Theory of Critical Distances: a discussion on concepts and applications

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Theory of Critical Distances: a discussion on concepts and applications

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Abstract

Theory of Critical Distances (TCD) collects several methods adopted in failure prediction of components provided with stress concentration features. The idea of evaluating stress effect in a zone rather than in a single point was proposed decades ago but, only thanks to relatively recent works, TCD concepts showed to be a successful extension of Linear Elastic Fracture Mechanics (LEFM), able to assess strength and fatigue life. The increasing computational power has made Finite Element Method (FEM) widespread, hence stress fields can be easily extracted and used as input data for fatigue post-processing and durability analyses. In this scenario, TCD reveals as a powerful tool which, thanks to the introduction of a single material parameter (critical distance, *L*), integrates classical fracture models by considering the presence of microscale phenomena acting in fracture process. In this sense, TCD behaves as a link between continuum mechanics and LEFM. Modalities and reasons for this connection to occur are interesting points of further investigations. Literature on TCD and its theoretical-experimental background is quite extended, nevertheless few industrial

applications are available in literature to the best of authors' knowledge. In this paper, an overview of concepts and applications related to TCD are reported highlighting the relevance of theoretical arguments in actual applications.

1. Introduction

Fatigue assessment in many engineering components is frequently related to the behavior of notched structures. When fatigue life results from a Finite Element Analysis (FEA) post-processing, two kinds of approaches are possible:

- Local approaches;
- Non-local approaches.

Local approaches manage different fatigue life criteria [1]. They focus on the stress state in critical points and can be adopted basing on FEA results. This way, most complex stress distributions and geometries are calculated. By using local approaches for fatigue life assessment, stress gradients effect is weakly taken into account [1] since fatigue strength at each point is considered independently.

Few questions easily arise from this discussion:

- What is the effect of surrounding stress state in the study of the fatigue behavior of the hot-spot?
- How can stress gradient effect be included in engineering fatigue analyses?

Answering these questions is not trivial, however non-local methods proved to be suitable [1–3] when stress gradients are not negligible as it occurs for notched components.

Non-local approaches appear as effective tools for stress gradients evaluation [1]; this process performs a stress correction, which considers the entire stress distribution in a limited area.

In the field of non-local approaches, Theory of Critical Distances (TCD) [1,4,5] finds a wide range of applications in fracture and fatigue life estimation.

Theory of Critical Distances is the name originally used by Taylor and Susmel [4,5] to refer to that group of theories adopted for the study of stress concentration features. However, the story of TCD started when Neuber understood that fatigue limits of notched elements could only be predicted by averaging the stress state, close to the stress concentration feature, over material characteristic units. From a TCD point of view, this idea results in the so called Line Method (LM), whereas Point Method (PM) stems from Peterson's idea that reference stress for components' strength estimation is located at a certain distance from the notch [6,7].

Over the past decade a considerable literature has grown up around the theme of TCD, as emphasized in Fig. 1. Namely, many Authors have developed research branches devoted to theoretical investigations as much as to the enhancement of the fields of TCD application. However, a literature contribution providing a wide-ranging account of the different ramifications in which TCD is evolving is not currently available. This motivated the Authors of this work to provide this kind of contribution. Indeed, this work aims to discuss the main TCD concepts and applications which arise from a literature review. In this sense, it is beyond the scope of this work to focus on a specific application, but rather it explores the array of possibilities which have established yet. To this aim, the main TCD notions, initially introduced in the works of Taylor and Susmel [4,5,8], are at first presented. Then, the ever-growing fields of application are discussed together with different interpretations offered by the literature on some issues such as critical distance estimation. This literature portrait is accompanied by considerations which project TCD applications in an industrial analysis context where automatable structural integrity assessments are often performed by means of commercial fatigue post-processors.

2. TCD: origins and fundamentals

Fatigue notch concentration factor is one of the first parameters analyzed through this approach. It is well known that notch concentration factor K_t is generally higher than the corresponding fatigue notch concentration factor K_f. These quantities are related as follows:

$$q = \frac{K_f - 1}{K_t - 1} \qquad (1)$$

To explain this difference, Neuber assumed that the material around the notch can be modelled with a stack of layers and that each of them, being unable to support stress gradients, is subjected to a constant stress state. Therefore, the total force can be obtained as the sum of constant stress state distribution in each layer [9]. By following this hypothesis, it is possible to demonstrate that the stress in the surface layer (linked to K_f) is lower than the stress coming from the theoretical continuous curve one (linked to K_t) [9]. If layer thickness is considered material property:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{A}{r}}} \qquad (2)$$

where A is a quantity strictly related to the layer thickness and r is the notch radius. We refer to this example as one of the first attempts at including a material characteristic length in strength assessment.

2.1 TCD formalization: static

Let us introduce a material property:

$$L = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_0}\right)^2 \qquad (3)$$

where K_{IC} is the material toughness and σ_0 is called "inherent strength". For ceramics and generally for brittle materials [1] σ_0 corresponds to UTS (also indicated as σ_u), whereas for ductile materials $\frac{\sigma_0}{\sigma_u} > 1$ [1,5]. In this concept lies one of the degrees of freedom for experimental data correlation since the value of σ_0 can be seen as a calibrating parameter.

It is clear by looking at Eq. (3) that if both L and σ_0 are unknown, it is not possible to estimate L. Therefore different approaches are adopted in literature. One of them [1] uses a first approximation value of σ_0 (corresponding to UTS) to perform an analytical estimation of K_{IC} by using TCD. Then, the experimental value of K_{IC} is compared with the analytical one searching for the value of L which better fits data. Finally, the ratio σ_0/σ_u is defined. Through this approach, according to Taylor [8], no physical meaning is assigned to σ_0. Strictly speaking, σ_0 is certainly a stress but it refers to elastic properties in a region where plasticity could be involved. Indeed, the value of σ_0 was found to be even two or three time higher than σ_u in materials where plasticity was the main toughening mechanism, whereas, unitary value were attributable to the ratio σ_0/σ_u when plasticity did not play a key role in the fracture process. According to Taylor [8], this is explainable by the fact the mechanism involved in the fracture of plain specimen is slightly different from that of notched ductile components.

2.2 TCD formalization: fatigue

However, in many applications, especially in the field of fatigue, the first approximation inherent strength is employed as the number of parameters influencing it becomes larger. One of all is the number of cycles to failure N_f.

Fatigue equivalent of Eq. (3) is Eq. (4).

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_0}\right)^2 \qquad (4)$$

The threshold intensity factor is ΔK_{th}, $\Delta\sigma_0$ is the inherent strength of the material that in general corresponds to the fatigue limit. L values from Eq. (3) and Eq. (4) are the scale lengths used to introduce four different TCD approaches. These methodologies, initially reported by Taylor [8], found a wide range of application in many structural integrity problems which will be presented and discussed in Section 3. Static and fatigue analyses may be carried out by referring to the corresponding L.

2.3 Point Method (PM) and Line Method (PM)

The Point Method (PM) relies on the assumption that reference stress to account for failure assessment is located at a distance $r = \frac{L}{2}$ from the notch. This fact results evident by linking TCD to LEFM. It is considered a sharp notch whose behavior is comparable to cracks' one. Stress curve close to crack tip is expressed as $(r \ll a)$:

$$\sigma(r) = \sigma\sqrt{\frac{a}{2r}} \qquad (5)$$

Failure condition is reached when nominal stress σ, far from crack, equals fracture stress σ_f. In this circumstance, there will be a distance r for which $\sigma(r) = \sigma_0$. Eq. (5) therefore becomes:

$$\frac{\sigma_0}{\sigma_f} = \sqrt{\frac{a}{2r}} \qquad (6)$$

Fracture stress σ_f is known from LEFM:

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi a}} \qquad (7)$$

Then, by substituting Eq. (7) and Eq. (3) in Eq. (6), it appears that: $r = L/2$. Thus, the occurring link between TCD and LEFM provides a possible interpretation of PM.

Line Method (LM) is similar to PM but stress distribution is averaged over a length $r = 2L$. In this case, a PM-like explanation is achieved by performing stress field integrations and searching for the length at which the averaged stress equals σ_0 (incipient failure). By extending these concepts to area and volume integrals, Area Method (AM) and Volume Method (VM) emerge as natural consequences. However, PM and LM are mostly applied because of they represent a trade-off between good accuracy and simple use.

2.4 Imaginary Crack Method (ICM) and Finite Fracture Mechanics (FFM)

Imaginary Crack Method (ICM) assumes that an imaginary crack is placed at the notch root. Failure is assumed to occur when stress intensity factor K reaches material toughness K_{IC}. Then, LEFM concepts are applied to assess fracture behavior.

Finite Fracture Mechanics (FFM) does not find actual applications in the industrial world to the best of the authors' knowledge. Nevertheless, according to Taylor [1,10] one of the possible explanation for TCD ability to predict failure conditions in notched structures lies in FFM.

FFM is essentially a reinterpretation of LEFM in which the mathematical integrations required to express fracture energy are performed in finite steps. The main hypothesis behind this argument is that crack growth occurs discontinuously developing by discrete quanta [10], namely with a Δa increment rather than a da. This is not unrealistic if we think of microstructural obstacles as discontinuity factors. By applying these concepts [10] it is possible to demonstrate:

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi\left(a + \frac{\Delta a}{2}\right)}} \quad (8)$$

Eq. (8) describes fracture stress behavior both in the small cracks and in the long cracks zone, this is not trivial as emphasized in the following.

Let's suppose a crack of length a in a component. Continuum mechanics, as the name suggests, is not able to properly manage discontinuities because of the hypothesis on which it is based. LEFM, on the contrary, uses energy to describe discontinuities such as cracks and can predict failure condition and fracture stress by using Eq. (7). This way, singularities such as cracks can be managed. If crack length changes, σ_f varies as well because an energy-based criterion is adopted and fracture is supposed to be K-governed. Instead, if there were no crack or a small one, a stress-based approach would be used, since failure would be defined by using the critical stress as threshold quantity ($\sigma_f = \sigma_0$). Indeed, this corresponds to the classic engineering failure approach which limits the allowable stress conditions to assess structural integrity of plain specimen. Fracture stress would be, in this case, constant and not influenced by crack length. In the field of fatigue and crack propagation analysis the same considerations may be done by using fatigue limit in place of UTS and ΔK_{th} in place of K_{IC}, as expressed in the Kitagawa-Takahashi diagram [11] (Fig. 2). This diagram shows the aforementioned concepts. In particular, when crack length decreases, the stress amplitude leading to crack propagation increases according to LEFM. Moreover, the stress-based and the LEFM models intersect in the critical length L, thus defining the region of propagating and non-propagating cracks. Short cracks behavior is, in any case, more complex to describe in terms of crack growth because of continuum mechanics limits in modelling entities with the same order of magnitude of microstructure. FFM, just as PM, LM and ICM, is able to depict the whole range of crack size behavior with Eq. (8). Indeed the fracture stress behavior of Eq. (8) is retraced by applying PM, LM and ICM as well. However, this capability is explainable only for FFM since a relation between Δa and L is identifiable. Indeed, when crack size becomes very small, Eq. (8) turns into:

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi\left(\frac{\Delta a}{2}\right)}} \tag{9}$$

163 it is worth noticing that if LEFM alone were applied, Eq. (7) would lead to an infinitely strong material for a close-to-zero crack size. Considering fracture stress equal to inherent strength $\sigma_f = \sigma_0$ in Eq. (9):

$$\sigma_0 = \frac{K_{IC}}{\sqrt{\pi\left(\frac{\Delta a}{2}\right)}} \tag{10}$$

that turns to be Eq. (3) for $L = \frac{\Delta a}{2}$. This provides an interpretation of the occurring relation between FFM and TCD.

This is one of the examples from which the link between LEFM, microstructural mechanisms (represented by discrete quanta Δa) and TCD emerges, candidating TCD as a more general fracture theory which embraces several multi-scale aspects.

One of the TCD limits, highlighted by Taylor [8,10], may be emphasized in this point. Whenever component characteristic dimensions become comparable with critical distance value, FFM is modeling a crack that is passing through the whole component that, consequently, will be predicted to be infinitely unable to support stress. LM, on the contrary, is averaging stress over the entire component, resulting in a quasi-zero averaged stress. In this case, LM would model a component infinitely able to support stresses. Therefore, LM and FFM assessments would diverge and combined methods (LM+FFM) would be necessary [10]. Hence, when component characteristic dimensions become comparable with critical distance value, both LM and FFM collapse and only combined methods are applicable.

Recently, Liu et al. [12] handled the coupled approaches to estimate fatigue limits of notched specimens. Namely, it was concluded that for blunt notches the difference between LM and LM+FFM estimations is insensitive to notch radius. Conversely, for sharp notches the variability is appreciable but, especially for very sharp notches, it is negligible from an engineering point of view. Then, Naimark [13] analyzed the variety of crack paths from a FFM point of view.

It results clear that LM and PM are stress based methods whereas ICM and FFM are energy-based methods directly coming from LEFM. In most common fatigue post-processors, critical distance methods are implemented in form of PM and LM as stresses coming from FEA are ready to use for this kind of approaches. To the best of the authors' knowledge, FFM and ICM are not currently implemented in commercial fatigue post-processor. Therefore, in an industrial context their application is bounded by the possibility of developing own codes.

3. TCD in Multiaxial Fatigue

In many applications [2,14–17] multiaxiality of loads affects fatigue behavior. The problem of TCD implementation in these problems has been addressed by several authors.

As in static conditions, multiaxial fatigue criteria assume that an equivalent stress to compare with uniaxial material properties can be found in order to solve complex stress states. Since fatigue damage is related to a localized plasticity, yield criteria were originally introduced also in fatigue analyses and equivalent stress amplitudes were originally defined by combining principal stresses amplitudes [18].

However, stress variability in time is what distinguishes fatigue from static failure. This adds some complexities since stresses are essentially vectors and their direction becomes an influent parameter in fatigue analyses.

3.1 Critical Plane Criteria

In order to take into account this phenomenon, critical plane criteria were introduced. Indeed, when components experience out-of-phase loadings, principal stress directions vary in time even if load directions are constant. In this case, mathematical combinations of principal stress amplitudes may be meaningless. For instance, Gough and Sines methods are static yield criteria [18–20].

Then, Findley [21] was the first to introduce the concept of critical plane referring to the plane on which a certain damage parameter is maximized. The damage parameter for fatigue crack initiation and growth is usually related to the shear stress/strain amplitude, to the normal stress/strain or, as in the case of Findley criterion, a combination of these. Brown and Miller developed the same concept in the Low Cycle Fatigue (LCF) regime [22]. The applicability of these criteria in the form of local approach in actual automotive applications has been proved yet [23,24].

However, in the most general cases, the maximum shear stress plane may be different in each instant because of principal stresses directions relocation. The computation of shear stress amplitude in a specific plane is not always a simple task. The longest chord method, longest projection method, minimum circumscribed circle method [25,26] are among the most used algorithms for the determination of the shear stress amplitude starting from the path described by the resolved shear stress vector. In the last decade, Susmel [27] proposed an algorithm based on the Maximum Variance Method (MVM) whose numerical advantage lies in the time needed to calculate the critical plane as it does not depend on the stress history length. As suggested by the name, MVM acts by searching for the plane experiencing the maximum variance of the resolved shear stress. Moreover, Constant Amplitude (CA) and Variable Amplitude (VA) time histories can be treated in the same manner.

3.2 Modified Wöhler Curve Method (MWCM)

In order to apply TCD to multiaxial fatigue, Susmel and Taylor [14,16] proposed a methodology for implementing the MWCM along with TCD with the aim of estimating fatigue failure of notched components under VA and CA loadings. Remarkably, this can be applied both in the case of in phase and out-of-phase stress histories. In many of the presented applications the conjunction between MWCM and TCD showed to be a powerful tool for multiaxial fatigue assessment. Therefore, a short account of this methodology is reported in the following.

In a CA case, once the critical plane is defined as the maximum shear stress amplitude plane, it is defined the stress ratio:

$$\rho_{eff} = \frac{m\sigma_{n,m} + \sigma_{n,a}}{\tau_a} \qquad (11)$$

where m is the mean stress sensitivity and it is defined as a material constant, $\sigma_{n,m}$ is the mean normal stress acting on the critical plane, $\sigma_{n,\,a}$ and τ_a are the normal stress amplitude and the shear stress

234 amplitude computed on the critical plane. By using Mohr's circles, it is simple to show that ρ_{eff} may vary between 0 and 1 respectively in a fully reversed torsion and fully reversed uniaxial loading case. It is also assumed that a threshold value ρ_{lim} actually exists in order to model the change in the physical phenomena behind crack initiation [16] process.

In the medium-high cycle range ρ_{eff} contains the information about multiaxiality and, as reported in Eq. (12) and Eq. (13), it is used to modify the fully-reversed torsion Wöhler curves. The slope k_τ and the reference stress $\tau_{A,\ ref}$ at N_A cycles change through linear laws and their values are obtained interpolating between the fully-reversed tension and the fully-reversed torsion conditions:

$$k_\tau = a\rho_{eff} + b \quad (12)$$

$$\tau_{A,ref} = c\rho_{eff} + d \quad (13)$$

where a, b, c, d are material constants that can be calculated, for instance, from fatigue properties at $\rho_{eff} = 1$ and $\rho_{eff} = 0$.

The notch bisector is taken as focus path and the critical plane shear stress amplitude τ_a is computed by a linear-elastic FEA as a function of r, distance from the hot-spot.

Since τ_a depends on the coordinate r, the number of cycles to failure N_f is computed as a function of r:

$$N_f(r) = N_A\left[\frac{\tau_{A,ref}(\rho_{eff})}{\tau_a}\right]^{k_\tau(\rho_{eff})} \quad (114)$$

PM and is adopted together with the hypothesis that L depends on N_f with a power law :

$$r = \frac{AN_f^B}{2} \quad (15)$$

where A and B are calibrating constant. The point on the focus path for which Eq. (16) is solved:

$$\frac{AN_f(r)^B}{2} - r = 0 \quad (16)$$

represents the coordinate at which the reference stress, according to TCD, is located and from which number of cycles to failure is estimated.

It is clear that, in a VA case, this methodology needs some corrections since stress history will contain more than a single τ_a and a cycle counting method (e.g. Rainflow) must be adopted.

Critical plane is identified through MVM [27], whereas Rainflow [28] may be adopted to extract several τ_a experienced by the component for a number of cycles n. For each $\tau_{a,i}$, a number of cycles to failure $N_{f,i}$ is estimated and cumulative damage D_{tot} is computed.

$$\sum_{i=1}^{j}\frac{n_i}{N_{f,i}} = D_{tot} \qquad (1712)$$

$N_{f,eq}$ is then estimated:

$$N_{f,eq} = \frac{n_{tot}}{D_{tot}} \qquad (1813)$$

where n_{tot} is the sum of the extracted n_i.

As in the CA case, $N_{f,eq}$ is a function of the coordinate r and, by using PM, the reference stress point is found:

$$\frac{AN_{f,eq}(r)^B}{2} - r = 0 \qquad (19)$$

In this procedure it is implied a unitary critical cumulative damage. Calibrating constants can be added to consider the variability of the critical damage sum that will lead component to failure.

4. Applications of TCD

TCD applicability field is not restricted to notched components: examples of its use are found in the study of fretting fatigue [2,3]. In this context, the so called "notch analogue" [2,3,29] is noteworthy: this term refers to similarities of stress states both around notches tip and contact surfaces in fretting fatigue. This is the starting point for applying the same fatigue concepts in two apparently different situations linked by the presence of a multiaxial stress state. Indeed Kouanga et al. [2] studied the crack initiation process in fretting fatigue loadings by using TCD. In this case, TCD showed to be able to predict finite lifetime in metals subjected to Constant Amplitude (CA) loads. In this case, the use of linear-elastic FE models and multiaxial fatigue analyses paved the way for a design methodology suitable for real mechanical components subjected to fretting fatigue loadings. Then, Zabala et al. [30]

adopted a mesh control approach to analyze fretting fatigue from a TCD point of view. This resulted in a reduction of the simulation time with respect to the original TCD. Indeed, the use of FE models in conjunction with TCD concepts call for attention on the relation between mesh refinement and the size of the area in which stresses are averaged. The mesh control approach works on element sizes relatable to L in such a way that the stress calculated in the hot spot retraces the one of TCD. In this way, the computational time related to small element sizes is reduced. Recently, Pinto et al. [31] investigated the problem of fretting fatigue in aeronautical Al 7075-7651 alloy proposing a life assessment procedure which involved TCD. This contribution introduced the possibility of estimating fretting fatigue life under variable amplitude load conditions which included wear effect. TCD has not to be addressed to as a static or fatigue failure criterion itself but as a standpoint from which most of fatigue life criteria may be applied. Indeed, in the case of fretting fatigue, Araújo et al. [3] compared two different multiaxial critical plane criteria [18] using them both in form of local and non-local (TCD) approach. In particular, the mesoscopic scale Dang Van criterion [32–34] and the Modified Wöhler Curve Method (MWCM) [14,16] applied along TCD exhibited their capability to predict size effect. In this case, MWCM revealed to be a more accurate critical plane criterion. Moreover, it was emphasized the noteworthy advantage of achieving promising results by simply post-processing linear-elastic FE analyses thanks to the use of TCD [35]. This feature broadens the applicability of TCD in estimating fatigue strength of real mechanical assemblies. Critical distance application along with critical plane analysis was further investigated in multiaxial fatigue by Liao et al. [36]. It was found that better accuracy may be achieved if critical plane analysis is applied before TCD. Similarly, a combined critical distance/plane approach was employed for predicting fatigue crack initiation in superalloy components [37].

In recent times, then, TCD found application in random fatigue assessments. Luo et al. [38,39] proposed a damage gradient model to estimate fatigue lives of notched metallic components subjected to random biaxial fatigue loadings. The model was further validated by experimental results.

Looking at industrial applications, TCD concepts implementation is present in engines durability analyses [15]. In these cases, FEA results, coming from thermal-structural simulations, are post-processed for fatigue life assessment even if complex loading paths and geometries are involved. For instance, Bishop [15] applied non-local approaches, very similarly to what was later formalized in TCD, to fatigue analyses of the cylinder head and block of automotive engines. Of course, this kind of analyses imply a higher degree of complexity with respect to the study of test specimens. For this reason, a compromise between analysis complexities, theory implications and material behavior model must be found introducing some simplifications.

However, some aspects cannot be completely neglected; i.e. by reducing TCD to a simple stress assessment far from hot-spots, TCD ability to predict fatigue behavior of notched components inevitably fails. Certainly, by means of TCD, stress status is investigated in an area and not only in the stress concentration points, but what is the size of this area? The answer is one of the TCD key points. It depends on material [1,4,5] just like Ultimate Tensile Strength (UTS) and Fatigue Endurance Limit and generally cannot be arbitrarily defined to expand components limits of acceptability.

Then, TCD applications are present in cast iron components, bones, welded joints [40,41]. In this context, Taylor presented several case-studies [40,42]. For instance, the improvement of the notch radius in cast iron components did not show fatigue benefits since critical distance value was larger than the radius itself. Further, the relation between notch radius and critical distance was investigated to avoid brittle fracture in bones modified by orthopedic operations. Among biological materials, remarkable considerations were advanced for what concerns eggshells, providing interpretations of commercial interest since many eggs may break prematurely. Taylor et al. [43] measured the fracture toughness of notched eggshells estimating the effect of stress concentration features by means of TCD. Instead, Schimdt et al. [44] offered an insight into the toughening mechanisms occurring in insect wings. In this study, TCD showed capable to assess crack growth and crack arrest, laying the foundation to investigate crack-resistant materials for engineering applications in a TCD perspective.

Then, in welded joints applications, the use of TCD allows to FE model a null notch radius without affecting the results as long as the radius real value is smaller than L. More recently, Fang et al. [45] addressed the problem of stress concentration in wind-induced fatigue assessment of welded structures by adopting TCD. This study confirmed TCD reliability in this field of application.

High level of accuracy were obtained in the cases of torsional fatigue [46], aluminum tubular beams [47] and environmentally assisted cracking [48,49] (EAC). Guitérrez et al. [48,49] analyzed EAC process in notched steel specimen by means of PM. In this context, TCD was successfully applied to analyze Hydrogen Embrittlement conditions which caused cleavage-dominated fracture micromechanisms. Then, notch effect was observed in Stress Corrosion Cracking (SCC) and it was effectively predicted by means of TCD [50]. Next, PM and LM accuracy was emphasized in estimating fatigue lives of high-strength steel wires with corrosion pits [51].

Also, Critical Distance approaches were applied to the study of the fracture behavior of rocks. Cicero et al. [52] investigated the notch effect on biotite granite and oolitic limestone through an experimental campaign involving 84 notched specimen tested in 4-points bending. The TCD approach revealed defect sizes much smaller than L to be harmless and a good agreement between critical distance estimation and experimental evidence was achieved. Similar results were obtained on different type of rocks such as Carrara marble [53]. More recently, Justo et al. [54] analyzed the fracture behavior of rocks subjected to Mode I loads in different temperature conditions. Interestingly, temperature influenced material toughness but had not clear effect on critical distance. Instead, the effect of temperature on metallic materials was studied by Louks and Susmel [55] . In this case, TCD showed to be successful in estimating high-cycle fatigue strength at elevated temperatures. The remarkable advantage of modeling the non-linear behavior of metals subjected to high temperature with a linear elastic stress field stem, even more in this context, as one of the main TCD benefits. Also at high temperature, the notch geometry showed to have no influence on critical distance values. However, the effect of geometry on L is one of the main issue which emerges from literature, since some research activities are devoted to the development of notch-dependent critical distances. The variability of L with the number of cycles to failure has been discussed as well [14,56]. A more in-depth discussion of this topic is provided in Section 4.

Applications are also found in the field of ceramic materials [57] and concrete structures [58]. In a recent work of Alanazi and Susmel [58], TCD linear-elastic modeling was proficiently applied in notched un-reinforced concrete subjected to mixed mode static and dynamic loadings.

Further research projects are devoted to the study of notch fatigue in additively manufactured materials [59] and 3D printed components [60]. Benedetti and Santus [59] employed critical distance approaches for the fatigue analysis of Ti-6-Al-4V additively manufactured via selective laser melting. In this case, defects strongly affect the fatigue behavior and TCD efficacy is linked to the knowledge of the actual defectiveness. The work of Ahmed [60] investigated the static strength of Polylactic acid (PLA) notched components obtained by means of fused deposition modeling. The application of TCD concepts showed to be promising, estimating static strength within +/-20% error range. Fatigue strength of PLA was investigated by Ezeh and Susmel [61]. This work showed that, in 3D-printed polymers, fatigue cracks follow irregular paths which retrace the orientation of the extruded filaments. Besides, TCD exhibited its capabilities to asses notch fatigue strength. The same considerations were advanced thanks to the study of the static strength of ABS [62]. Overall, good experimental agreement was achieved by applying TCD to polymers, such as PMMA [63].

Composite materials were analyzed from a TCD perspective as well. Namely, the structural integrity of short glass fiber reinforced polyamide 6 (SGFR-PA6) was investigated basing on Failure Assessment Diagrams [64] and LM stress corrections. Whereas, Nguyen-Hoang and Becker explored the behavior of composite bolted joints for aeronautical applications [65].

Next, fracture toughness was analyzed from a TCD point of view [66], apparent fracture toughness of notched structures was investigated by Cicero et al. [67] and, more recently, Taylor provided an interpretation of the multiscale toughening mechanism relying on TCD concepts [68].

5. Remarks on Critical Distance

It was introduced a scale length whose ability of predicting fatigue failure was widely proved [1–3,14,16]. Actually, explaining the meaning of this length is not straightforward. Thanks to FFM, L was related to crack growth discrete quanta but its correlation with fracture mechanism is not fully assessed. Critical distance can be considered as a good working tool but how to relate it to other fracture theories? Different interpretations come out of this discussion [1].

5.1 Physical interpretations of critical distance parameter

For instance, process zone models [1,69,70] represent fracture nonlinearities by entrusting this task to special stress-displacement curves which model material behavior close to stress concentrations features. The zone in which these properties are assigned is the so-called "process zone". Interestingly, a close correlation of the size of this zone with critical distance is recognizable.

Then, fracture statistical models [71] link probability of failure to material constants. Weibull model is one of these:

$$P_f = 1 - e^{\left(-\frac{\sigma}{\sigma_m}\right)^b} \qquad (2014)$$

where P_f is the cumulative probability of failure, the ratio $\frac{\sigma}{\sigma_m}$ is descriptive of how much the component is stressed (σ) in relation with its strength properties (σ_m) and b describes how much stochastic phenomena affect strength.

When b is very large, stress state at the hot-spot is alone responsible for component failure, on the other hand low values of b indicate that failure process involves a wider material volume, since probability of failure is significant also far from hot-spots. This concept, as suggested by Taylor [1], sounds very similar to what happens when critical distance argument is applied.

Then, microstructure properties [72], such as grain size, appear closely related to critical distance values. The presence of an intrinsic material length is detectable in other fracture theories such as Gradient Mechanics models [73,74] that may themselves be related to TCD.

5.2 Critical Distance Estimation

In light of these ideas, it is clear that a practical application of TCD requires an estimation methodology for L. Basically, by using Eq. (3) or Eq. (4), critical distance estimation calls for the knowledge of two different material properties. However, two main issues arise: the first is related to the fact that ΔK_{th} may be difficult to measure [75], the second concerns the use of σ_0. Indeed, for some materials the value of σ_0 corresponds to UTS or, equivalently, to the fatigue limit in the cycling loading case. Nevertheless, being σ_0 physical significance strictly related to whether plasticity phenomena are involved in fracture process, it should be considered as a dependent variable coming from L and ΔK_{th} or K_C.

Following TCD main assumptions, L is considered a material constant in most of the methodologies adopted for its estimation [2,75–79] and this is explainable by the following. If it is reasonable to link L to microstructure, no geometry effect is expected to influence L since TCD itself, as a more inclusive interpretation of LEFM, should be able to predict size effect rather than be affected by that. Nevertheless, in some cases theoretical constraints can be relaxed to allow mathematical models to fit physical reality, especially in practical applications.

On the other hand, Wang et al. [80] obtained a higher accuracy level by modifying PM and LM to consider critical distance geometry effect in Ti-6Al-4V.

Moving on to the substance, critical distance estimation is obtained from the post-processing of the linear elastic stress field by using PM. The procedure is applied to a notched component of which a FEM model is built for stress field extraction (Fig. 3).

L can be computed for any number of cycles to failure. Essentially, the plain and the notch fatigue curves are employed. From the latter it is extracted the $\Delta\sigma_{nom}$ at the given N_f, whereas $\Delta\sigma_1$ is extracted from the first curve. Then, literature on TCD suggests a sharp notch to be employed [75] for critical distance estimation. The stress field resulting from the application of $\Delta\sigma_{nom}$ is studied along the "focus path" represented by the notch bisector. According to PM, this condition of incipient failure

421 occurs when, at a certain distance from stress concentration feature ($r = L/2$), linear elastic stress $\Delta\sigma$ equals inherent strength $\Delta\sigma_1$. Then, the coordinate r for which the linear elastic stress-distance curve has the value of $\Delta\sigma_1$ is $r = \frac{L}{2}$ and L may be extracted.

Referring to Fig. 3, the computational problem related to mesh size has not to be neglected since L should be considerably greater than the element size [81]. Indeed, Braun et al. [82] analyzed the mesh requirements for stress gradients methods.

However, two different notch geometries exhibit two different critical distance values when the presented procedure is applied because different sharpness corresponds to different stress gradients. As a matter of fact, this seems to endorse the existence of a possible geometry effect on L. For sake of TCD generality, it should be accepted that, when plasticity phenomena are involved in fracture process, inherent strength cannot be assessed a priori, as it is not always true that it coincides with fatigue limit (or UTS in static loadings). Therefore, two notches with different levels of sharpness (e.g. sharp and blunt notches) may be used (Fig. 4) [17]. Figure 4 shows the stress curves resulting from the application of $\Delta\sigma_{nom}$ to blunt and sharp notched specimen. Therefore, the value of $\Delta\sigma_0$ is assessed on the basis of the intersection between different curves. This way, it is searched for the combination of the parameters L and $\Delta\sigma_0$ which are invariant with respect to notch geometry.

The inherent strength value now acts as a calibrating constant thanks to which the same critical distance for both geometries is found, canceling any geometry effect.

A different approach is followed by Yang et al. in the study of DS superalloy [83] where authors found non-constant critical distances when standard TCD is applied while the product of K_t and L remained constant.

Therefore, conventional TCD was be modified [80,83] by introducing K_t and by considering a new critical distance value:

$$L^* = K_t L \qquad (21)$$

At a fixed number of cycles, experimental evidence of this quantity to remain constant is declared.

Then, notch factor is used as a geometry describer in order to include size effect in critical distance evaluation. Nevertheless, when loads and geometry complexities starts to approach actual industrial applications, K_t definition is not straightforward, especially if multiaxiality is involved. Indeed, even if multiaxiality were solved by equating multiaxial stress state to an equivalent stress, a possible theoretical inconsistency may occur as reported in the following.

Principal stresses directions do not remain constant in time when non proportional loadings act on components, and this makes equivalence stress criteria unadvisable for the fatigue assessment [84]. Critical plane criteria, on the contrary, consider only some stress components related to fatigue crack initiation (e.g. shear stress, normal stress, shear strain, normal strain, and combinations of these) and their variability is studied along precise directions searching for the critical one.

Since critical plane criteria are most used in actual industrial applications, by using Eq. (12) it may occur that an equivalence criterion is used for K_t evaluation whereas a critical plane criterion is used for fatigue life assessment. The present authors raise an issue on this point as it may result both in a harmless event and in, at least theoretically, inconsistency. Moreover, the definition of K_t for components subjected to complex stress states may be different with respect to the adopted standard. In this sense, the definition of a K_t-dependent critical distance may erase the generality of TCD since geometry would affect theory formulation.

However, in the field of automotive industry some research activities are devoted to Variable Critical Distance Methods [85], providing promising results in the field of automotive components design. Other works investigated critical distance estimation methods. For instance, Santus et al. [75,79,86] proposed a refined methodology for critical distance estimation based on the inverse search method and they provided a sensitivity analysis based on the specimen geometry. In this method, critical distance is estimated considering an, up to this point, unmentioned aspect. By using relatively low stress gradients, small variations of the fatigue limit introduce large alterations in critical distance value whereas, in sharp notches, contained error on L bring to large error in the reference stress and, consequently, on strength assessment. Moreover correction functions are introduced in [75,79]. Then, Benedetti and Santus [86] studied the statistical properties of the thus determined critical distance.

As earlier anticipated, critical distance can be evaluated for a certain N_f and at least two cycle regimes are necessary to build Eq. (22) which, as often occurs in fatigue problem, is a power law:

$$L = AN_f^B \tag{22}$$

where A and B are material constants that can be tuned by performing suitable experiments [16,78].

Then, when critical distance arguments are employed to compare reference stress with allowable stress, the knowledge of L with respect to a prescribed N_f for safety factor computation is enough, but what if N_f were unknown?

This is the case of finite life estimation. In this condition the size of the averaged stress zone depends on the number of cycles to failure itself and an iterative process is required.

One of the great advantages of Eq. (22) power law [14] lies in the fact that it makes TCD available in the medium-high cycles regime by using a linear elastic stress field as explained in the following. Given a negative B constant, L increases as N_f decreases following actual alterations that plastic zone around notch tip undergoes when applied load increases. Thanks to these assumptions, cyclic plastic behavior is modelled through a linear elastic field by changing the size of process zone we are looking at [14]. These models lose in accuracy when N_f is lower than $10^3 - 10^4$ cycles since this hypothesis is no more sufficient. In this cases, an elasto-plastic reformulation of TCD is required [87,88].

To the best of the authors' knowledge, commercial fatigue post-processors currently do not implement an iterative procedure for L and N_f computation and fixed High Cycle Fatigue (HCF) critical distance is usable. For this reason, fatigue life assessment in terms of number of cycles to failure, for what concerns the medium-high and medium-low cycle regime by TCD, is not possible by using commercial codes. On the contrary, safety factor computation for infinite life is usually available through TCD algorithms since fixed critical distances can be used.

6. Discussion and conclusions

This is the result of the recollecting work performed by the authors in order to link two aspects. The first is the ever-growing industrial need to develop accurate as much as simple and ready-to-use models for failure prediction in real components, especially if they are subjected to complex load distributions both in space and time. The second is the academic world where theoretical arguments acquire primary relevance. These aspects are otherwise connected by the necessity of portraying a current frame of TCD applications and implications.

By analyzing TCD from both points of view, many questions arose.

1. What is TCD? If we were looking for a method suitable for strength assessment in an industrial component, we could propose TCD as a stress evaluation carried out far from hot-spots. As presented in this work, there is something more beyond this simplification. Indeed, it is quite complex to describe cracking micro-mechanism by using equations and it becomes even more difficult to apply those to real cases because microstructure needs to be considered. On the other hand, continuum mechanics models allow us to solve the problem by using stresses, strains and energy as parameters. Explaining the reasons why continuum mechanics shows better accuracy when applied from the TCD point of view is not straightforward. However, if critical distance is actually linked to microstructure, it starts to be clear that we are somehow including micro-mechanism in continuum mechanics models reinforcing them. This candidates TCD as a general fracture theory.
2. What is L and how is it used? It is introduced a material constant that may be linked as much to microstructure as well as to other fracture theories. Practically, this quantity is used to define the size of the zone over which continuum mechanics quantities are averaged. Critical distance measurement may be done in static and cyclic loading conditions. In the second case a distinction between infinite and finite life regime must be done in order to consider the possible variability of L with the number of cycles to failure. Commercial fatigue post-processors bound TCD applicability to a fixed L, allowing Safety Factor computations through

critical distance argument but not cycles to failure estimation in the finite life regime. This fact represents a limitation in the industrial world. In fact, Safety Factors estimation in terms of stresses is for sure simpler to correlate with experimental data but this quantity actually represents a measure of what we do not know about physical phenomena occurring in failure process. Consequently, threshold values are defined upon experience. That is, the more our model is good, the more threshold Safety Factor will resemble unitary value. On the contrary, number of cycles to failure can be seen as a measure of what we know, being directly comparable with experimental data. In fact, the prediction of N_f is a more refined information telling us when failure will occur and not simply how much, on the basis of experience, we are far from it. Finally, critical distance estimation problem and its dependence on geometry is an open issue.

3. How to apply TCD? Most of the failures occurring in machines are related to fatigue. In the most general, but not unlikely case, multiaxial non-proportional loadings occur. Therefore, critical plane criteria should be applied by considering stress vector paths besides stress values. MWCM is presented as one of the possible methods because it is applicable in the most complex case of a multiaxial loading path with VA non-proportional loadings in the finite life regime.
4. Limitations in the available commercial fatigue post-processors employed in the industrial field narrow TCD applicability in this context.

TCD fields of application are growing and the accuracy of the method is continuously confirmed. Many works were initially devoted to the study of metallic materials, whereas recent applications investigated additively manufactured materials, polymers and random fatigue loadings. The available literature offer an insight into several issues regarding critical distance. In particular, several studies which considered complex mechanical assemblies, such as in the field of automotive industry, developed their considerations on the variability of critical distance with geometry. On the other hand, although the problem of notch geometry is deeply discussed in literature, critical distance variability is mainly studied with respect to material properties, temperature conditions and number of cycles to failure but geometry effect on material strength is expected to be assessed by means of *L* rather than the opposite. This is reasonable if TCD generality as fracture theory is embraced, since it is expected a more inclusive fracture model by introducing a material property linked also to microstructure. In this sense, TCD proposes itself as a link between micromechanical models and continuum-mechanics models.

Then, a growing research body is moving in the direction of TCD issues related to FE modeling. This could be an interesting point for the full implementation of TCD in fatigue post-processors. Although the computational power is always increasing, the problem of mesh refinement with respect to critical distance values is indeed not negligible.

The validity of TCD is somehow being undermined by component dimensions. Therefore, the application of these concepts to micromechanical components, such as MEMS, is still challenging and proposes itself for future developments.

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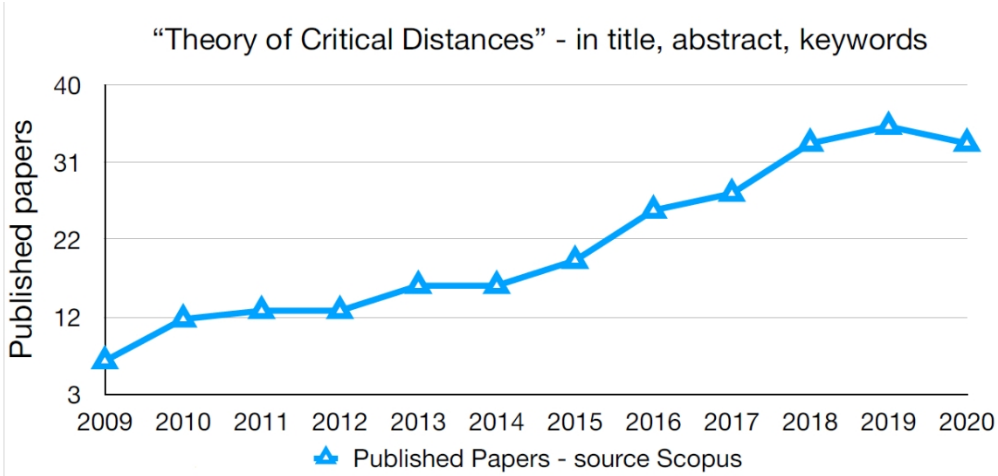
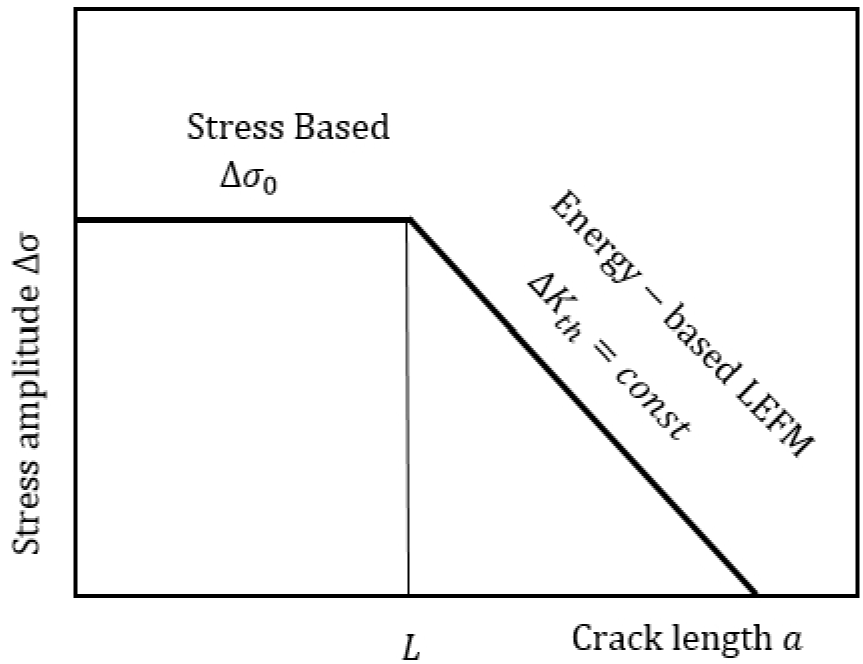
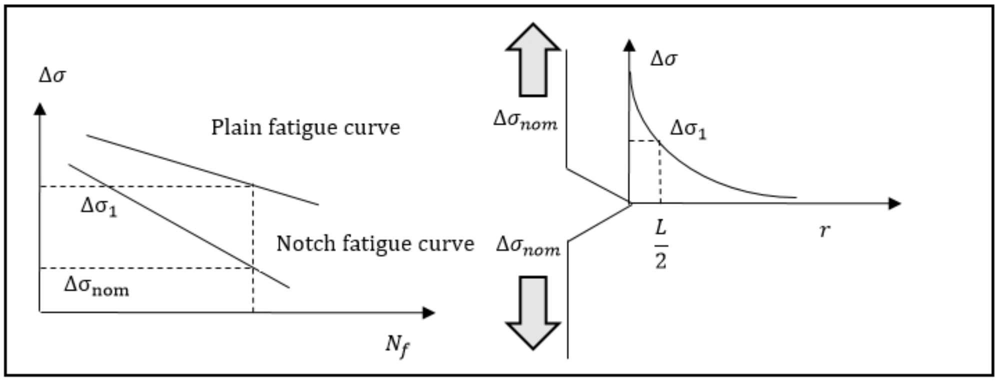


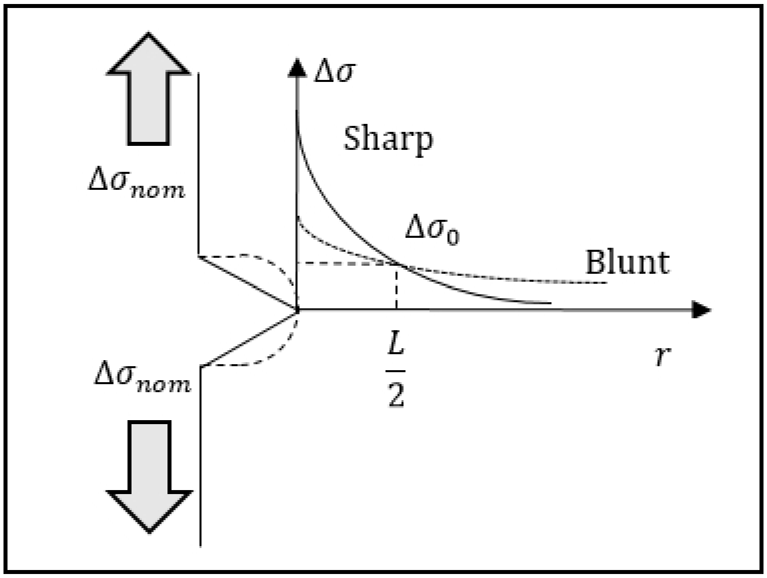
Theory of Critical Distances in literature



Kitagawa-Takahashi diagram



Critical distance estimation procedure



Notch sharpness and critical distance estimation