

# Volume Integral Equation Methods for Forward and Inverse Bioelectromagnetic Problems

This dissertation describes doctoral research activities on new computational techniques based on volume integral equations specifically tailored for bioelectromagnetic applications such as electroencephalography (EEG) source localization, microwave imaging, and inverse scattering synthesis for improved dosimetric assessment.

This thesis work began with an in-depth analysis of the conditioning properties of full-wave volume integral equations, particularly in a scenario with a low frequency electromagnetic source illuminating an inhomogeneous object with high material contrast, which corresponds to the operating regime of numerous bioelectromagnetic applications. This analysis led to the introduction of a new set of volume quasi-Helmholtz projectors that, with proper re-scaling, renders the electric flux volume integral equation (D-VIE) accurate and stable at low frequencies in lossy dielectric objects. This new method was successfully applied in EEG source localization for solving the forward problem.

In parallel, bioelectromagnetic applications requiring higher frequency modeling were investigated. For instance, the usage of the D-VIE formulation in inverse scattering scenarios required the full derivation of a new inversion algorithm based on modified gradient methods. The practical use of this new inverse solver is illustrated through its application in microwave imaging and inverse source synthesis. Regarding the former application, numerical examples show that it compares to several standard inverse scattering schemes in a canonical setting. The latter responds to the need of understanding the impact of mobile phones on brain activity. To remedy one significant limitation of the existing techniques that attempt to assess this risk with an EEG helmet, which acts as a shield for the incoming radiations of the phone, an inverse synthesis algorithm was introduced for the design of a deformation source enabling EEG recordings with a non perturbed field distribution in the head.