Wiener-hopf formulation of the scattering by a PEC wedge over an half dielectric grounded slab

URSI EM Theory Symposium, EMTS 2019, San Diego, CA, 27– 31 May 2019

WIENER-HOPF FORMULATION OF THE SCATTERING BY A PEC WEDGE OVER AN HALF DIELECTRIC GROUNDED SLAB

Vito Daniele(1) (2), Guido Lombardi* (1) (2), and Rodolfo S. Zich(1) (2)
(1) DET-Politecnico di Torino, Italy, http://www.polito.it
(2) ISMB, Torino, Italy, http://www.ismb.it

Abstract

This paper presents the formulation of electromagnetic problems constituted of inhomogeneous coupled angular and planar regions by using the Generalized Wiener-Hopf Technique (GWHT). In particular the paper is focused on the scattering of a perfectly electrically conducting (PEC) wedge in contact with an half dielectric grounded slab. The solution method is based on deriving the Wiener-Hopf formulation and on using the Fredholm factorization. In this case the presence of inhomogeneous regions introduces further difficulties.

1. Introduction

The study of electromagnetic scattering problems constituted of isolated angular regions or of layered regions is consolidated by a number of different approaches in spectral domain. However these techniques are not able to analyze complex scattering problems where inhomogeneous angular and planar regions are combined.

The Generalized Wiener-Hopf Technique (GWHT) has been introduced to extend the classical Wiener-Hopf technique first to solve electromagnetic problems involving angular regions [1-4]. According to our opinion the GWHT is not only a new powerful technique to solve isolated wedge problems as done by the popular Sommerfeld-Malyuzhinets (SM) method, see [5-7] and references therein. In fact the GWHT allows to handle new complex problems [8-14].

Moreover, in case of absence of exact closed-form solutions, the Generalized Wiener-Hopf equations (GWHEs) can be approximately solved via an efficient approximate factorization technique known as Fredholm factorization [15-16]. The Fredholm factorization consists of eliminating one kind of unknowns (either plus or minus) of the Wiener-Hopf (WH) equations by applying the Cauchy decomposition formula. The elimination of the minus (plus) functions yields integral representations in plus (minus) unknowns. Combining the integral representations of the problem, we can obtain the solution via approximation of Fredholm integral equations of second type. We note that Fredholm factorization avoids the steps of factorization, decomposition and application of Liouville's theorem of the classical WH solution procedure [17].

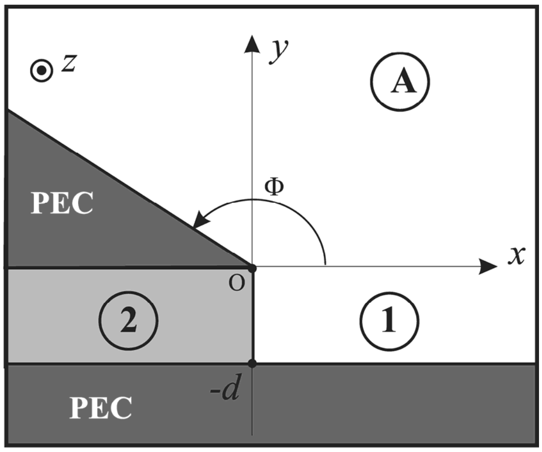

Figure 1. The PEC wedge over an half dielectric grounded slab.

Taking inspiration from [18],[19],[10], we introduce the use of network modelling/representation of the relevant GWHEs an their integral representations. This framework allows to order and systematize the mathematical procedure avoiding redundancy.

The aim of this paper is to introduce the GWHT to solve the problem constituted of a PEC Wedge over an Half Dielectric Grounded Slab, see Fig.1. In this case, due to the presence of inhomogeneous regions, we need to start the procedure by analyzing the wave equations with the help of the characteristic Green function procedure as applied in [18],[19]. The procedure presents some difficulties that were not encountered studying a similar problem, see [11-12].

2. Definitions and Geometry of the Problem

In this work we consider time harmonic electromagnetic field with a time dependence specified by $e^{j\omega t}$ which is omitted. Figure 1 illustrates the problem. A PEC wedge is in contact of a half dielectric grounded slab. We subdivide the geometry into three regions and the structure is with translational symmetry along the z axis. In particular, according to cylindrical coordinates centered in O, the PEC wedge is delimited by PEC boundary conditions at

$\varphi=\pi$ and $\varphi=\Phi$. Using Cartesian coordinates centered in O, the half dielectric grounded slab is located at $-d<y<0$, filled by free space with free-space propagation constant k and free space impedance Zo for x>0 (region1) and filled by dielectric material with relative permittivity εr for x<0 (region 2). The angular region $0<\varphi<\Phi$ is filled by free space (region A).

We note that region 2 ($-d<y<0$, $x<0$) constitutes a planar waveguide filled by dielectric material. For the sake of simplicity, we suppose that the source is constituted of the progressive first TE mode:

$$H_x^i(x,y)=\cos\frac{\pi}{d}y\,e^{-j\chi_1 x}\quad(-\mathrm{d}<\mathrm{y}<0,\mathrm{x}<0) \qquad (1).$$

where χ_n with $n\in\mathbb{N}_0$ are the TE mode propagation constants

$$\chi_n=\sqrt{\varepsilon_r k^2-\left(\frac{n\pi}{d}\right)^2} \qquad (2).$$

We note that the discontinuity at x=0, $-d<y<0$ excites all the TE modes that propagate along the negative x direction.

The formulation of the problem is defined in terms of Laplace transforms $V_+(\eta),\ I_+(\eta),\ I_{a+}(-m)$ respectively of the E_z at $\varphi=0$, H_x at $\varphi=0$ and H_ρ at $\varphi=\Phi$:

$$V_+(\eta)=\int_0^\infty E_z(x,0)e^{j\eta x}dx$$
$$I_+(\eta)=\int_0^\infty H_x(x,0)e^{j\eta x}dx \qquad (3).$$
$$I_{a+}(-m)=\int_0^\infty H_\rho(\rho,\Phi)e^{-jm\rho}d\rho$$

3. Formulation

3.1 Angular Region

With reference to [1] the GWHE of region A is

$$Y_\infty(\eta)V_+(\eta)-I_+(\eta)=-I_{a+}(-m) \qquad (4).$$

where $Y_\infty(\eta)=\xi(\eta)/kZ_o$, $\xi(\eta)=\sqrt{k^2-\eta^2}$, $m=-\eta\cos\Phi+\xi(\eta)\sin\Phi$. Eq. (4) is a Generalized WH Equation (GWHE) since the unknown $I_{a+}(-m)$ is a minus function in the m-plane and not in the η-plane. We resort to a Classical Wiener-Hopf equation by introducing the mapping

$$\eta(\bar{\eta})=-k\cos\left[\frac{\Phi}{\pi}\arccos(-\frac{\bar{\eta}}{k})\right] \qquad (5).$$

yielding

$$\bar{Y}_c(\bar{\eta})\bar{V}_{1+}(\bar{\eta})-\bar{I}_{1+}(\bar{\eta})=-\bar{I}_{a+}(-\bar{\eta}) \qquad (6).$$

By applying the Fredholm factorization [15] we eliminate the minus function $\bar{I}_{a+}(-\bar{\eta})$ and mapping back the integral representation to η plane we obtain the integral representation [10]

$$I_+(\eta)=Y_\infty(\eta)V_+(\eta)+\mathscr{Y}[V_+(\eta')],\quad \eta\in\mathbb{R} \qquad (7).$$

where $\mathscr{Y}[\bullet]=\frac{1}{2\pi j}\int_{-\infty}^{\infty}y(\eta,\eta'][\bullet]d\eta$ with

$$y(\eta,\eta')=\frac{Y_\infty(\eta')}{\bar{\eta}(\eta')-\bar{\eta}(\eta)}\frac{d\bar{\eta}}{d\eta'}-\frac{Y_\infty(\eta)}{\eta'-\eta}+\sum_{n=1}^{n_{at}}\frac{q_{2n}^{\Phi_a}(\eta)}{\eta'-\mathrm{p}_{2n}^{\Phi_a}(\eta)}u(\frac{\pi}{2}-n\Phi)$$

$$p_{2n}^{\Phi}[\eta]=\eta\cos 2n\Phi-\sqrt{k^2-\eta^2}\sin 2n\Phi$$

$$q_{2n}^{\Phi}[\eta]=\frac{1}{\mathrm{k}\,\mathrm{Z_o}}(\eta\sin 2n\Phi+\sqrt{k^2-\eta^2}\cos 2n\Phi)$$

and u(t) is the unit step function.

In (7) we have used an abstract notation that is associated to a network interpretation of the integral representation [10]. In particular region A is modelled via algebraic-integral operator admittance that relates the plus unknowns $V_+(\eta),\ I_+(\eta)$ see Fig. 2.

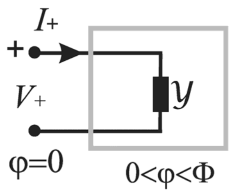

Figure 2. Equivalent network model of region A corresponding to (7).

3.2 Half Dielectric Grounded Slab

Region 1 and 2 constitute the half dielectric grounded slab. In this subsection we give the hints to build an integral representation that relates the plus unknowns $V_+(\eta),I_+(\eta)$. Since we are dealing with an inhomogeneous combined region we resort to the characteristic Green's function procedure to model the half slabs via the solution of the wave equation. Continuity at the interface x=0 yields the final representation.

The wave equations of the two regions are

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_i^2\right)E_z(x,y)=0 \qquad (8).$$

with i=1,2 and $k_1=k$ and $k_2=k\sqrt{\varepsilon_r}$.

By applying the unilateral Fourier transforms we get equations

$$\left(\frac{d^2}{dy^2}+\tau^2\right)\tilde{\Phi}(\alpha,y)=f_\alpha(y) \qquad (9).$$

in α plane where the forcing term is related to the field components at the interface x=0 (initial conditions) and $\tau=\sqrt{k^2-\alpha^2}$.

Focusing the attention on region 1, using the characteristic Green's function procedure [18],[19] and enforcing the

PEC boundary condition at y=-d we obtain an equation that relates $V_+(\eta), I_+(\eta)$ to the initial conditions $f_{1\alpha}(y)$ for x=0+:

$$\left(\frac{\int_{-d}^{0} \sin\left(\tau\left(y'+d\right)\right) f_{1\alpha}(y')dy}{jkZ_o \sin\left(\tau d\right)} \right) - I_+(\alpha) = Y_d(\alpha)V_+(\alpha) \quad (10).$$

with $Y_d(\alpha) = -j\frac{\sqrt{k^2-\alpha^2}}{k\,Z_o}\cot\left[\sqrt{k^2-\alpha^2}\,d\right]$,

To have a closed mathematical problem we explicit $f_{1\alpha}(y)$ by considering the modal representation of the field in region 2 at x=0-:

$$E_z(y,x) = E_o \sin\left(\frac{\pi}{d}y\right)e^{-j\chi_1 x} + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{d}y\right)e^{j\chi_n x} \quad (11).$$

Analytical properties of the integral term in (10) after the introduction of (11) allows to obtain a WH equation where the right hand side of (10) is equal to a forcing term that depends on the source (first TE mode), the modal field expansion of region 2 (poles χ_n) and $V_+(\alpha)$ estimated for $\alpha = -\alpha_n = -\sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}, n \in \mathbb{N}_0$:

$$\psi_-(\alpha) + \psi_+(\alpha) + \psi_+^i(\alpha) = I_+(\alpha) + Y_d(\alpha)V_+(\alpha) \quad (12).$$

where $\psi_+^i(\alpha) = 2\frac{\pi\chi_1}{dkZ_o(\alpha-\alpha_1)(\alpha_1+\chi_1)}E_o$.

In particular, taking into account that $I_+(\alpha)$ and $V_+(\alpha)$ are regular at $\alpha = -\alpha_n$ and assuming the residue at $\alpha = -\alpha_n$, the above equations relate the unknown coefficients C_n to $V_+(-\alpha_n)$.

Eq. (12) models regions 1-2. The application of Fredholm factorization eliminates the minus function $\psi_-(\alpha)$ and it yields the integral representation

$$\psi_+^i(\alpha) + \psi_+(\alpha) - I_+(\alpha) = Y_d(\alpha)V_+(\alpha) + \mathcal{Y}_d[V_+(\alpha')] \quad (13).$$

Also for (13) we can introduce a network interpretation. In particular regions 1-2 are modelled via algebraic-integral operator admittance (right hand side), a source term $\psi_+^i(\alpha)$ and a dependent source term $\psi_+(\alpha)$.

3.3 The Complete Problem

By substituting (13) into (7) (noting that η and α are the same spectral variable) we eliminate I_+ obtaining a Fredholm integral equation of second kind of the following kind:

$$V_+(\alpha) + \frac{1}{2\pi j}\int_{-\infty}^{\infty} M(\alpha,\alpha')\cdot V_+(\alpha')d\alpha' = N_o(\alpha) + \sum_{m=1}^{\infty} h_m(\alpha)V_+(-\alpha_m) \quad (14).$$

Solution may be obtained by truncating the infinite series to M terms and by using superposition.
Simple discretization technique of (14) can be applied according to Fredholm theory [20].
Further developments will be reported during the presentation at the conference and solution will be given in terms of modal field components inside the waveguide region 2 and in terms of far field components in region A.

4. Acknowledgements

This work was partially supported by Politecnico di Torino and the Istituto Superiore Mario Boella (ISMB), Torino, Italy.

5 References

1. V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle", *SIAM Journal of Applied Mathematics*. Vol **63**, pp. 1442-1460, 2003

2. V.G. Daniele and G. Lombardi, "Wiener–Hopf Solution for Impenetrable Wedges at Skew Incidence", *IEEE Trans. Antennas and Propagat.*, vol. **54**, pp. 2472-2485, 2006.

3. V.G. Daniele and G. Lombardi, "The Wiener-Hopf Solution of the Isotropic Penetrable Wedge Problem: Diffraction and Total Field", *IEEE Trans. Antennas and Propagat.*, vol. **59**, pp. 3797-3818 , 2011

4. G. Lombardi, "Skew Incidence on Concave Wedge With Anisotropic Surface Impedance," *IEEE Antennas Wireless Propag. Lett.*, vol. **11**, pp. 1141-1145, 2012
5. B. Budaev, Diffraction by wedges, London, UK: Longman Scient., 1995.

6. T.B.A. Senior, and J.L. Volakis, Approximate boundary conditions in electromagnetics, London, UK: IEE, 1995.

7. A.V. Osipov, A.N. Norris, "The Malyuzhinets theory for scattering from wedge boundaries: a review," *Wave motion*, vol **29**, pp. 313-340, 2006.

8. V.G. Daniele, "Electromagnetic fields for PEC wedge over stratified media. Part I," *Electromagnetics*, vol. **33**, pp. 179–200, 2013.

9. V.G. Daniele and G. Lombardi, "Arbitrarily Oriented Perfect Conducting Wedge over a Dielectric Half-Space: Diffraction and Total Far Field," *IEEE Trans. Antennas and Propagation*, Vol. **64** No. 4, pp. 1416-1433, 2016, doi: 10.1109/TAP.2016.2524412

10. V.G. Daniele, G. Lombardi, R.S. Zich, "Network representations of angular regions for electromagnetic scattering," *Plos One*, vol. **12**, n. 8, e0182763, pp. 1-53, 2017, https://doi.org/10.1371/journal.pone.0182763.

11. V.G. Daniele, G. Lombardi, R.S. Zich, "The electromagnetic field for a PEC wedge over a grounded dielectric slab: 1. Formulation and validation,"' *Radio Science*, vol. **52**, pp. 1-20, 2017, https://doi.org/10.1002/2017RS006355.

12. V.G. Daniele, G. Lombardi, R.S. Zich, "The electromagnetic field for a PEC wedge over a grounded dielectric slab: 2. Diffraction, Modal Field, Surface Waves, and Leaky Waves," *Radio Science*, vol. **52**, pp.1-18, 2017, https://doi.org/10.1002/2017RS006388.

13. V.G. Daniele, G. Lombardi, R.S. Zich, "The scattering of electromagnetic waves by two opposite staggered perfectly electrically conducting half-planes," *Wave Motion*, vol. **83**, pp. 241-263, 2018, doi: 10.1016/j.wavemoti.2018.09.017

14. V.G. Daniele, G. Lombardi, R.S. Zich, "The Double PEC Wedge Problem: Diffraction and Total Far Field,", *IEEE Trans. Antennas and Propagation*, in press, 2018, doi: 10.1109/TAP.2018.2877260

15. V.G. Daniele and G. Lombardi, "Fredholm Factorization of Wiener-Hopf scalar and matrix kernels," *Radio Science*, vol. **42**: RS6S01, 2007

16. V.G.Daniele, and R.S. Zich, The Wiener Hopf method in Electromagnetics, Scitech Publishing, 2014

17. B. Noble, Methods Based on the Wiener-Hopf Technique: For the Solution of Partial Differential Equations, London,UK:Pergamon, 1958.

18. L. B. Felsen and N. Marcuvitz, Radiation and Scattering of Waves, Englewood Cliffs, NJ: Prentice-Hall, 1973.

19. V.G. Daniele, R.D. Graglia, G. Lombardi, P.L.E. Uslenghi, "Cylindrical resonator sectorally filled with DNG metamaterial and excited by a line source," *IEEE Antennas Wireless Propag. Lett.*, vol. **11**, pp. 1060-1063, 2012.

20. L.V. Kantorovich and V.I. Krylov, Approximate methods of higher analysis, Groningen, The Netherlands: Noordhoff, 1964.