N = 2 supersymmetric S-folds

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$\mathcal{N} = 2$ supersymmetric S-folds

Adolfo Guarino,[a,b] **Colin Sterckx**[c] **and Mario Trigiante**[d]

[a] *Departamento de Física, Universidad de Oviedo, Avda. Federico García Lorca 18, 33007 Oviedo, Spain*
[b] *Instituto Universitario de Ciencias y Tecnologías Espaciales de Asturias (ICTEA), Calle de la Independencia 13, 33004 Oviedo, Spain*
[c] *Université Libre de Bruxelles (ULB) and International Solvay Institutes, Service de Physique Théorique et Mathématique, Campus de la Plaine, CP 231, B-1050, Brussels, Belgium*
[d] *Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24, I-10129 Turin, Italy and INFN, Sezione di Torino, Italy*

E-mail: adolfo.guarino@uniovi.es, colin.sterckx@ulb.ac.be, mario.trigiante@polito.it

Abstract: Multi-parametric families of AdS_4 vacua with various amounts of supersymmetry and residual gauge symmetry are found in the $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes \mathbb{R}^{12}$ maximal supergravity that arises from the reduction of type IIB supergravity on $\mathbb{R} \times \mathrm{S}^5$. These provide natural candidates to holographically describe new strongly coupled three-dimensional CFT's which are localised on interfaces of $\mathcal{N} = 4$ super-Yang-Mills theory. One such AdS_4 vacua features a symmetry enhancement to $\mathrm{SU}(2) \times \mathrm{U}(1)$ while preserving $\mathcal{N} = 2$ supersymmetry. Fetching techniques from the $\mathrm{E}_{7(7)}$ exceptional field theory, its uplift to a class of $\mathcal{N} = 2$ S-folds of type IIB supergravity of the form $\mathrm{AdS}_4 \times \mathrm{S}^1 \times \mathrm{S}^5$ involving S-duality twists of hyperbolic type along S^1 is presented.

Keywords: Flux compactifications, String Duality, Supergravity Models, Superstring Vacua

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1 Introduction

Electromagnetic duality in four-dimensional maximal supergravity has provided a very rich phenomenology as far as the existence of new gaugings and vacuum solutions are concerned. The prototypical example is the dyonically-gauged SO(8) supergravity where the action of electromagnetic duality on the gauging generates a one-parameter family of inequivalent theories parameterised by a continuous parameter $c \in \left[0, \sqrt{2} - 1\right]$ [1]. Setting the parameter to $c = 0$ then the standard (electric) SO(8) supergravity of de Wit and Nicolai [2] is recovered which is known to arise upon dimensional reduction of eleven-dimensional supergravity on a seven-sphere S^7. The various AdS$_4$ vacua of the $c = 0$ theory [3] (see also [4] for an updated encyclopedic reference) get generalised to one-parameter families of vacua when turning on c and, more importantly, new and genuinely dyonic AdS$_4$ vacua also appear which do not have a well defined (electric) $c \to 0$ limit [1, 5–7]. Other types of four-dimensional solutions, like domain-walls [8, 9] or black holes [10–12], have also been investigated using instead a phase-like parameterisation $\omega = \arg(1 + ic) \in [0, \pi/8]$ of the electromagnetic deformation parameter. However, and despite the rich structure of new solutions at $c \neq 0$, the question about the eleven-dimensional interpretation of the electromagnetic parameter c remains elusive and various no-go theorems have been stated against the existence of such a higher dimensional origin [13, 14]. Also, for the new supersymmetric

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AdS_4 vacua at $c \neq 0$, the holographic interpretation of the deformation parameter remains obscure from the perspective of the AdS_4/CFT_3 correspondence.

Unlike for the SO(8) theory, much more is by now known about the dyonically-gauged ISO(7) supergravity that arises from the reduction of massive IIA supergravity on a six-sphere S^6 [15]. In this case the electromagnetic deformation parameter is a discrete (on/off) deformation, namely, it can be set to $c = 0$ or 1 without loss of generality [16]. Various AdS_4 [17–19], domain-wall [19, 20], and black hole [21–24] solutions have been constructed which necessarily require a non-zero electromagnetic deformation parameter c. Within this massive IIA context, the electromagnetic parameter is identified with the Romans mass parameter $\hat{F}_0$ of the ten-dimensional theory [25], and has a holographic interpretation as the Chern-Simons level k of a three-dimensional super-Chern-Simons dual theory [26].

The role of the electromagnetic deformation c has been much less investigated in the context of type IIB supergravity. The relevant dyonically-gauged supergravity in this case is the $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes \mathbb{R}^{12}$ theory which arises from the reduction of type IIB supergravity on the product $\mathbb{R} \times S^5$ [27]. As for the ISO(7) theory, the electromagnetic deformation is again a discrete (on/off) deformation, namely, $c = 0$ or 1 [16]. This four-dimensional supergravity has been shown to contain various types of AdS_4 vacua preserving different amounts of supersymmetry as well as of residual gauge symmetry. In particular, an $\mathcal{N} = 4$ and SO(4) symmetric solution was reported in [28] and subsequently, in [27], uplifted to a class of $AdS_4 \times S^1 \times S^5$ S-fold backgrounds of type IIB supergravity using the $E_{7(7)}$ exceptional field theory ($E_{7(7)}$-EFT). These S-folds involve S-duality twists $A_{(k)}$ ($k \geq 3$) that induce $\mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$ monodromies $\mathfrak{M}(k) = -\mathcal{S}\mathcal{T}^k$ of hyperbolic type along S^1, and can be systematically constructed as quotients of degenerate Janus-like solutions of the type IIB theory [29, 30] where the string coupling g_s diverges at infinity. Together with the $\mathcal{N} = 4$&SO(4) solution, additional $\mathcal{N} = 0$&SO(6) [31] and $\mathcal{N} = 1$&SU(3) [32] solutions have been found and uplifted to similar S-fold backgrounds of type IIB supergravity with hyperbolic monodromies in [32]. From a holographic perspective, these AdS_4 vacua describe new strongly coupled three-dimensional CFT's, referred to as J-fold CFT's in [33] (see also [34, 35] and [36]), which are localised on interfaces of $\mathcal{N} = 4$ super-Yang-Mills theory (SYM) [37]. In the $\mathcal{N} = 4$ case [33], a hyperbolic monodromy $J = -\mathcal{S}\mathcal{T}^k \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$ was shown to introduce a Chern-Simons level k in the dual J-fold CFT which, in turn, is constructed from the $T(\mathrm{U}(N))$ theory [38] upon suitable gauging of flavour symmetries. A diagram illustrating this type IIB construction is depicted in figure 1.

On the other hand, a classification of interface SYM theories was performed in [39] (see also [40]) in correspondence to the various amounts of supersymmetry, as well as the largest possible global symmetry, preserved by the interface operators. Three supersymmetric cases were identified: interfaces with $\mathcal{N} = 4$&SO(4) symmetry, $\mathcal{N} = 2$&SU(2) $\times$ U(1) symmetry and $\mathcal{N} = 1$&SU(3) symmetry. While the S-folds in [27] and [32] respectively match the symmetries of the $\mathcal{N} = 4$ and $\mathcal{N} = 1$ cases, the gravity duals of the would be $\mathcal{N} = 2$ J-fold CFT's localised on the interface with SU(2) $\times$ U(1) symmetry remain missing. In this work we fill this gap and present a new family of $AdS_4 \times S^1 \times S^5$ S-folds with $\mathcal{N} = 2$ supersymmetry, SU(2) $\times$ U(1) symmetry and, as in the previous cases, involving S-duality twists that induce monodromies of hyperbolic type along S^1.

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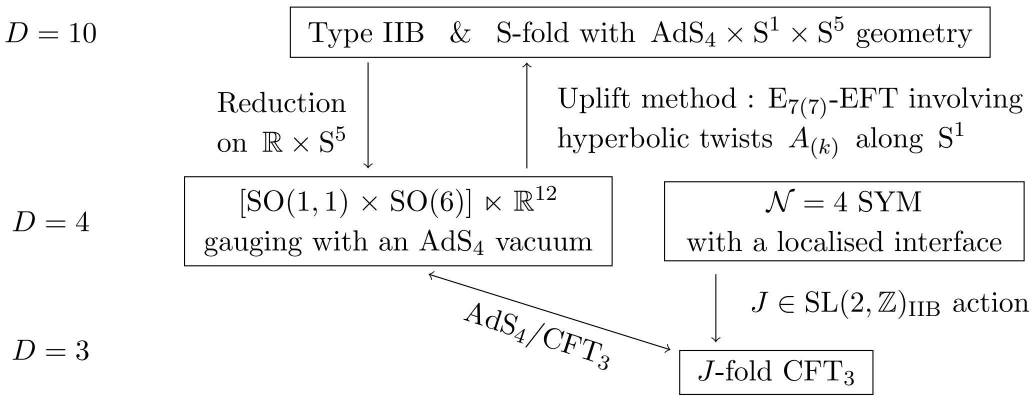

Figure 1. Type IIB S-folds with hyperbolic monodromies $\mathfrak{M}(k) = -\mathcal{S}\mathcal{T}^k$ along S^1 and connection with three-dimensional J-fold CFT's.

The paper is organised as follows. In section 2 we perform a study of multi-parametric families of AdS_4 vacua in the $[SO(1,1) \times SO(6)] \ltimes \mathbb{R}^{12}$ maximal supergravity. We find four families of vacua, one of them being $\mathcal{N} = 2$ supersymmetric and containing a vacuum with a residual symmetry enhancement to $SU(2) \times U(1)$. In section 3, by implementing a generalised Scherk-Schwarz (S-S) ansatz in $E_{7(7)}$-EFT, we uplift such an AdS_4 vacuum to a class of $AdS_4 \times S^1 \times S^5$ $\mathcal{N} = 2$ S-folds of type IIB supergravity with $SU(2) \times U(1)$ symmetry and a non-trivial hyperbolic monodromy along S^1. In section 4 we present our conclusions and discuss future directions.

2 AdS_4 vacua of $[SO(1,1) \times SO(6)] \ltimes \mathbb{R}^{12}$ maximal supergravity

We continue the study of AdS_4 vacua initiated in [31], and further investigated in [28] and [32], for the dyonically-gauged maximal supergravity with non-abelian gauge group

$$G = [SO(1,1) \times SO(6)] \ltimes \mathbb{R}^{12}\,. \tag{2.1}$$

We will show how the AdS_4 vacua of [28, 31, 32] actually correspond to very special points (featuring residual symmetry enhancements) within multi-parametric families of solutions. Each of these families preserves a given amount supersymmetry, namely, $\mathcal{N} = 0, 1, 2$ or 4. More specifically we find:

- A three-parameter family of $\mathcal{N} = 0$&$SU(2)$ symmetric AdS_4 vacua with symmetry enhancements to $SU(2) \times U(1)^2$, $SU(3) \times U(1)$ and $SO(6) \sim SU(4)$ at specific values of the three arbitrary parameters.

- A two-parameter family of $\mathcal{N} = 1$&$U(1)^2$ symmetric AdS_4 vacua with symmetry enhancements to $SU(2) \times U(1)$ and $SU(3)$ at specific values of the two arbitrary parameters.

- A one-parameter family of $\mathcal{N} = 2$&$U(1)^2$ symmetric AdS_4 vacua with a symmetry enhancement to $SU(2) \times U(1)$ at a special value of the arbitrary parameter.

- A single $\mathcal{N} = 4$&$SO(4)$ symmetric AdS_4 vacuum.

The $\mathcal{N}=2$ family of AdS_4 vacua is new and we will uplift the solution with $\mathrm{SU}(2)\times\mathrm{U}(1)$ enhanced residual symmetry to a new and analytic family of S-fold backgrounds of type IIB supergravity in section 3.

2.1 The $\mathcal{N}=8$ theory: gauging and scalar potential

We follow the conventions and notation of [32], which slightly differ from those of [27], to describe the dyonically-gauged maximal supergravity with gauge group G in (2.1). For the purposes of this work, i.e. the study of AdS_4 vacua, we set to zero all the vector and (auxiliary [41]) tensor fields of the theory, so that the bosonic Lagrangian reduces to the following one

$$\mathcal{L}_{\mathcal{N}=8} = \left(\frac{R}{2} - V_{\mathcal{N}=8}\right) * 1 + \frac{1}{96}\mathrm{Tr}\left(dM \wedge *dM^{-1}\right), \tag{2.2}$$

which describes the scalar fields M_{MN} coupled to Einstein gravity in the presence of a scalar potential. The scalar fields serve as coordinates on the coset space of maximal supergravity

$$M_{MN} = \mathcal{V}\mathcal{V}^t \in \frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)}, \tag{2.3}$$

with $M = 1,\ldots,56$ being a fundamental index of $\mathrm{E}_{7(7)}$. The coset representative $\mathcal{V}$ is constructed by direct exponentiation of the 70 non-compact generators $t_A{}^B$ (with $t_A{}^A = 0$) and $t_{ABCD} = t_{[ABCD]}$ generators of $\mathrm{E}_{7(7)}$ in the $\mathrm{SL}(8)$ basis.[1] The scalar potential in (2.2), which survives our truncation to the Einstein-scalar sector, is induced by the gauging of the group G in (2.1) within the maximal theory and has the following general form:

$$V_{\mathcal{N}=8} = \frac{g^2}{672} X_{MN}{}^R X_{PQ}{}^S M^{MP}\left(M^{NQ}M_{RS} + 7\delta_R^Q\delta_S^N\right), \tag{2.4}$$

which depends on the gauge coupling g, the scalar matrix M_{MN} (and its inverse M^{MN}) and on a constant *embedding tensor* $X_{MN}{}^P$ living in the **912** of $\mathrm{E}_{7(7)}$ [43]. This tensor codifies how the gauge group G is embedded into the $\mathrm{E}_{7(7)}$ duality group of maximal supergravity. Moreover, it also specifies the gauge connection which involves both electric and magnetic vector fields transforming under the $\mathrm{Sp}(56)$ group of electromagnetic transformations of the theory (for reviews see [44, 45]).

Under $\mathrm{SL}(8)\subset\mathrm{E}_{7(7)}$ the index M decomposes into antisymmetric pairs ${}_M = ({}_{[AB]}, {}^{[AB]})$ where $A=1,\ldots,8$ denotes a fundamental index of $\mathrm{SL}(8)$. This implies that, for gaugings of subgroups of $\mathrm{SL}(8)$, the non-vanishing electric and magnetic components of the embedding tensor are given by [31]

$$\begin{aligned} \text{electric} &: \quad X_{[AB][CD]}{}^{[EF]} = -X_{[AB]}{}^{[EF]}{}_{[CD]} = -8\delta^{[E}_{[A}\eta_{B][C}\delta^{F]}_{D]}, \\ \text{magnetic} &: \quad X^{[AB]}{}_{[CD]}{}^{[EF]} = -X^{[AB][EF]}{}_{[CD]} = -8\delta^{[A}_{[C}\tilde{\eta}^{B][E}\delta^{F]}_{D]}, \end{aligned} \tag{2.5}$$

in terms of two symmetric matrices η_{AB} and $\tilde{\eta}^{AB}$. For the gauging of $\mathrm{G}\subset\mathrm{SL}(8)$ in (2.1) these are

$$\eta_{AB} = \mathrm{diag}(0, \mathbb{I}_6, 0) \qquad \text{and} \qquad \tilde{\eta}^{AB} = c\ \mathrm{diag}(-1, 0_6, 1)\,. \tag{2.6}$$

[1] We adopt the conventions in the appendix of [42] for the explicit form of the $t_A{}^B$ and t_{ABCD} matrices.

As stated in the introduction, the magnetic part of the embedding tensor in (2.5) allows for an (on/off) electromagnetic parameter c so that $\tilde{\eta}^{AB} \propto c$.

2.2 $\mathbb{Z}_2^3$ invariant sector

In order to efficiently search for extrema of the scalar potential (2.4), we will now construct a $\mathbb{Z}_2^3$ invariant sector of the $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes \mathbb{R}^{12}$ maximal supergravity. This sector can be recast as a minimal $\mathcal{N} = 1$ supergravity coupled to seven chiral multiplets z_i with $i = 1, \ldots, 7$. The same invariant sector has recently been explored in the dyonically-gauged ISO(7) theory [19] and the purely electric SO(8) theory [46], and it originally appeared in the context of type II orientifold compactifications with generalised fluxes [47, 48].

To describe this sector of the maximal theory, we first focus on a four-element Klein subgroup of SL(8). Its action on the fundamental index A is given by

$$\begin{aligned}
\mathbb{Z}_2^{(1)} &: (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \to (x_1, x_2, x_3, -x_4, -x_5, -x_6, -x_7, x_8)\,, \\
\mathbb{Z}_2^{(2)} &: (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \to (x_1, -x_2, -x_3, x_4, x_5, -x_6, -x_7, x_8)\,,
\end{aligned} \tag{2.7}$$

together with the remaining generators $\mathbb{I}$ and $\mathbb{Z}_2^{(1)}\mathbb{Z}_2^{(2)}$. In addition, we will also require invariance under an extra $\mathbb{Z}_2^{*}$ generator acting as

$$\mathbb{Z}_2^{*} : (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \to (x_1, -x_2, x_3, -x_4, x_5, -x_6, x_7, -x_8)\,. \tag{2.8}$$

The resulting $\mathbb{Z}_2^3$ invariant sector describes $\mathcal{N} = 1$ supergravity coupled to seven chiral multiplets (and no vector multiplets)

$$z_i = -\chi_i + i e^{-\varphi_i} \qquad \text{with} \qquad i = 1, \ldots, 7\,. \tag{2.9}$$

The fourteen real spinless fields are associated with generators ${t_A}^B$ (scalars) and $t_{[ABCD]}$ (pseudo-scalars) of $\mathrm{E}_{7(7)}$ in the SL(8) basis. The former have associated generators of the form

$$\begin{aligned}
g_{\varphi_1} &= -{t_1}^1 - {t_2}^2 - {t_3}^3 + {t_4}^4 + {t_5}^5 + {t_6}^6 + {t_7}^7 - {t_8}^8, \\
g_{\varphi_2} &= -{t_1}^1 + {t_2}^2 + {t_3}^3 - {t_4}^4 - {t_5}^5 + {t_6}^6 + {t_7}^7 - {t_8}^8, \\
g_{\varphi_3} &= -{t_1}^1 + {t_2}^2 + {t_3}^3 + {t_4}^4 + {t_5}^5 - {t_6}^6 - {t_7}^7 - {t_8}^8, \\
g_{\varphi_4} &= {t_1}^1 - {t_2}^2 + {t_3}^3 + {t_4}^4 - {t_5}^5 + {t_6}^6 - {t_7}^7 - {t_8}^8, \\
g_{\varphi_5} &= {t_1}^1 + {t_2}^2 - {t_3}^3 - {t_4}^4 + {t_5}^5 + {t_6}^6 - {t_7}^7 - {t_8}^8, \\
g_{\varphi_6} &= {t_1}^1 + {t_2}^2 - {t_3}^3 + {t_4}^4 - {t_5}^5 - {t_6}^6 + {t_7}^7 - {t_8}^8, \\
g_{\varphi_7} &= {t_1}^1 - {t_2}^2 + {t_3}^3 - {t_4}^4 + {t_5}^5 - {t_6}^6 + {t_7}^7 - {t_8}^8,
\end{aligned} \tag{2.10}$$

whereas the latter correspond with generators given by

$$\begin{aligned}
&g_{\chi_1} = t_{1238}, \qquad g_{\chi_4} = t_{2578}, \\
&g_{\chi_2} = t_{1458}, \qquad g_{\chi_5} = t_{4738}, \qquad g_{\chi_7} = t_{8246}\,. \\
&g_{\chi_3} = t_{1678}, \qquad g_{\chi_6} = t_{6358},
\end{aligned} \tag{2.11}$$

Exponentiating (2.10) and (2.11) with coefficients φ_i and χ_i as

$$\mathcal{V} = \text{Exp}\left[-12\sum_{i=1}^{7}\chi_i g_{\chi_i}\right]\text{Exp}\left[\frac{1}{4}\sum_{i=1}^{7}\varphi_i g_{\varphi_i}\right], \tag{2.12}$$

yields a parameterisation of an $M_{MN} = \mathcal{V}\mathcal{V}^t \in [\text{SL}(2)/\text{SO}(2)]^7$ subspace of the coset space in (2.3). The kinetic terms in the resulting $\mathcal{N} = 1$ sector follow from (2.2) and (2.12), and are given by

$$\mathcal{L}_{kin} = -\frac{1}{4}\sum_{i=1}^{7}\left[(\partial\varphi_i)^2 + e^{2\varphi_i}(\partial\chi_i)^2\right]. \tag{2.13}$$

These match the standard kinetic terms $\mathcal{L}_{kin} = -(\partial^2_{z_i,\bar{z}_j}K)dz_i \wedge *d\bar{z}_j$ for a set of seven chiral fields z_i with Kähler potential

$$K = -\sum_{i=1}^{7}\log[-i(z_i - \bar{z}_i)]\,. \tag{2.14}$$

Lastly, when restricted to the $\mathbb{Z}_2^3$ invariant sector entering (2.12), the scalar potential, as computed from (2.4), can be recovered from a holomorphic superpotential

$$W = 2g\big[z_1z_5z_6 + z_2z_4z_6 + z_3z_4z_5 + (z_1z_4 + z_2z_5 + z_3z_6)z_7\big] + 2gc(1 - z_4z_5z_6z_7)\,, \tag{2.15}$$

using the standard $\mathcal{N} = 1$ formula

$$V_{\mathcal{N}=1} = e^K\left[K^{z_i\bar{z}_j}D_{z_i}WD_{\bar{z}_j}\bar{W} - 3W\bar{W}\right], \tag{2.16}$$

where $D_{z_i}W \equiv \partial_{z_i}W + (\partial_{z_i}K)W$ is the Kähler derivative and $K^{z_i\bar{z}_j}$ is the inverse of the Kähler metric $K_{z_i\bar{z}_j} \equiv \partial^2_{z_i,\bar{z}_j}K$. Note that only the last term in the superpotential (2.15) turns out to be sensitive to the electromagnetic parameter c.

2.3 New families of AdS_4 vacua

A thorough study of the structure of extrema of the scalar potential (2.4), restricted to the $\mathbb{Z}_2^3$ invariant sector, reveals a rich structure of (fairly) symmetric AdS_4 vacua. We find four families of vacua preserving $\mathcal{N} = 0, 1, 2$ or 4 supersymmetry as well as various residual gauge symmetries ranging from $\text{U}(1)^2$ to $\text{SO}(6) \sim \text{SU}(4)$. The three supersymmetric families are also supersymmetric within the $\mathcal{N} = 1$ model with seven chirals presented in the previous section, and therefore satisfy the F-flatness conditions

$$D_{z_i}W = 0\,, \tag{2.17}$$

that follow from the superpotential (2.15) and Kähler potential (2.14). Importantly, all the AdS_4 vacua we will present in this section are genuinely dyonic, namely, they disappear if taking the limit $c \to 0$ to a purely electric gauging of G in (2.1).

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2.3.1 $\mathcal{N}=0$ vacua with $\mathrm{SU}(2) \to \mathrm{SU}(2)\times\mathrm{U}(1)^2 \to \mathrm{SU}(3)\times\mathrm{U}(1) \to \mathrm{SO}(6)$ symmetry

There is a three-parameter family of $\mathcal{N}=0$ solutions that preserves SU(2) and is located at

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{1}{\sqrt{2}}\right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = i\,, \tag{2.18}$$

with $\chi_{1,2,3}$ being arbitrary (real) parameters. This family of solutions has a vacuum energy given by

$$V_0 = -2\sqrt{2}g^2c^{-1}\,, \tag{2.19}$$

and a spectrum of normalised scalar masses of the form

$$\begin{aligned} m^2L^2 = {} & 6(\times 2)\,, \quad -3(\times 2)\,, \quad 0(\times 28)\,, \\ & -\frac{3}{4}+\frac{3}{2}\chi^2(\times 2)\,, \\ & -\frac{3}{4}+\frac{3}{2}\left(\chi-2\chi_i\right)^2(\times 2) \qquad i=1,2,3\,, \\ & -\frac{3}{4}+\frac{3}{2}\chi_i^2(\times 4) \qquad i=1,2,3\,, \\ & -3+6\chi_i^2(\times 2) \qquad i=1,2,3\,, \\ & -3+\frac{3}{2}(\chi_i\pm\chi_j)^2(\times 2) \qquad i<j\,, \end{aligned} \tag{2.20}$$

where $\chi \equiv \chi_1+\chi_2+\chi_3$ and $L^2 = -3/V_0$ is the AdS_4 radius. This family of solutions is perturbatively unstable due to the mass eigenvalue -3 lying below the Breitenlohner-Freedman bound for stability in AdS_4 [49]. The computation of the vector masses yields

$$\begin{aligned} m^2L^2 = {} & 0(\times 3)\,, \quad 6(\times 1)\,, \\ & \frac{9}{4}+\frac{3}{2}\chi_i^2(\times 4) \qquad i=1,2,3\,, \\ & \frac{3}{2}\left(\chi_i\pm\chi_j\right)^2(\times 2) \qquad i<j\,. \end{aligned} \tag{2.21}$$

Note that a generic solution in this family preserves an SU(2) symmetry as three vectors are generically massless. Therefore, out of the 28 massless scalars in (2.20), only 3 of them correspond to physical directions in the scalar potential. An additional $\mathrm{U}(1)^2$ factor appears when imposing a pairwise identification between the free axions $\chi_{1,2,3}$, thus resulting in a symmetry enhancement to $\mathrm{SU}(2)\times\mathrm{U}(1)^2$. A further identification $\chi_1=\chi_2=\chi_3\neq 0$ implies a symmetry enhancement to $\mathrm{SU}(3)\times\mathrm{U}(1)$. Lastly, setting $\chi_{1,2,3}=0$ enhances the symmetry to $\mathrm{SU}(4)\sim\mathrm{SO}(6)$. This SO(6) symmetric solution was originally studied in [29] from a ten-dimensional perspective and, more recently, connected with a family of type IIB S-fold backgrounds in [32].

2.3.2 $\mathcal{N}=1$ vacua with $\mathrm{U}(1)^2 \to \mathrm{SU}(2)\times\mathrm{U}(1) \to \mathrm{SU}(3)$ symmetry

There is a two-parameter family of $\mathcal{N}=1$ supersymmetric AdS_4 solutions that preserves $\mathrm{U}(1)^2$ and is located at

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{\sqrt{5}}{3}\right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = \frac{1}{\sqrt{6}}(1+i\sqrt{5})\,, \tag{2.22}$$

subject to the constraint

$$\chi_1 + \chi_2 + \chi_3 = 0\,. \tag{2.23}$$

This family of AdS_4 solutions has a vacuum energy given by

$$V_0 = -\frac{162}{25\sqrt{5}} g^2 c^{-1}\,, \tag{2.24}$$

and a spectrum of normalised scalar masses of the form

$$\begin{aligned} m^2L^2 = \,& 0(\times 28)\,, \quad 4 \pm \sqrt{6}(\times 2)\,, \quad -2(\times 2)\,, \\ & -\frac{14}{9} + 5\chi_i^2 \pm \frac{1}{3}\sqrt{4+45\chi_i^2}(\times 2) \qquad i=1,2,3\,, \\ & -\frac{14}{9} + \frac{5}{4}\chi_i^2 \pm \frac{1}{6}\sqrt{16+45\chi_i^2}(\times 2) \qquad i=1,2,3\,, \\ & \frac{7}{9} + \frac{5}{4}\chi_i^2(\times 2) \qquad i=1,2,3\,, \\ & -2 + \frac{5}{4}\left(\chi_i - \chi_j\right)^2(\times 2) \qquad i<j\,, \end{aligned} \tag{2.25}$$

where $L^2 = -3/V_0$ is the AdS_4 radius. The computation of the vector masses yields

$$\begin{aligned} m^2L^2 = \,& 0(\times 2)\,, \;\; 6(\times 1)\,, \;\; 2(\times 1)\,, \\ & \frac{16}{9} + \frac{5}{4}\chi_i^2 \pm \frac{1}{6}\sqrt{64+45\chi_i^2}(\times 2) \qquad i=1,2,3\,, \\ & \frac{25}{9} + \frac{5\chi_i^2}{4}(\times 2) \qquad i=1,2,3\,, \\ & \frac{5}{4}\left(\chi_i - \chi_j\right)^2(\times 2) \qquad i<j\,. \end{aligned} \tag{2.26}$$

Note that a generic solution in this family preserves $U(1)^2$ as only two vectors are generically massless. Therefore, out of the 28 massless scalars in (2.25), only 2 of them correspond to physical directions in the potential. The residual symmetry gets enhanced to $SU(2) \times U(1)$ when imposing a pairwise identification between the axions $\chi_{1,2,3}$ so that a total of four vectors become massless. Finally there is a symmetry enhancement to $SU(3)$ when setting $\chi_{1,2,3} = 0$ so that a total of eight vectors become massless. The $SU(3)$ symmetric solution was recently uplifted to a ten-dimensional family of type IIB S-fold backgrounds in [32].

2.3.3 $\mathcal{N} = 2$ vacua with $U(1)^2 \to SU(2) \times U(1)$ symmetry

There is a one-parameter family of $\mathcal{N} = 2$ supersymmetric AdS_4 solutions that preserves $U(1)^2$ and is located at

$$z_1 = -\bar{z}_3 = c\left(-\chi + i\frac{1}{\sqrt{2}}\right), \quad z_2 = ic, \quad z_4 = z_6 = i \quad \text{and} \quad z_5 = z_7 = \frac{1}{\sqrt{2}}(1+i)\,. \tag{2.27}$$

This family of AdS_4 solutions has a vacuum energy given by

$$V_0 = -3g^2c^{-1}\,, \tag{2.28}$$

and a spectrum of normalised scalar masses of the form

$$\begin{aligned} m^2L^2 = 0(\times 30)\,, \quad 3\pm\sqrt{17}(\times 2)\,, \quad -2(\times 4)\,, \quad 2(\times 6)\,, \quad -2+4\chi^2(\times 6) \\ -1+4\chi^2\pm\sqrt{16\chi^2+1}(\times 2)\,, \quad \chi^2\pm\sqrt{\chi^2+2}(\times 8)\,, \end{aligned} \tag{2.29}$$

where $L^2 = -3/V_0$ is the AdS_4 radius. The computation of the vector masses yields

$$\begin{aligned} m^2L^2 = 0(\times 2)\,, \quad 6(\times 2)\,, \quad 4(\times 2)\,, \quad 2(\times 4)\,, \\ 4\chi^2(\times 2)\,, \quad 2+\chi^2\pm\sqrt{\chi^2+2}(\times 8)\,. \end{aligned} \tag{2.30}$$

Note that a generic solution in this family preserves $\mathrm{U}(1)^2$ as only two vectors are generically massless. Therefore, out of the 30 massless scalars in (2.29), only 4 of them correspond to physical directions in the scalar potential. However, the residual symmetry gets enhanced to $\mathrm{SU}(2)\times\mathrm{U}(1)$ when $\chi = 0$ and two additional vectors become massless. This special AdS_4 vacuum will be uplifted to a ten-dimensional family of type IIB S-fold backgrounds in section 3.

2.3.4 $\mathcal{N} = 4$ vacuum with SO(4) symmetry

There is an $\mathcal{N} = 4$ supersymmetric AdS_4 solution that preserves SO(4) and is located at

$$z_1 = z_2 = z_3 = ic \qquad \text{and} \qquad z_4 = z_5 = z_6 = -\bar{z}_7 = \frac{1}{\sqrt{2}}(1+i)\,. \tag{2.31}$$

This AdS_4 solution has a vacuum energy given by

$$V_0 = -3g^2c^{-1}\,, \tag{2.32}$$

as for the previous solution, and a spectrum of normalised scalar masses of the form

$$m^2L^2 = 0(\times 48)\,, \quad 10(\times 1)\,, \quad 4(\times 10)\,, \quad -2(\times 11)\,, \tag{2.33}$$

where $L^2 = -3/V_0$ is the AdS_4 radius. The computation of the vector masses yields

$$m^2L^2 = 0(\times 6)\,, \quad 6(\times 7)\,, \quad 2(\times 15)\,, \tag{2.34}$$

thus reflecting the SO(4) residual symmetry at the AdS_4 solution. Therefore, out of the 48 massless scalars in (2.33), only 26 of them correspond to physical directions in the scalar potential. This $\mathcal{N} = 4$ solution was first reported in [28], and then uplifted to a ten-dimensional family of type IIB S-fold backgrounds in [27].

3 S-folds with $\mathcal{N} = 2$ supersymmetry

From this moment on we will set

$$g = c = 1\,, \tag{3.1}$$

without loss of generality. From (2.18), (2.22), (2.27) and (2.31) it becomes clear that varying c amounts to a rescaling of the vacuum expectation values of $z_{1,2,3} \propto c$ at the AdS_4

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vacua. After c has been set to unity, varying g simply corresponds to a rescaling of the vacuum energy $V_0 \propto g^2 c^{-1}$ and thus to a redefinition of the AdS_4 radius $L^2 = -3/V_0$. Let us emphasise again that all the AdS_4 vacua in section 2.3 are genuinely dyonic as they do not survive the limit $c \to 0$ to implement a purely electric gauging. In this limit one has that $\mathrm{Im}(z_{1,2,3}) \to 0$ or, by virtue of (2.9), a runaway behaviour towards the boundary of moduli space $\varphi_{1,2,3} \to \infty$.

Going back to the goal of this section, the $\mathcal{N} = 2$ family of solutions in section 2.3.3 is new and preserves a $\mathrm{U}(1)^2$ symmetry. It is a one-parameter family of AdS_4 vacua and, in the special case of the parameter vanishing $\chi = 0$, there is an enhancement of symmetry to $\mathrm{SU}(2) \times \mathrm{U}(1)$. Following [27], and implementing a generalised S-S ansatz in $\mathrm{E}_{7(7)}$-EFT [50], we will uplift such an $\mathcal{N} = 2$&$\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetric AdS_4 vacuum to a class of ten-dimensional S-fold backgrounds of type IIB supergravity of the form $\mathrm{AdS}_4 \times \mathrm{S}^1 \times \mathrm{S}^5$ with an S-duality hyperbolic monodromy along S^1.

3.1 Type IIB uplift using $\mathbf{E_{7(7)}}$-EFT

Generalised Scherk-Schwarz (S-S) reductions of exceptional field theory (EFT) have proved a very efficient method to perform consistent truncations of eleven-dimensional and type IIB supergravity on spheres and hyperboloids [51]. Here we are interested in the uplift of an AdS_4 vacuum of a four-dimensional gauged maximal supergravity, which thus selects the $\mathrm{E}_{7(7)}$-EFT of [50] as the natural framework to carry out this mission.

The $\mathrm{E}_{7(7)}$-EFT lives in an extended space-time that consists of an external four-dimensional space with coordinates x^μ ($\mu = 0, \ldots, 3$) and a 56-dimensional generalised internal space with coordinates Y^M ($M = 1, \ldots, 56$) in the fundamental representation **56** of $\mathrm{E}_{7(7)}$, subject to the action of the $\mathrm{E}_{7(7)}$-covariant generalised diffeomorphisms. In order to uplift an AdS_4 vacuum amongst those in section 2.3 to a ten-dimensional background of type IIB supergravity, the relevant field content of $\mathrm{E}_{7(7)}$-EFT reduces to the external metric $g_{\mu\nu}(x, Y)$ and the internal generalised metric $\mathcal{M}_{MN}(x, Y)$ (vector and tensor fields are consistently set to zero). These are connected with the metric $g_{\mu\nu}(x)$ and the scalar fields $M_{MN}(x)$ of the four-dimensional maximal supergravity in (2.2) via a generalised S-S ansatz [51]

$$
\begin{aligned}
g_{\mu\nu}(x, Y) &= \rho^{-2}(Y) g_{\mu\nu}(x) \,, \\
\mathcal{M}_{MN}(x, Y) &= U_M{}^K(Y) U_N{}^L(Y) M_{KL}(x) \,.
\end{aligned}
\tag{3.2}
$$

The entire dependence on the Y^M coordinates is then encoded in a twist matrix $U_M{}^K(Y)$ and a scaling function $\rho(Y)$ satisfying

$$
\begin{aligned}
(U^{-1})_M{}^P (U^{-1})_N{}^Q \partial_P U_Q{}^K \big|_{\mathbf{912}} &= \frac{1}{7} \rho X_{MN}{}^K \,, \\
\partial_N (U^{-1})_M{}^N - 3\rho^{-1} \partial_N \rho (U^{-1})_M{}^N &= 2\rho \vartheta_M \,,
\end{aligned}
\tag{3.3}
$$

where $X_{MN}{}^K$ is the embedding tensor specifying the gauging in the four-dimensional supergravity, ϑ_M is a constant scaling tensor and $|_{\mathbf{912}}$ denotes projection onto the **912** irreducible representation of $\mathrm{E}_{7(7)}$ where the embedding tensor lives.

For the dyonic gauging of G $\subset$ SL(8) in (2.1) the non-vanishing components of the embedding tensor were given in (2.5) and the tensor ϑ_M vanishes identically. The generalised S-S ansatz depends on six physical coordinates $(y^i, \tilde{y}) \in Y^M$: five of them are electric y^i $(i = 2, \ldots, 6)$ and one is magnetic $\tilde{y}$. Considering the electric-magnetic splitting of generalised coordinates $Y^M = (Y^{AB}, Y_{AB})$ under $\mathrm{SL}(8) \subset \mathrm{E}_{7(7)}$, one has

$$y^i = Y^{i7} \in Y^{AB} \qquad \text{and} \qquad \tilde{y} = Y_{18} \in Y_{AB} \,. \tag{3.4}$$

In terms of the physical coordinates $(y^i, \tilde{y})$ the scaling function ρ in (3.2)–(3.3) reads

$$\rho(y^i, \tilde{y}) = \hat{\rho}(y^i)\mathring{\rho}(\tilde{y}) \,, \tag{3.5}$$

where the two factors in (3.5) are given by

$$\hat{\rho}^4 = 1 - |\vec{y}|^2 \qquad \text{and} \qquad \mathring{\rho}^4 = 1 + \tilde{y}^2, \tag{3.6}$$

and $\vec{y} \equiv (y^i)$. On the other hand, the generalised twist matrix $(U^{-1})_M{}^N$ in (3.2)–(3.3) is SL(8)-valued and possesses a block diagonal structure

$$(U^{-1})_M{}^N = \begin{pmatrix} (U^{-1})_{[AB]}{}^{[CD]} & 0 \\ 0 & (U^{-1})^{[AB]}{}_{[CD]} = U_{[CD]}{}^{[AB]} \end{pmatrix} , \tag{3.7}$$

with components

$$(U^{-1})_{[AB]}{}^{[CD]} = 2(U^{-1})_{[A}{}^{[C}(U^{-1})_{B]}{}^{D]} \,, \tag{3.8}$$

and

$$(U^{-1})_A{}^B = \left(\frac{\mathring{\rho}}{\hat{\rho}}\right)^{\frac{1}{2}} \begin{pmatrix} 1 & 0 & 0 & \mathring{\rho}^{-2}\tilde{y} \\ 0 & \delta^{ij} + \hat{K} y^i y^j & \hat{\rho}^2 y^i & 0 \\ 0 & \hat{\rho}^2 y^j \hat{K} & \hat{\rho}^4 & 0 \\ \mathring{\rho}^{-2}\tilde{y} & 0 & 0 & \mathring{\rho}^{-4}(1+\tilde{y}^2) \end{pmatrix} . \tag{3.9}$$

The twist matrix in (3.9) also depends on a function $\hat{K}(y^i)$ which is given in this case by a hypergeometric function [27]

$$\hat{K} = -{}_2F_1\left(1, 2, \frac{1}{2}; 1 - |\vec{y}|^2\right) . \tag{3.10}$$

Using the dictionary between the fields of type IIB supergravity and those of $\mathrm{E}_{7(7)}$-EFT [52, 53], together with the S-S ansatz (3.2) involving generalised twist parameters (3.5)–(3.9), one arrives at the final uplift formulae

$$\begin{aligned} G^{mn} &= G^{\frac{1}{2}} \mathcal{M}^{mn} \,, \\ \mathbb{B}_{mn}{}^{\alpha} &= G^{\frac{1}{2}} G_{mp} \epsilon^{\alpha\beta} \mathcal{M}^p{}_{n\beta} \,, \\ C_{klmn} - \frac{3}{2}\epsilon_{\alpha\beta}\mathbb{B}_{k[l}{}^{\alpha}\mathbb{B}_{mn]}{}^{\beta} &= -\frac{1}{2} G^{\frac{1}{2}} G_{kp} \mathcal{M}^p{}_{lmn} \,, \\ m_{\alpha\beta} &= \frac{1}{6} G\left(\mathcal{M}^{mn}\mathcal{M}_{m\alpha n\beta} + \mathcal{M}^m{}_{k\alpha}\mathcal{M}^k{}_{m\beta}\right) , \end{aligned} \tag{3.11}$$

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for the purely internal components of the type IIB fields: (inverse) metric G^{mn}, two-form potentials $\mathbb{B}^{\alpha} = (B_2, C_2)$ with $\alpha = 1, 2$, four-form potential C_4 and axion-dilaton $m_{\alpha\beta}$. The various blocks $\mathcal{M}^{mn}$, $\mathcal{M}^{p}{}_{n\beta}$, $\mathcal{M}^{p}{}_{lmn}$ and $\mathcal{M}_{m\alpha n\beta}$ entering the r.h.s. of (3.11) can be extracted from the internal generalised metric $\mathcal{M}_{MN}(x, y^i, \tilde{y})$ by performing the group-theoretical decomposition that is relevant for the embedding of type IIB supergravity into $\mathrm{E}_{7(7)}$-EFT:

$$\begin{aligned} \mathrm{E}_{7(7)} \supset & \qquad \mathrm{GL}(6) \times \mathrm{SL}(2)_{\mathrm{IIB}} \times \mathbb{R}^{+} \\ \mathbf{56} \; \rightarrow & \; (\mathbf{6}, \mathbf{1})_{+2} + (\mathbf{6'}, \mathbf{2})_{+1} + (\mathbf{20}, \mathbf{1})_{0} + (\mathbf{6}, \mathbf{2})_{-1} + (\mathbf{6'}, \mathbf{1})_{-2} \\ Y^{M} \; \rightarrow & \qquad y^{m} + y_{m\alpha} + y^{mnp} + y^{m\alpha} + y_{m} \end{aligned} \tag{3.12}$$

The physical coordinates are identified as $y^m = (y^i, \tilde{y})$, with $m = (i, 7)$ and $i = 2, \dots, 6$, which implies a further group-theoretical branching $\mathrm{GL}(6) \rightarrow \mathrm{GL}(1) \times \mathrm{GL}(5)$ compatible with the $\mathbb{R}(\text{or } \mathrm{S}^1) \times \mathrm{S}^5$ factorisation of the geometry we are behind of. The various mappings between coordinates discussed above are summarised as

y^m		$y_{m\alpha}$		y^{mnp}		$y^{m\alpha}$		y_m	
y^i	y^7	$y_{i\alpha}$	$y_{7\alpha}$	y^{ijk}	y^{ij7}	$y^{i\alpha}$	$y^{7\alpha}$	y_i	y_7
Y^{i7}	Y_{18}	$Y_{i\alpha}$	$\epsilon_{\alpha\beta} Y^{\beta 7}$	$\epsilon^{ijkj'k'} Y_{j'k'}$	Y^{ij}	$Y^{i\alpha}$	$\epsilon^{\alpha\beta} Y_{\beta 7}$	Y_{i7}	Y^{18}

(3.13)

We refer the reader to the original works [52, 53] (and also [27, 32]) for more details on the generalised S-S reductions of $\mathrm{E}_{7(7)}$-EFT and their connection with the gauged maximal supergravities.

We now move to the uplift of the AdS_4 vacuum with $\mathcal{N} = 2\&\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry discussed in section 2.3.3 to a ten-dimensional background of type IIB supergravity using (3.11). We have explicitly verified that the ten-dimensional equations of motion and Bianchi identities of type IIB supergravity are satisfied.[2]

Ten-dimensional metric. We adopt the conventions of [32] to describe the geometry of the round five-sphere S^5. Using coordinates y^i $(i = 2, \dots, 6)$ to parameterise S^5, the metric and its inverse are given by

$$\hat{G}_{ij} = \delta_{ij} + \frac{\delta_{ik}\delta_{jl} y^k y^l}{1 - y^m \delta_{mn} y^n} \qquad \text{and} \qquad \hat{G}^{ij} = \delta^{ij} - y^i y^j \,. \tag{3.14}$$

However it will also be convenient to introduce a set of embedding coordinates $\mathcal{Y}_{\underline{m}}$ on $\mathbb{R}^6$ $(\underline{m} = 2, \dots, 7)$ of the form

$$\mathcal{Y}_{\underline{m}} = \left\{ y^i, \mathcal{Y}_7 \equiv \left(1 - |\vec{y}|^2\right)^{\frac{1}{2}} \right\} \qquad \text{with} \qquad \delta^{\underline{mn}} \mathcal{Y}_{\underline{m}} \mathcal{Y}_{\underline{n}} = 1 \,, \tag{3.15}$$

so that the Killing vectors on S^5 are constructed as

$$\mathcal{K}_{\underline{mn}}{}^{i} \equiv \hat{G}^{ij} \partial_j \mathcal{Y}_{[\underline{m}} \mathcal{Y}_{\underline{n}]} = \delta^{i}_{[\underline{m}} \mathcal{Y}_{\underline{n}]} \,. \tag{3.16}$$

[2] We adopt the type IIB conventions in the appendix B of [32].

Following the derivation of [27], the internal part of the ten-dimensional metric has components in (3.11) given by

$$
\begin{aligned}
G^{11} &= \Delta \mathring{\rho}^4 M_{1818} = 2\Delta(1+\tilde{y}^2)\,, \\
G^{1k} &= \Delta \mathring{\rho}^2 \mathcal{K}_{\underline{ij}}{}^{k} M^{\underline{ij}}{}_{18} = 0\,, \\
G^{ij} &= \Delta \mathcal{K}_{\underline{kl}}{}^{i} \mathcal{K}_{\underline{mn}}{}^{j} M^{\underline{klmn}} = \Delta\left(\hat{G}^{ij} + L^{ij}\right)\,,
\end{aligned}
\tag{3.17}
$$

where $M^{\underline{ij}}{}_{18} = 0$ as a consequence of having set $\chi = 0$ in the $\mathcal{N} = 2$ AdS_4 vacuum, and where we have defined

$$
L^{ij} = \begin{pmatrix}
\mathcal{Y}_{\underline{4}}^2 + \mathcal{Y}_{\underline{5}}^2 + \mathcal{Y}_{\underline{6}}^2 & -\mathcal{Y}_{\underline{6}}\mathcal{Y}_{\underline{7}} & -\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{4}} & -\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{5}} & -\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{6}} \\
-\mathcal{Y}_{\underline{6}}\mathcal{Y}_{\underline{7}} & \mathcal{Y}_{\underline{4}}^2 + \mathcal{Y}_{\underline{5}}^2 + \mathcal{Y}_{\underline{7}}^2 & -\mathcal{Y}_{\underline{3}}\mathcal{Y}_{\underline{4}} & -\mathcal{Y}_{\underline{3}}\mathcal{Y}_{\underline{5}} & \mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{7}} \\
-\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{4}} & -\mathcal{Y}_{\underline{3}}\mathcal{Y}_{\underline{4}} & 1 - \mathcal{Y}_{\underline{4}}^2 & -\mathcal{Y}_{\underline{4}}\mathcal{Y}_{\underline{5}} & -\mathcal{Y}_{\underline{4}}\mathcal{Y}_{\underline{6}} \\
-\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{5}} & -\mathcal{Y}_{\underline{3}}\mathcal{Y}_{\underline{5}} & -\mathcal{Y}_{\underline{4}}\mathcal{Y}_{\underline{5}} & 1 - \mathcal{Y}_{\underline{5}}^2 & -\mathcal{Y}_{\underline{5}}\mathcal{Y}_{\underline{6}} \\
-\mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{6}} & \mathcal{Y}_{\underline{2}}\mathcal{Y}_{\underline{7}} & -\mathcal{Y}_{\underline{4}}\mathcal{Y}_{\underline{6}} & -\mathcal{Y}_{\underline{5}}\mathcal{Y}_{\underline{6}} & \mathcal{Y}_{\underline{4}}^2 + \mathcal{Y}_{\underline{5}}^2 + \mathcal{Y}_{\underline{2}}^2
\end{pmatrix}\,.
\tag{3.18}
$$

The warping factor Δ in (3.17) is nowhere vanishing and reads

$$
\Delta = (\det G)^{\frac{1}{2}} \rho^2 = \frac{1}{\sqrt{2}} \left(1 + \mathcal{Y}_{\underline{4}}^2 + \mathcal{Y}_{\underline{5}}^2\right)^{-\frac{1}{4}}\,.
\tag{3.19}
$$

The six-dimensional internal metric becomes more transparent if first introducing a new variable for the magnetic coordinate

$$
\tilde{y} = \sinh \eta \qquad \text{with} \qquad \eta \in (-\infty, \infty)\,,
\tag{3.20}
$$

and then a set of angular variables for S^5 of the form

$$
\begin{aligned}
y^2 &= \cos\theta \cos\left(\frac{\beta}{2}\right) \cos\left(\frac{\alpha+\gamma}{2}\right), & y^3 &= \cos\theta \cos\left(\frac{\beta}{2}\right) \sin\left(\frac{\alpha+\gamma}{2}\right)\,, \\
y^4 &= \cos\phi \sin\theta, & y^5 &= \sin\phi \sin\theta, \\
y^6 &= \cos\theta \sin\left(\frac{\beta}{2}\right) \cos\left(\frac{\alpha-\gamma}{2}\right)\,,
\end{aligned}
\tag{3.21}
$$

with ranges given by

$$
\theta \in \left[0, \frac{\pi}{2}\right]\,, \quad \phi \in [0, 2\pi]\,, \quad \alpha \in [0, 2\pi]\,, \quad \beta \in [0, \pi]\,, \quad \gamma \in [0, 2\pi]\,.
\tag{3.22}
$$

In this manner, and upon introducing a set of SU(2) left-invariant one-forms

$$
\begin{aligned}
\sigma_1 &= \frac{1}{2}(-\sin\alpha d\beta + \cos\alpha \sin\beta d\gamma), \\
\sigma_2 &= \frac{1}{2}(\cos\alpha d\beta + \sin\alpha \sin\beta d\gamma), \\
\sigma_3 &= \frac{1}{2}(d\alpha + \cos\beta d\gamma),
\end{aligned}
\tag{3.23}
$$

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the internal six-dimensional metric takes a simple $\mathbb{R}\times \mathrm{S}^5$ form

$$ds_6^2 = \frac{1}{2}\Delta^{-1}\left[d\eta^2 + ds_{\mathrm{S}^2}^2 + \cos^2\theta\, ds_{\mathrm{S}^3}^2\right] , \tag{3.24}$$

with a warping factor

$$\Delta^{-1} = \left(6 - 2\cos(2\theta)\right)^{\frac{1}{4}} , \tag{3.25}$$

and where we have introduced S^2 and (squashed) S^3 metrics to describe the deformation of the internal S^5. These metrics are explicitly given by

$$ds_{\mathrm{S}^2}^2 = d\theta^2 + \sin^2\theta d\phi^2 \qquad \text{and} \qquad ds_{\mathrm{S}^3}^2 = \sigma_2^2 + 8\Delta^4\left(\sigma_1^2+\sigma_3^2\right) . \tag{3.26}$$

Bringing together (3.24) and the external AdS_4 part of the geometry, one obtains a ten-dimensional metric of the form[3]

$$ds^2 = \frac{1}{2}\Delta^{-1}\left[ds_{\mathrm{AdS}_4}^2 + d\eta^2 + ds_{\mathrm{S}^2}^2 + \cos^2\theta\, ds_{\mathrm{S}^3}^2\right] . \tag{3.27}$$

This metric has an $\mathrm{SU}(2)\times \mathrm{U}(1)_\phi \times \mathrm{U}(1)_\sigma$ symmetry, where $\mathrm{U}(1)_\sigma$ acts as a rotation on the (σ_1,σ_3)-plane. Finally, our choice of *undeformed* frames for the metric (3.27) is

$$\begin{array}{llll}
ds_{\mathrm{AdS}_4}^2 : & \hat{e}^0 = \frac{L}{r}dr, & \hat{e}^i = \frac{L}{r}dx^i \quad (i=1,2,3) & \text{and} \quad \eta_{ij} = (-1,1,1) \\
ds_{\mathbb{R}}^2 : & \hat{e}^4 = d\eta & & \\
ds_{\mathrm{S}^2}^2 : & \hat{e}^5 = d\theta, & \hat{e}^6 = \sin\theta d\phi & \\
ds_{\mathrm{S}^3}^2 : & \hat{e}^7 = \sigma_1, & \hat{e}^8 = \sigma_2, \quad \hat{e}^9 = \sigma_3 &
\end{array} \tag{3.28}$$

with $L^2 = -3/V_0 = 1$ being the AdS_4 radius at the four-dimensional $\mathcal{N}=2\&\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetric AdS_4 vacuum.

B_2 and C_2 potentials. The two-form potentials $\mathbb{B}^\alpha = (B_2, C_2)$ in (3.11) transform as a doublet under the global S-duality group $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ of type IIB supergravity. An explicit computation along the lines of [27] shows that

$$\begin{aligned}
\mathbb{B}_{1j}{}^\alpha &= 0 , \\
\mathbb{B}_{ij}{}^\alpha &= \Delta G_{ik}\mathcal{K}_{\underline{kl}}{}^k \partial_j \mathcal{Y}^{\underline{m}} \epsilon^{\alpha\beta}(A^{-1})^\gamma{}_\beta M^{\underline{kl}}{}_{\underline{m}\gamma} ,
\end{aligned} \tag{3.29}$$

in terms of a local $\mathrm{SO}(1,1)\subset \mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ twist matrix

$$A^\alpha{}_\beta \equiv \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} , \qquad (A^{-1})^\gamma{}_\beta \equiv \begin{pmatrix} \cosh\eta & -\sinh\eta \\ -\sinh\eta & \cosh\eta \end{pmatrix} . \tag{3.30}$$

[3] Restoring the explicit dependence of the warping factor (3.25) on the parameter c one finds $\Delta \propto c$, so the (electric) limit $c\to 0$ of the metric (3.27) becomes pathological. In other words, the ten-dimensional solution is genuinely dyonic, namely, it requires $c\neq 0$, as for its associated AdS_4 vacuum in (2.27) with $\chi = 0$.

This matrix encodes the dependence of the two-form potentials on the direction η. Using the scalar block $M^{\underline{kl}}{}_{\underline{m}\gamma}$ at the $\mathcal{N}=2$ AdS_4 vacuum under consideration, and using differential form notation, one finds

$$\mathbb{B}^{\alpha} = A^{\alpha}{}_{\beta}\mathfrak{b}^{\beta}\,, \tag{3.31}$$

with

$$\begin{aligned}\mathfrak{b}^1 &= \frac{1}{\sqrt{2}}\cos\theta\left[\left(\cos\phi d\theta + \frac{1}{2}\sin(2\theta)d(\cos\phi)\right)\wedge\sigma_2 + \cos\phi\frac{4\sin(2\theta)}{6-2\cos(2\theta)}\sigma_1\wedge\sigma_3\right],\\ \mathfrak{b}^2 &= -\frac{1}{\sqrt{2}}\cos\theta\left[\left(\sin\phi d\theta + \frac{1}{2}\sin(2\theta)d(\sin\phi)\right)\wedge\sigma_2 + \sin\phi\frac{4\sin(2\theta)}{6-2\cos(2\theta)}\sigma_1\wedge\sigma_3\right].\end{aligned} \tag{3.32}$$

The two-form potentials in (3.32) preserve $\text{SU}(2)\times\text{U}(1)_\sigma$ but break the $\text{U}(1)_\phi$ factor due to the explicit dependence on the coordinate ϕ.

C_4 potential. The internal component of the four-form potential C_4 can be explicitly obtained from the third uplift formula in (3.11). Computing the associated (purely internal) five-form field strength, and imposing ten-dimensional self-duality, one gets

$$\begin{aligned}\widetilde{F}_5 &= dC_4 - \frac{1}{2}\epsilon_{\alpha\beta}\mathbb{B}^{\alpha}\wedge\mathbb{H}^{\beta}\\ &= (1+\star)\Big[6\sqrt{2}\Delta^{5/2}\text{vol}_{\text{M}_5}\\ &\qquad -4\Delta^4\sin\theta\cos^3\theta d\eta\wedge\left(\cos(2\phi)d\theta - \frac{1}{2}\sin(2\theta)\sin(2\phi)d\phi\right)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big],\end{aligned} \tag{3.33}$$

where

$$\text{vol}_{\text{M}_5} = \sqrt{2}\Delta^{3/2}\sin\theta\cos^3\theta d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\,, \tag{3.34}$$

denotes the volume of the deformed five-sphere. Note that $\text{U}(1)_\phi$ is also broken by $\widetilde{F}_5$ due to its explicit dependence on the coordinate ϕ.

Axion-dilaton. The axion-dilaton matrix $m_{\alpha\beta}$ can be obtained from the last equation in (3.11). Transforming linearly under S-duality, a direct computation shows an explicit dependence of $m_{\alpha\beta}$ on the A-twist in (3.30) of the form

$$m_{\alpha\beta} = \frac{1}{\text{Im}\tau}\begin{pmatrix}|\tau|^2 & -\text{Re}\tau\\ -\text{Re}\tau & 1\end{pmatrix} = (A^{-t})_{\alpha}{}^{\gamma}\mathfrak{m}_{\gamma\delta}(A^{-1})^{\delta}{}_{\beta}\,, \tag{3.35}$$

with $\tau = C_0 + ie^{-\Phi}$ and

$$\mathfrak{m}_{\gamma\delta} = 2\Delta^2\begin{pmatrix}1+\sin^2\theta\cos^2\phi & -\frac{1}{2}\sin^2\theta\sin(2\phi)\\ -\frac{1}{2}\sin^2\theta\sin(2\phi) & 1+\sin^2\theta\sin^2\phi\end{pmatrix}. \tag{3.36}$$

Again $\mathrm{U}(1)_\phi$ is broken by the explicit dependence of (3.36) on the angle ϕ. This concludes the uplift of the AdS_4 vacuum with $\mathcal{N} = 2$ and $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry discussed in section 2.3.3 to a ten-dimensional background of type IIB supergravity. It is worth emphasising that, if trivialising the A-twist in (3.30), i.e. $A^\alpha{}_\beta = \delta^\alpha{}_\beta$, then the ten-dimensional equations of motion of type IIB supergravity are no longer satisfied.

3.2 S-fold interpretation

The dependence of the full type IIB solution on the coordinate η along the $\mathbb{R}$ direction of the geometry (3.27) is totally encoded in the local $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ A-twist in (3.30). This twist matrix is of hyperbolic type and thus induces a non-trivial monodromy

$$\mathfrak{M}_{\mathrm{S}^1} = A^{-1}(\eta) A(\eta + T) = \begin{pmatrix} \cosh T & \sinh T \\ \sinh T & \cosh T \end{pmatrix} , \tag{3.37}$$

when forcing the η coordinate to be periodic $\eta \to \eta + T$ with period T, namely, when replacing $\mathbb{R} \to \mathrm{S}^1$ in the geometry. Generalising the A-twist in (3.30) to a discrete k-family ($k \in \mathbb{N}$ with $k \geq 3$) of new ones

$$A_{(k)} = A g(k) \qquad \text{with} \qquad g(k) = \begin{pmatrix} \dfrac{(k^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{k}{\sqrt{2}(k^2-4)^{\frac{1}{4}}} & \dfrac{\sqrt{2}}{(k^2-4)^{\frac{1}{4}}} \end{pmatrix} , \tag{3.38}$$

the monodromy (3.37) gets generalised to a k-family of $\mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$ hyperbolic monodromies

$$\mathfrak{M}(k) = A^{-1}_{(k)}(\eta) A_{(k)}\big(\eta + T(k)\big) = \begin{pmatrix} k & 1 \\ -1 & 0 \end{pmatrix} \qquad , \qquad k \geq 3 , \tag{3.39}$$

with $T(k) = \log(k + \sqrt{k^2-4}) - \log(2)$ and $\mathrm{Tr}\mathfrak{M}(k) > 2$. Therefore, as discussed in [27] (see also [32]), these backgrounds can be interpreted as locally geometric compactifications on $\mathrm{S}^1 \times \mathrm{S}^5$ involving a k-family of S-duality monodromies (3.39). These monodromies can be written as

$$\mathfrak{M}(k) = -\mathcal{S}\mathcal{T}^k \qquad \text{with} \qquad \mathcal{S} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathcal{T} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} , \tag{3.40}$$

and thus define a k-family of S-fold backgrounds. Moreover, the argument wielded in [27] for the straightforward uplift of the four-dimensional supersymmetries to ten dimensions relied on the monodromy (3.37) being in the hyperbolic conjugacy class of $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$. This is still our case, so the S-folds presented here preserve $\mathcal{N} = 2$ supersymmetry.

Lastly, various holographic aspects of both $\mathcal{N} = 4$ [27] and $\mathcal{N} = 1$ [32, 36] S-folds with hyperbolic monodromies have respectively been investigated in [33–35] and [36] within the context of three-dimensional quiver theories involving $\mathcal{N}=4$ $T(\mathrm{U}(N))$ theories [38], and their potential generalisation to $\mathcal{N}=1$ SCFT's. It would be interesting to extend these holographic studies to the $\mathcal{N}=2$ S-folds with hyperbolic monodromies (3.39) presented in this work.

3.3 Connection with Janus-like solutions

The type IIB solution with $\mathcal{N}=2\&\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry we just obtained can be mapped to a new (but equivalent) solution with a linear dilaton profile along the coordinate η upon performing a global $\Lambda\in\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ transformation, equivalently a change of duality frame, based on the matrix element

$$\Lambda=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}. \tag{3.41}$$

The composed action of $\Lambda A^{-1}(\eta)$ on (3.36) yields a shift of the form $\Phi\to\Phi-2\eta$. Therefore, a degenerate Janus-like behaviour with a linear dilaton Φ running from $-\infty$ to ∞ becomes manifest

$$g_s=e^{\Phi}\propto e^{-2\eta}, \tag{3.42}$$

giving rise to a varying string coupling g_s that interpolates between the singular values 0 and ∞.

Upon performing the $\Lambda\in\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ transformation (3.41) on the original solution found in section 3.1, a new type IIB background is generated. The metric and self-dual five-form flux are $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$ singlets and are not affected by the transformation. Therefore, they take the same form as in (3.27) and (3.33), namely,

$$\begin{aligned}
ds^2&=\frac{1}{2}\Delta^{-1}\left[ds^2_{\mathrm{AdS}_4}+d\eta^2+d\theta^2+\sin^2\theta\, d\phi^2+\cos^2\theta\left(\sigma_2^2+8\Delta^4\left(\sigma_1^2+\sigma_3^2\right)\right)\right],\\
\widetilde{F}_5&=4\Delta^4\sin\theta\cos^3\theta(1+\star)\Big[3d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\\
&\qquad-d\eta\wedge\left(\cos(2\phi)d\theta-\frac{1}{2}\sin(2\theta)\sin(2\phi)d\phi\right)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big].
\end{aligned} \tag{3.43}$$

The axion-dilaton matrix $m_{\alpha\beta}$ in (3.35) transforms linearly under $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$. Reading off the new components of τ one finds

$$\begin{aligned}
\Phi&=-2\eta+\log\left[\frac{1}{2}\Delta^2\left(5-\cos(2\theta)-2\sin^2\theta\sin(2\phi)\right)\right],\\
C_0&=-2e^{2\eta}\frac{\cos(2\phi)\sin^2\theta}{5-\cos(2\theta)-2\sin^2\theta\sin(2\phi)}.
\end{aligned} \tag{3.44}$$

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The two-form potentials $\mathbb{B}^{\alpha} = (B_2, C_2)$ in (3.31)–(3.32) transform as an $\text{SL}(2,\mathbb{R})_{\text{IIB}}$ doublet and take the new form[4]

$$\begin{aligned} B_2 &= e^{-\eta}\Big[\frac{1}{2}\cos\theta\big((\cos\phi+\sin\phi)d\theta+\frac{1}{2}\sin(2\theta)(\cos\phi-\sin\phi)d\phi\big)\wedge\sigma_2 \\ &\qquad + 2\Delta^4\cos\theta\sin(2\theta)(\cos\phi+\sin\phi)\sigma_1\wedge\sigma_3\Big]\,, \\ C_2 &= e^{\eta}\Big[\frac{1}{2}\cos\theta\big((\cos\phi-\sin\phi)d\theta-\frac{1}{2}\sin(2\theta)(\cos\phi+\sin\phi)d\phi\big)\wedge\sigma_2 \\ &\qquad + 2\Delta^4\cos\theta\sin(2\theta)(\cos\phi-\sin\phi)\sigma_1\wedge\sigma_3\Big]\,. \end{aligned} \tag{3.45}$$

The nowhere vanishing warping factor still reads

$$\Delta^{-4} = 6 - 2\cos(2\theta)\,. \tag{3.46}$$

In the asymptotic region at $\eta \to -\infty$ one has that g_s in (3.42) diverges (strong coupling) and B_2 dominates over other gauge potentials, e.g., $C_0 \to 0$ and $C_2 \to 0$. On the contrary, in the asymptotic region at $\eta \to \infty$, the solution becomes dominated by C_0 and C_2 whereas $g_s \to 0$ (weak coupling) and $B_2 \to 0$. At intermediate values of the coordinate η one has an interpolating behaviour between these two regimes. Finally, it is also worth noticing that, unlike for the $\mathcal{N}=4$ [27] and $\mathcal{N}=1$ [32] S-folds, there is no $\text{SL}(2,\mathbb{R})_{\text{IIB}}$ frame in which the axion C_0 (and thus the dual θ-angle) vanishes identically or becomes independent of the coordinate η.

4 Conclusions

In this work we have extended the study of AdS_4 vacua in [28, 31, 32] for the dyonically-gauged $[\text{SO}(1,1)\times\text{SO}(6)]\ltimes\mathbb{R}^{12}$ maximal supergravity and found multi-parametric families of new AdS_4 vacua. Within one such families, all the solutions preserve the same amount of supersymmetry but, importantly, residual symmetry enhancements occur at particular values of the parameters. The previously known $\mathcal{N}=0\,\&\,\text{SO}(6)$ [31], $\mathcal{N}=1\,\&\,\text{SU}(3)$ [32] and $\mathcal{N}=4\,\&\,\text{SO}(4)$ [28] AdS_4 vacua are shown to correspond to the points of largest symmetry enhancement within their respective families. This is in line with the analysis of (global) symmetry breaking patterns of three-dimensional interface SYM theories presented in [39].

In the second part of the paper we focused on the new family of $\mathcal{N}=2$ supersymmetric AdS_4 vacua and, more concretely, on the vacuum within this family featuring the largest possible residual symmetry, which turns to be $\text{SU}(2)\times\text{U}(1)$. By implementing a generalised

[4] The two terms in B_2 and C_2 which are proportional to $\sigma_1\wedge\sigma_3$ can be eliminated by means of a gauge transformation of the form

$$\begin{aligned} B_2 &\to B_2 - d\big(2\sqrt{2}\Delta^4 e^{-\eta}\sin(2\theta)\cos\theta\cos\psi\sigma_2\big)\,, \\ C_2 &\to C_2 + d\big(2\sqrt{2}\Delta^4 e^{\eta}\sin(2\theta)\cos\theta\sin\psi\sigma_2\big)\,, \end{aligned}$$

where we have shifted the coordinate $\phi \to \psi + \frac{\pi}{4}$. However, since these terms are generated by the generalised S-S ansatz discussed in section 3.1, we will retain them here.

S-S ansatz in $E_{7(7)}$-EFT, we uplifted the AdS_4 vacuum to a new family of $AdS_4 \times S^1 \times S^5$ S-folds of type IIB supergravity with hyperbolic monodromies $\mathfrak{M}(k) = -\mathcal{S}\mathcal{T}^k$ (with $k \geq 3$) along S^1. The residual $SU(2) \times U(1)$ symmetry and $\mathcal{N} = 2$ supersymmetry of the AdS_4 vacuum are realised on the S-folds: the internal S^5 is deformed into a product of S^2 and (squashed) S^3 with $SU(2) \times U(1)_\sigma \times U(1)_\phi$ isometries and a warping factor, whereas the background fluxes break the $U(1)_\phi$ factor explicitly by introducing a dependence on the coordinate ϕ. In many aspects, the realisation of symmetries is much alike the $AdS_5 \times S^5$ background by Pilch and Warner [54] that uplifts the $\mathcal{N} = 2$ and $SU(2) \times U(1)$ symmetric AdS_5 vacuum of the five-dimensional SO(6) maximal supergravity presented in [55].

Finally it would be interesting to investigate the brane setups underlying the families of S-folds presented here (and in [32]), especially due to the non-trivial $SL(2,\mathbb{Z})_{IIB}$ hyperbolic monodromies $\mathfrak{M}(k) = -\mathcal{S}\mathcal{T}^k$. It would also be interesting to investigate holographic aspects of such $\mathcal{N} = 2$ and $\mathcal{N} = 1$ S-folds (in the spirit of the J-fold CFT's of [33–36] with $J = -\mathcal{S}\mathcal{T}^k$), as well as to study holographic RG flows by explicitly constructing domain-wall solutions interpolating between the various families of AdS_4 vacua presented in this work. Lastly, since the S-folds here and in [32] display SU(2) isometries in the internal geometry, it would also be interesting to apply non-abelian T-duality in order to generate new analytic type IIA backgrounds. We plan to address these and related issues in the future.

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