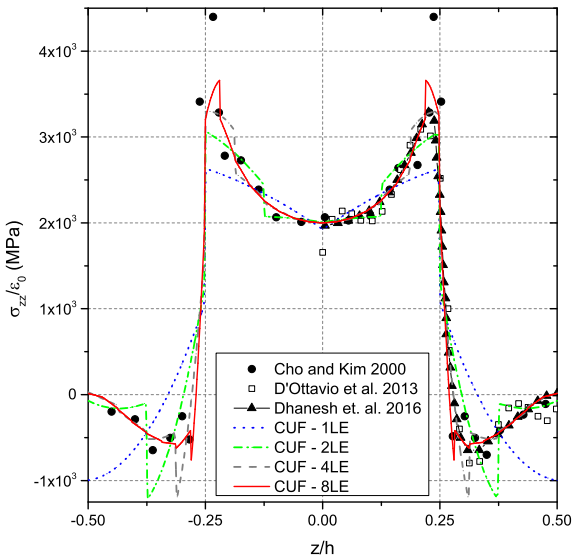


Figure 1: Transverse stresses along $z$ of the $[0,90]_s$ laminate under extension at $x = b/2$.

## COMPOSITE COUPON UNDER TRACTION

For representative purpose, we propose here the free-edge analysis of a composite coupon subjected to unitary axial strain, $\varepsilon_0 = 1$. The material and geometry are equivalent to that of the pioneering work of Pipes and Pagano [7], who considered a symmetric cross-ply with four layers of equal thickness, $t = h/4$.

The model generated for the present assessment consists of 6 cubic 1D elements along the beam axis and a distribution of quadratic LE over the cross-section domain which in all cases consists of 10 LE in the $x$-axis with a graded distribution towards both the free edges. A convergence study regarding the number of mathematical layers in the direction of the stacking sequence is perform, accounting from one LE per layer up to eight LE per layer. The total number of degrees of freedom goes from 10,431 for the coarsest model, to 77,805 for the finest. All the stress solutions reported in the following are obtained at half-length of the beam, $y = L/2$.

Fig. 1 shows the transverse normal stresses, $\sigma_{zz}$, across the thickness at the free-edge for an increasing number of mathematical layers per ply, from one (CUF - 1LE) to eight (CUF - 8LE). The convergence study proves that highly refined kinematics are required to compute accurate free-edge stresses by means of LW theories. Since the only unknowns are displacements and no recovery of the 3D stress fields is performed, the trasnverse stress solutions may show some discontinuities between the expansion domains (corresponding to the vertical lines in the graphs), which are more pronounced in the vicinities of the interfaces between layers due to the high stress gradients. By increasing the number of mathematical layers in the beam model, the stress distributions tend to those obtained from the exact theories, approximating well both the stress-free boundary conditions and the interlaminar continuity.

## CONCLUSIONS

This paper has introduced the use of CUF to study free-edge stress concentrations of composite structures. Preliminary results demonstrate that LW models may provide accurate and efficient results at coupon scale. Further analyses will show that the method can be successfully employed for the study of complex composite structural assemblies, in a global/local sense. Particular attention will be given to, but not limited to, the analysis of composite wings and composite truss structures for space applications.

## References

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