Local affection of weak gravitational field from supercondensates

Local affection of weak gravitational field from supercondensates

Antonio Gallerati

Politecnico di Torino, Dipartimento di Scienza Applicata e Tecnologia, corso Duca degli Abruzzi 24, 10129 Torino, Italy

Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via Pietro Giuria 1, 10125 Torino, Italy

antonio.gallerati@polito.it

Abstract

We study the mutual interaction between a superconducting sample and the weak, static Earth's gravitational field, exploiting the gravito-Maxwell formalism combined with the time-dependent Ginzburg-Landau model. We will also determine the appropriate conditions to enhance the desired gravity/superfluid interplay, analysing the effects of thermal fluctuations and optimizing the superconductor parameters and sample geometry.

Contents

1 Introduction

The possible interaction between superconductors and gravitational field is an intriguing field of research, providing an interesting connection between condensed matter systems and gravitational interaction, with beneficial effects both in theoretical and applied physics. The seminal paper [1] set the stage for a deeper analysis of the phenomenon, while, in the following years, a certain amount of scientific literature on the subject was produced [2–21]. The underlying idea behind this line of research is that, under certain conditions, a gravity/supercondensate interplay should exist, resulting in a slight affection of the local gravitational field through the interaction with suitable condensate systems. Finally, in 1992 Podkletnov and Nieminen proposed a laboratory experimental configuration to detect the conjectured mutual interplay [22, 23].

The above ideas led many researchers to the discuss various theoretical explanation of the described effect. First of all, it is clear that there are no dielectric-like effects that can affect the gravitational interaction, since, in the standard classical picture, there are no charges of (gravitational) opposite sign to be redistributed in the medium to counteract the external field. If, on the other hand, we consider the medium as a standard quantum system, we find a suppressed probability of (graviton) excitation of a medium particle, due to the very small gravitational coupling. The only possibility left is therefore to consider an interplay with an unconventional state of matter, like a Bose–condensate or a more general superfluid, the hypothetical effect consisting in some kind of interaction between the gravitational field and the superfluid constituents, claiming for a more specific quantum description of the phenomenon.

The clearest theoretical interpretation of the proposed interaction dates 1996 [24] and was obtained exploiting a quantum gravity model coupling the superfluid to the gravitational field. Let us briefly discuss this approach.

Quantum gravity framework. Let us imagine we have a superconducting sample immersed in a weak gravitational background. The corresponding metric $g_{\mu\nu}(x)$ can be expanded as

$$g_{\mu\nu}(x) \;=\; \eta_{\mu\nu} + h_{\mu\nu}(x)\,, \tag{1}$$

sum of the flat Lorentz background $\eta_{\mu\nu}$ plus small fluctuations, encoded in the $h_{\mu\nu}(x)$ contribution. The Cooper pairs inside the sample give rise to a Bose condensate, that we can describe by means of a bosonic field ϕ having non-vanishing vacuum expectation value (v.e.v.) $\phi_0 = \langle 0|\phi|0\rangle$.

The total Lagrangian describing the quantum system has the form

$$\mathscr{L} \;=\; \mathscr{L}_{\rm EH} \;+\; \mathscr{L}_\phi\;. \tag{2}$$

The first term gives the standard Einstein-Hilbert contribution

$$\mathscr{L}_{\rm EH} \;=\; \frac{1}{8\pi \mathrm{G}}\;(R - 2\,\Lambda)\;, \tag{3}$$

where R is the Ricci scalar and Λ the cosmological constant[1]. The second term describes the coupling of the bosonic field ϕ to gravity and is written as:

$$\mathscr{L}_\phi \;=\; -\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi^*\,\partial_\nu\phi + \frac{1}{2}\,m^2\,\phi^*\phi\,, \tag{4}$$

[1] we work in the "mostly plus" convention, $\eta = \mathrm{diag}(-1,+1,+1,+1)$, and set $c = \hbar = 1$

m being the mass of the Cooper pair [24].

If we now expand the bosonic field as $\phi = \phi_0 + \bar{\phi}$, the ϕ_0 v.e.v. can be considered as an external source, depending on the sample characteristics and on the (possible) presence of external electromagnetic fields, while the $\bar{\phi}$ component is included in the integration variables. The $\mathscr{L}_\phi$ contribution can be then expanded as

$$\mathscr{L}_\phi \,=\, \mathscr{L}_{\bar{\phi}} \,+\, \mathscr{L}_h \,+\, \mathscr{L}_0 \,. \tag{5}$$

The first term of the above expansion includes contributions coming from the $\bar{\phi}$ components; these contributions are involved in (negligible) graviton emission-absorption processes and can be then dropped in the Lagrangian. The second term describes the interaction between the $h_{\mu\nu}$ metric fluctuations and the condensate, with the form

$$\mathscr{L}_h \,\propto\, h^{\mu\nu}\, \partial_\mu {\phi_0}^* \, \partial_\nu \phi_0 \,, \tag{6}$$

and determines corrections to the gravitational propagator, which are again irrelevant. Finally, we have the term

$$\mathscr{L}_0 \,=\, -\frac{1}{2}\, \partial_\mu {\phi_0}^* \, \partial^\mu \phi_0 + \frac{1}{2}\, m^2 \, |\phi_0|^2 \,\,, \tag{7}$$

which turns out to be a superfluid contribution to the total effective cosmological term.

The coupling described by the above eq. (7) may then produce localized instabilities in superfluid regions with larger condensate density, giving rise to observable effects [24, 25]. In these particular regions, some physical cutoff should be at work, preventing arbitrary growth of the gravitational field, while the physical effect on field propagation and static potential turns out to be a kind of slight partial shielding. The introduced superfluid density $\phi_0(x)$ is related to the internal (microscopic) structure of the involved sample, as well as to the presence of magnetic fields and currents in the supercondensate.

Although being a solid and elegant formulation offering a general, theoretical explanation for the described interplay, the described quantum gravity approach involves a formalism that makes it hard to extract quantitative predictions. One is then led to consider also alternative, phenomenological researches to better understand the proposed interplay, trying to obtain a formulation leading to more explicit experimental predictions.

Generalized EM fields. Parallel to DeWitt (and subsequent) studies about the coupling between supercondensates and gravity, other theoretical [26, 27] and experimental [28–30] researches were conducted about generalized electric-type fields induced in (super)conductors by the presence of the Earth's weak gravitational field. The main result of those studies was the introduction of a fundamental, generalized electric-type field, featuring an electrical component and a gravitational one, leading to detectable corrections to the free fall of charged particles.

In the following, we are going to obtain the same results in a more general formulation, making use of the gravito-Maxwell formalism [31–33]. This approach exploits the introduction of generalized Maxwell fields, featuring both electromagnetic and gravitational components. These new generalized fields are involved in quantum effects originating from the interaction with the weak gravitational background. The formalism turns out to be powerful in the study of the gravity/superconductivity interplay, since the emerging formal analogy between the Maxwell and weak gravity equations allows to use the Ginzburg–Landau model for the description of the physics, resulting in a mean-field theory for the system thermodynamics including the effects of thermal fluctuations. In particular, we will analyse how the local gravitational field can be affected by the presence of the supercondensate, using the time-dependent Ginzburg–Landau equations in the regime of fluctuations. We will also study which parameters could be optimized to enhance the desired effect, choosing appropriate sample physical characteristics, geometrical properties and range of temperatures.

2 Weak static gravitational field expansion

Let us now briefly derive the generalized form of fields and potential coming from the gravito-Maxwell formulation in weak field approximation.

Linearized gravity. Let us consider a nearly-flat, static spacetime configuration. The associated metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ can be expanded as

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} \,, \qquad\qquad g^{\mu\nu} \simeq \eta^{\mu\nu} - h^{\mu\nu} \,, \tag{8}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$ is the flat Minkowski metric in the "mostly plus convention" and $h_{\mu\nu}$ is a small perturbation. If we consider an inertial coordinate system, the Ricci tensor (see Appendix A) in the weak field limit can be expanded as [34]

$$R_{\mu\nu} \simeq \partial^\rho \partial_{(\mu} h_{\nu)\rho} - \frac{1}{2}\,\partial^2 h_{\mu\nu} - \frac{1}{2}\,\partial_\mu \partial_\nu h \,, \tag{9}$$

where $h = h^\sigma{}_\sigma$. The Einstein equation are written as [34, 35]

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R \;=\; 8\pi \mathrm{G}\;T_{\mu\nu} \,, \tag{10}$$

where the l.h.s. second term in linear approximation reads

$$-\,\frac{1}{2}\,g_{\mu\nu}\,R \;\simeq\; -\frac{1}{2}\,\eta_{\mu\nu}\,\left(\partial^\rho \partial^\sigma h_{\rho\sigma} - \partial^2 h\right) \,, \tag{11}$$

$R = g^{\mu\nu} R_{\mu\nu}$ being the Ricci scalar. In first order approximation, equations (10) can be then rewritten as

$$\partial^\rho \partial_{(\mu} h_{\nu)\rho} - \frac{1}{2}\,\partial^2 h_{\mu\nu} - \frac{1}{2}\,\partial_\mu \partial_\nu h - \frac{1}{2}\,\eta_{\mu\nu}\left(\partial^\rho \partial^\sigma h_{\rho\sigma} - \partial^2 h\right) \;=\; 8\pi \mathrm{G}\;T_{\mu\nu} \,. \tag{12}$$

Let us now introduce the traceless symmetric tensor

$$\bar{h}_{\mu\nu} \;=\; h_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\,h \,, \tag{13}$$

so that the linearized equations (12) can be rewritten as

$$\frac{1}{2}\left(\partial^\rho \partial_\mu \bar{h}_{\nu\rho} + \partial^\rho \partial_\nu \bar{h}_{\mu\rho} - \partial^\rho \partial_\rho \bar{h}_{\mu\nu} - \eta_{\mu\nu}\,\partial^\rho \partial^\sigma \bar{h}_{\rho\sigma}\right) \;=\; \partial^\rho \left(\partial_{[\nu} \bar{h}_{\rho]\mu} + \partial^\sigma \eta_{\mu[\rho}\,\bar{h}_{\nu]\sigma}\right) \;=\; 8\pi \mathrm{G}\;T_{\mu\nu} \,, \tag{14}$$

corresponding to the compact form [31–33]

$$\partial^\rho \mathscr{G}_{\mu\nu\rho} \;=\; 8\pi \mathrm{G}\;T_{\mu\nu} \,, \tag{15}$$

once introduced the tensor

$$\mathscr{G}_{\mu\nu\rho} \;\equiv\; \partial_{[\nu}\bar{h}_{\rho]\mu} + \partial^{\sigma}\eta_{\mu[\rho}\,\bar{h}_{\nu]\sigma}\,. \tag{16}$$

Gravito-Maxwell equations. We now decide to work in the De Donder gauge[2] $g^{\mu\nu}\,\Gamma^{\lambda}{}_{\mu\nu} = 0$. The latter implies the so-called *Lorentz condition*:

$$\partial^{\mu}\bar{h}_{\mu\nu} \;\simeq\; 0\,, \tag{17}$$

that further simplifies expression (16) for $\mathscr{G}_{\mu\nu\rho}$, giving

$$\mathscr{G}_{\mu\nu\rho} \;\simeq\; \partial_{[\nu}\bar{h}_{\rho]\mu}\,. \tag{18}$$

We then introduce the fields

$$\begin{aligned} \mathbf{E}_{\rm g} \;&=-\frac{1}{2}\,\mathscr{G}_{00i} \;=-\frac{1}{2}\,\partial_{[0}\bar{h}_{i]0}\,, \qquad \mathbf{A}_{\rm g} \;=\; \frac{1}{4}\,\bar{h}_{0i}\,,\\ \mathbf{B}_{\rm g} \;&=\; \frac{1}{4}\,\varepsilon_i{}^{jk}\,\mathscr{G}_{0jk} \;=\; \varepsilon_i{}^{jk}\,\partial_j A_k \;=\; \nabla\times\mathbf{A}_{\rm g}\,, \end{aligned} \tag{19}$$

that satisfy the set of equations [31–33, 36]:

$$\begin{aligned} &\nabla\cdot\mathbf{E}_{\rm g} = 4\pi{\rm G}\,\rho_{\rm g}\,, &&\nabla\cdot\mathbf{B}_{\rm g} = 0\,,\\ &\nabla\times\mathbf{E}_{\rm g} = -\frac{\partial\mathbf{B}_{\rm g}}{\partial t}\,, &&\nabla\times\mathbf{B}_{\rm g} = 4\pi{\rm G}\,\frac{1}{c^2}\,\mathbf{j}_{\rm g} + \frac{1}{c^2}\,\frac{\partial\mathbf{E}_{\rm g}}{\partial t}\,, \end{aligned} \tag{20}$$

having restored physical units and introduced the mass density $\rho_{\rm g} \equiv -T_{00}$ and the mass current density $\mathbf{j}_{\rm g} \equiv T_{0i}$.

As we can appreciate, the above (20) have the same formal structure of the Maxwell equations, once interpreted $\mathbf{E}_{\rm g}$ and $\mathbf{B}_{\rm g}$ as the gravitoelectric and gravitomagnetic field, respectively. For instance, for an observer on the Earth's surface, $\mathbf{E}_{\rm g}$ corresponds to the standard (Newtonian) gravity acceleration, while the $\mathbf{B}_{\rm g}$ field is related to angular momentum interactions [15, 31, 37–40].

Generalized fields and equations. Now we introduce the generalized electric/magnetic fields, scalar and vector potentials, featuring both electromagnetic and gravitational contributions:

$$\mathbf{E} = \mathbf{E}_{\rm e} + \frac{m}{e}\,\mathbf{E}_{\rm g}\,, \qquad \mathbf{B} = \mathbf{B}_{\rm e} + \frac{m}{e}\,\mathbf{B}_{\rm g}\,, \qquad \phi = \phi_{\rm e} + \frac{m}{e}\,\phi_{\rm g}\,, \qquad \mathbf{A} = \mathbf{A}_{\rm e} + \frac{m}{e}\,\mathbf{A}_{\rm g}\,, \tag{21}$$

where m and e identify the electron mass and charge, respectively. We then obtain the *generalized Maxwell equations* for the new fields, reading [31–33, 36, 41]:

$$\begin{aligned} &\nabla\cdot\mathbf{E} = \left(\frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_{\rm g}}\right)\rho\,, &&\nabla\cdot\mathbf{B} = 0\,,\\ &\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}\,, &&\nabla\times\mathbf{B} = (\mu_0 + \mu_{\rm g})\,\mathbf{j} + \frac{1}{c^2}\,\frac{\partial\mathbf{E}}{\partial t}\,, \end{aligned} \tag{22}$$

[2] this is equivalent to the harmonic coordinate condition $\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\right) = 0 \;\;\Leftrightarrow\;\; \Box x^\mu = 0$

where ε_0 and μ_0 are the standard vacuum electric permittivity and magnetic permeability, while ρ and $\mathbf{j}$ give the electric charge density and electric current density, respectively. We have also introduced the mass density ρ_{g} and the mass current density vector $\mathbf{j}_{\mathrm{g}}$, that we write in terms of ρ and $\mathbf{j}$ as

$$\rho_{\mathrm{g}} = \frac{m}{e}\,\rho\,, \qquad \mathbf{j}_{\mathrm{g}} = \frac{m}{e}\,\mathbf{j}\,. \tag{23}$$

Finally, we obtain for the vacuum *gravitational* permittivity ε_{g} and permeability μ_{g} the expressions

$$\varepsilon_{\mathrm{g}} = \frac{1}{4\pi \mathrm{G}}\,\frac{e^2}{m^2}\,, \qquad \mu_{\mathrm{g}} = \frac{4\pi \mathrm{G}}{c^2}\,\frac{m^2}{e^2}\,. \tag{24}$$

3 Thermodynamic fluctuations. Ginzburg-Landau formulation

Let us now imagine we have a superconductive sample in the vicinity of its critical temperature T_{c}. At the microscopical level, the superfluid undergoes thermodynamic fluctuations of the order parameter $\psi(\mathbf{x},t)$ describing superconducting electrons[3], giving rise to localized regions of accelerated charge carriers. For $T > T_{\mathrm{c}}$, the average size of the generated unstable regions is greater than the mean electron free path, causing in turn an increase of the sample resistivity[4]. This physical situation can be described, for T slightly greater than T_{c}, by using the time-dependent Ginzburg-Landau equations [42], that can be written in the gauge-invariant form [43, 44]:

$$\Gamma\,(\hbar\,\partial_t - 2\,i\,e\,\phi)\,\psi \;=\; \frac{1}{2m}\,(\hbar\,\nabla\phi - 2\,i\,e\,\mathbf{A})^2\,\psi + \alpha\,\psi \qquad (T > T_{\mathrm{c}})\,. \tag{25}$$

In the above expression, $\psi(\mathbf{x},t)$ is the order parameter, $\phi(\mathbf{x},t)$ is the electric potential and $A(\mathbf{x},t)$ is the vector potential, and we also introduce the following quantities:

$$\Gamma = \frac{\alpha}{\epsilon(T)}\,\frac{\pi}{8\,\mathrm{k_B}\,T_{\mathrm{c}}}\,, \qquad \epsilon(T) = \sqrt{\frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}}}\,, \qquad \xi(T) = \frac{\xi_0}{\sqrt{\epsilon(T)}}\,, \qquad \alpha = \frac{\hbar^2}{2\,m\,\xi(T)^{\,2}}\,, \tag{26}$$

ξ_0 being the BCS intrinsic coherence length, roughly characterizing the smallest size of a wave packet formed by superconducting charge carriers. The temperature-dependent Ginzburg-Landau coherence length $\xi(T)$ gives instead a measure of the distance over which the order parameter can vary without undue energy increase, for a given temperature T.

Let us now make the following ansatz for the solution:

$$\psi(\mathbf{x},t) \;=\; f(\mathbf{x},t)\,\exp\big(i\,g(\mathbf{x},t)\big)\,. \tag{27}$$

We then obtain from (25) the relations

$$\Gamma\,\hbar\,\frac{\partial f}{\partial t} \;=\; \alpha\,f - \frac{1}{2} m\,v_{\mathrm{s}}^2\,f + \frac{\hbar^2}{2m}\,\Delta f\,, \tag{28.i}$$

$$\Gamma\,\hbar\,f\,\frac{\partial g}{\partial t} \;=\; 2\,e\,\Gamma\,\phi\,f - \frac{\hbar^2}{2m}\,f\,\Delta g - 2\,\hbar\,\mathbf{v}_{\mathrm{s}}\cdot\nabla f\,, \tag{28.ii}$$

[3] from a physical point of view, ψ can be thought as the the pseudowavefunction for the center of mass motion of the Cooper pairs

[4] we suppose we work with sufficiently dirty materials, so that the effects of the fluctuations can be observed over a sizable range of temperature; this can be achieved if the electronic mean free path of the material in the normal state is $< 10\,\text{Å}$

where the superfluid speed $\mathbf{v}_{\mathrm{s}}$ is expressed as

$$\mathbf{v}_{\mathrm{s}} \;=\; \frac{1}{m}\left(\hbar\,\nabla g + 2\,\frac{e}{c}\,\mathbf{A}\right)\;, \tag{29}$$

while the associated (super)current density $\mathbf{j}_{\mathrm{s}}$ is given by

$$\mathbf{j}_{\mathrm{s}} \;=-2\,\frac{e}{m}\,|\psi|^2\left(h\,\nabla g + 2\,\frac{e}{c}\,\mathbf{A}\right)\;=-2\,e\,f^2\,\mathbf{v}_{\mathrm{s}}\,. \tag{30}$$

Fluctuations. The presence of a thermal energy (of the order of $\sim \mathrm{k_B}\,T$) implies that the system will fluctuate in different low-lying states with non-zero probability. Let us denote f_k the value of the f function for a fluctuation of the wave vector $\mathbf{k}$. We can then recast the above (28) as

$$\Gamma\,\hbar\,\frac{\partial f_k}{\partial t} \;=\; \alpha\,f_k - \frac{\hbar^2}{2m}\,k^2\,f_k - \frac{1}{2}\,m\,v_{\mathrm{s}}^2\,f_k\;, \tag{31.i}$$

$$\frac{\partial \mathbf{v}_{\mathrm{s}}}{\partial t} \;=\; -2\,\frac{e}{m}\,\mathbf{E} \tag{31.ii}$$

where the last equation is found using expressions eq. (29), (28.ii) and

$$\nabla\phi = -\mathbf{E} - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\;. \tag{32}$$

After integration of (31.ii), we can write for (31.i)

$$\Gamma\,\hbar\,\frac{\partial f_k}{\partial t} \;=\; \left(\alpha - \frac{\hbar^2}{2m}\,k^2 - 2\,\frac{e^2}{m}\,E^2\,t^2\right)f_k\;, \tag{33}$$

so that we find for f_k

$$f_k(t) \;=\; f_k(0)\,\exp\left(\frac{\left(\alpha - \frac{\hbar^2}{2m}k^2\right)t - \frac{2}{3}\,\frac{e^2}{m}\,E^2\,t^3}{\Gamma\,\hbar}\right)\;, \tag{34}$$

with

$$f_k^2(0) = \frac{\mathrm{k_B}\,T}{2\left(|\alpha| + \frac{\hbar^2}{2m}k^2\right)}\;, \tag{35}$$

and the associated current density $\mathbf{j}_{\mathrm{s}k}(t)$ can be written as

$$\mathbf{j}_{\mathrm{s}k}(t) \;=\; \frac{4\,e^2}{m}\,\mathbf{E}\;t\;f_k^2(0)\,\exp\left(2\,\frac{\left(\alpha - \frac{\hbar^2}{2m}k^2\right)t - \frac{2}{3}\,\frac{e^2}{m}\,E^2\,t^3}{\Gamma\,\hbar}\right)\;. \tag{36}$$

Now we can obtain the explicit expression for the physical (super)current density $\mathbf{j}_{\mathrm{s}}$ summing over $\mathbf{k}$:

$$\mathbf{j}_{\mathrm{s}}(t) \;=\; \frac{1}{8\pi^3}\int_0^{+\infty} dk\;4\pi\,k^2\,\mathbf{j}_{\mathrm{s}k}(k,t)\;, \tag{37}$$

where we assumed we are dealing with a three-dimensional sample of dirty material with dimensions greater than the correlation length.

Generalized fields. Once obtained the explicit form for the supercurrent density, the vector potential $\mathbf{A}(x, y, z, t)$ is given by:

$$\mathbf{A}(x, y, z, t) = \frac{1}{4\pi} \int \frac{\mathbf{j}_\mathrm{s}(t)\ dx'\, dy'\, dz'}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\ , \tag{38}$$

while the generalized electric field $\mathbf{E}(x, y, z, t)$ (21) is expressed as

$$\mathbf{E}(x, y, z, t)\ =\ -\frac{1}{c}\frac{\partial \mathbf{A}(x, y, z, t)}{\partial t} + \frac{m}{e}\,\mathbf{g}\ =\ -\frac{1}{c}\frac{\partial \mathbf{j}_\mathrm{s}(t)}{\partial t}\,\mathcal{C}(x, y, z) + \frac{m}{e}\,\mathbf{g}\ , \tag{39}$$

and features the contribution coming from the supercurrent coupled to the one from the weak Earth's gravitational field $\mathbf{g}$. The geometrical factor $\mathcal{C}(x, y, z)$ comes from the integral in (38), and depends on the space point where we measure the gravitational fluctuation and on the superconductor's shape.

4 Discussion

Let us now consider the case of a superconductive disk at a temperature very close to T_c. The sample is put it in its normal state by a weak magnetic field. The latter is then turned off at the time $t = 0$, so that the system goes in the superconductive state. The disk has bases parallel to the ground, and we are interested in the field variation measured above the sample. Since the material is at a temperature very close but higher than T_c, we are in the fluctuation regime and we can exploit the results obtained in Section 3.

In Figure 1 it is shown the variation of the local gravitational field as a function of time for a Sn sample, a low–T_c metallic superconductor with large intrinsic coherence length ξ_0. The variation is measured at a fixed distance d above the base surface, along the disk axis. We can note that the local gravitational field is initially reduced with respect to the unperturbed value g; after that, there is an increase up to a maximum value for $t = \tau_0$ and a final decrease to the standard external value. In Figure 2 we show the field variation as a function of distance from the base surface, measured along the axis of the disk at the fixed time $t = \tau_0$ that maximizes the effect. In Figures 3 and 4 the same calculations are displayed for a TlBaCaCuO (TlBCCO) sample, an high–T_c superconductor featuring smaller BCS coherence length.

It is possible to demonstrate that the maximum value Δ for the variation of the local gravitational field is proportional to $\xi(T)^{-1}$; this in turn implies that a larger contribution is present when dealing with high–T_c superconductors, having the latter smaller coherence lengths. On the other hand, it is easily shown that the discussed maximal perturbation takes place after a time interval $\tau_0 \propto (T - T_\mathrm{c})^{-1}$: this suggests that we can extend the time range in which the perturbation occurs if the sample is very close to its critical temperature. However, we pay this time range extension with a reduced alteration of the local field, since, if we put ourselves at a temperature very close T_c, we find a divergence of the Ginzburg-Landau coherence length $\xi(T)$, see eq. (26).

We hypothesize that the best choice would be to select a large, high–T_c superconductive sample (shorter intrinsic coherence length and consequent maximized local alteration) at a temperature very close to T_c, resulting then in an increase for the perturbation time range with beneficial effects for experimental detection. Clearly, large dimensions for the sample would enhance the interaction (and would also result in a larger contribution from the geometrical factor $\mathcal{C}(x, y, z)$ of eq. (39)), while choosing a dirty material would determine a wider temperature range in which the effects of fluctuations take place.

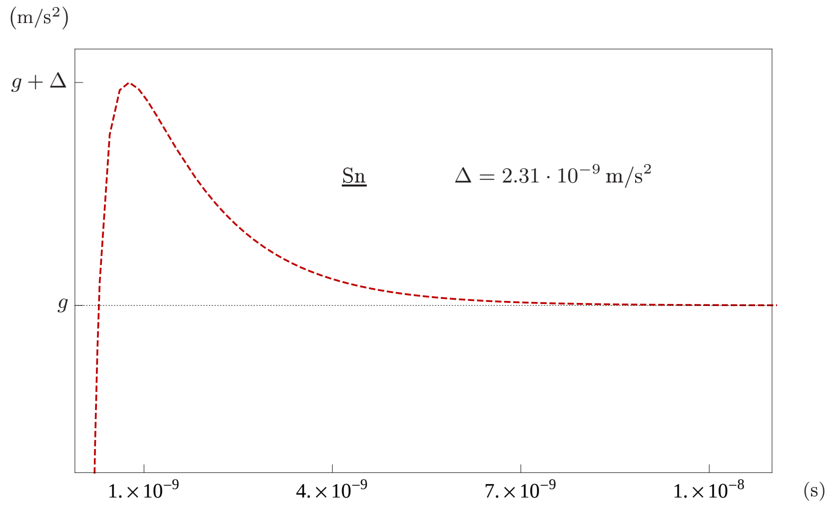

Fig. 1: Local gravitational field variation as a function of time for a Sn sample, measured along the axis of a superconductive disk at fixed distance $d = 0.25\,\mathrm{cm}$ above the base surface ($\xi_0 = 180\,\mathrm{nm}$, $T_c = 3.721\,\mathrm{K}$, $\Delta T = 10^{-3}\,\mathrm{K}$ [45]). The disk radius is $R = 15\,\mathrm{cm}$ and the disk thickness is $h = 2\,\mathrm{cm}$.

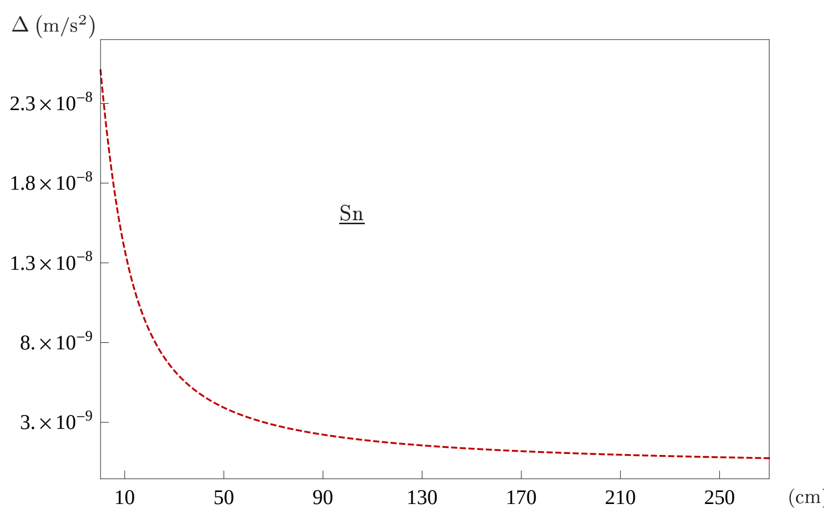

Fig. 2: Local gravitational field variation as a function of distance for the same Sn sample, measured along the disk axis above the base surface, at fixed time $t = \tau_0 = 0.745\,\mathrm{ns}$. The disk radius is $R = 15\,\mathrm{cm}$ and the disk thickness is $h = 2\,\mathrm{cm}$.

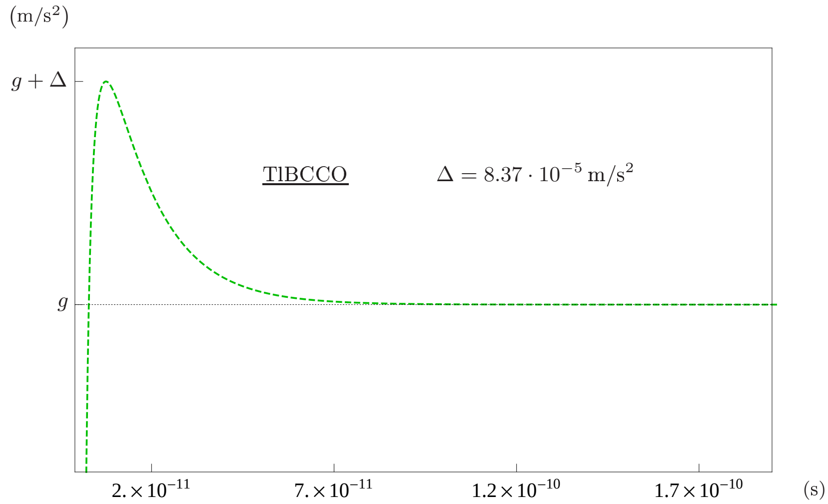

Fig. 3: Local gravitational field variation as a function of time for a TlBCCO sample, measured along the axis of a superconductive disk at fixed distance $d = 0.25\,\text{cm}$ above the base surface ($\xi_0 = 2.8\,\text{nm}$, $T_\text{c} = 100\,\text{K}$, $\Delta T = 0.1\,\text{K}$ [45]). The disk radius is $R = 15\,\text{cm}$ and the disk thickness is $h = 2\,\text{cm}$.

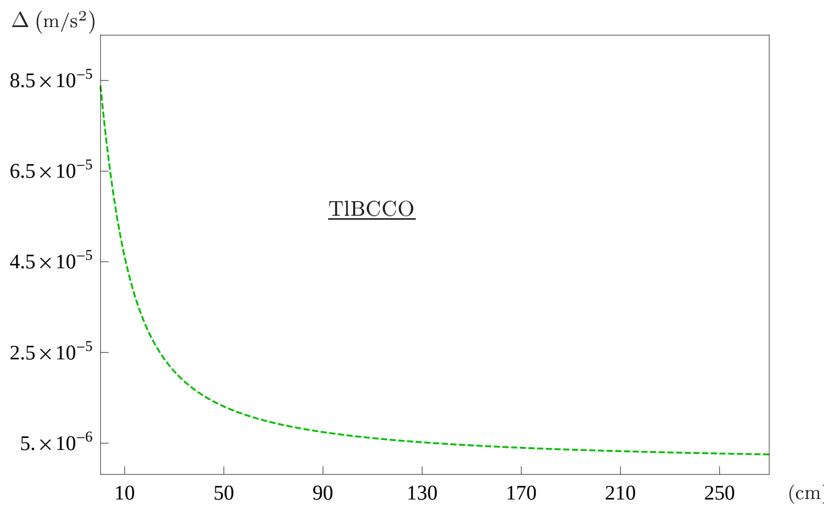

Fig. 4: Local gravitational field variation as a function of distance for the same TlBCCO sample, measured along the disk axis above the base surface, at fixed time $t = \tau_0 = 7.49 \cdot 10^{-3}\,\text{ns}$. The disk radius is $R = 15\,\text{cm}$ and the disk thickness is $h = 2\,\text{cm}$.

5 Concluding remarks

A deeper intertwining of different scientific areas can actually bring about a major step forward in our understanding of many fundamental aspects of our world. In general, a considerable gap between different branches of physics is generated by the independent mathematical formulations and developments in the individual research fields. This gap can be narrowed by taking advantage of a multidisciplinary approach, in which a fruitful exchange of ideas and techniques from different areas can provide new insights into the related unsolved issues (see e.g. [46–62]). Following this spirit, we have exploited along the paper different techniques from general relativity, quantum field theory and condensed matter physics, to describe the conjectured gravity/superfluid interplay. The latter was then combined with the gravito-Maxwell formulation to describe in detail how a detectable perturbation of the local gravitational field could be in principle obtained in a laboratory experiment. In particular, we have seen in Section 4 that a perceptible alteration is expected, especially in high–T_c superconductors. A careful arrangement of the experimental setup is very important: the material characteristic parameters, together with the sample geometry, dimensions and temperature, will determine the magnitude of the effect and the related time scales.

Future significant improvements could be obtained including the presence of suitable electric and magnetic fields, determining also the formation of moving vortices and giving rise to a possible further interaction with the gravitational field. It would be also interesting to perform analogous calculations at a temperature $T \lesssim T_c$ (rather than $T > T_c$) with an external magnetic field contribution or, alternatively, an applied external electric field parallel to a superconductor (plane) surface in the presence of defects.

As already pointed out in Section 1, a complete description of the proposed interaction would necessarily come from a quantum gravity formulation, but this in general implies a formalism from which it may be difficult to extract quantitative predictions: this was the main reason that led us to exploit the alternative gravito-Maxwell approach of section 2. However, in our opinion, at least as long as we limit ourselves to a weak gravitational background, no further significant contributions would come from the quantum gravity framework. The situation would be different if we considered, e.g., the interaction between the gravitational field of a neutron star and a hypothetical superfluid in its core, that is, an interplay involving a very strong gravitational background. Clearly, in this case the weak-field approach does not hold and, since the gravitational field is so intense that it also affects the dynamics of elementary particles, a quantum gravity formulation is called for[5].

Finally, we would also like to remark that the gravito-Maxwell formalism can be also exploited in other physical situation where generalized electric-magnetic fields of the form (21) are induced by the presence of a static, weak gravitational field; an example of application to the Josephson junction physics can be found in [32].

Acknowledgments

I would like to thank professor G. A. Ummarino for extremely helpful discussions during the preparation of this report. I would also like to thank prof. M. Trigiante, prof. F. Laviano and prof. A. Carbone that supported these studies with their funds.

[5] we should also note that those new contributions might be relevant if compared with the alterations we have considered for the Earth's gravity, but would likely be irrelevant as a correction to the very strong gravitational field from which they originate

A Conventions

The spacetime metric of a flat background in the "mostly plus" convention is given by the standard Minkowski

$$\eta \;=\; \mathrm{diag}(-1,+1,+1,+1)\;. \tag{A.1}$$

We define the Riemann tensor as:

$$\begin{aligned} R^{\sigma}{}_{\mu\lambda\nu} \;&=\; \partial_\lambda \Gamma^{\sigma}{}_{\mu\nu} \;-\; \partial_\nu \Gamma^{\sigma}{}_{\mu\lambda} \;+\; \Gamma^{\sigma}{}_{\rho\lambda}\,\Gamma^{\rho}{}_{\nu\mu} \;-\; \Gamma^{\sigma}{}_{\rho\nu}\,\Gamma^{\rho}{}_{\lambda\mu} \;= \\ &=\; 2\,\partial_{[\lambda}\Gamma^{\sigma}{}_{\nu]\mu} \;+\; 2\,\Gamma^{\sigma}{}_{\rho[\lambda}\,\Gamma^{\rho}{}_{\nu]\mu}\;, \end{aligned} \tag{A.2}$$

where the Christoffel symbols are given by

$$\Gamma^{\lambda}{}_{\nu\rho} \;=\; \frac{1}{2}\, g^{\lambda\mu}\,\Gamma_{\mu\nu\rho}\left(\partial_\rho g_{\mu\nu} + \partial_\nu g_{\mu\rho} - \partial_\mu g_{\nu\rho}\right)\;. \tag{A.3}$$

The Ricci tensor is defined as a contraction of the Riemann tensor

$$R_{\mu\nu} \;=\; R^{\sigma}{}_{\mu\sigma\nu}\;, \tag{A.4}$$

while the Ricci scalar is given by

$$R \;=\; g^{\mu\nu} R_{\mu\nu}\;. \tag{A.5}$$

The Einstein tensor $G^{(\mathrm{E})}_{\mu\nu}$ has the form

$$G^{(\mathrm{E})}_{\mu\nu} \;\equiv\; R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R\;, \tag{A.6}$$

and the Einstein equations are then written as

$$G^{(\mathrm{E})}_{\mu\nu} \;\equiv\; R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R \;=\; 8\pi \mathrm{G}\; T_{\mu\nu}\;, \tag{A.7}$$

where $T_{\mu\nu}$ is the total energy-momentum tensor. The contribution coming from the cosmological constant Λ can be pointed out splitting the $T_{\mu\nu}$ tensor in its matter and Λ component:

$$T_{\mu\nu} \;=\; T^{(\mathrm{M})}_{\mu\nu} + T^{(\Lambda)}_{\mu\nu} \;=\; T^{(\mathrm{M})}_{\mu\nu} - \frac{\Lambda}{8\pi\mathrm{G}}\, g_{\mu\nu}\;, \tag{A.8}$$

so that the Einstein equation can be rewritten as

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R \;=\; 8\pi\mathrm{G}\left(T^{(\mathrm{M})}_{\mu\nu} + T^{(\Lambda)}_{\mu\nu}\right)\;, \tag{A.9}$$

or, equivalently,

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\,(R - 2\,\Lambda) \;=\; 8\pi\mathrm{G}\; T^{(\mathrm{M})}_{\mu\nu}\;. \tag{A.10}$$

References

[1] Bryce S. DeWitt, *"Superconductors and gravitational drag"*, Phys. Rev. Lett. **16** (1966) 1092–1093. [P. 2]

[2] G. Papini, *"Detection of inertial effects with superconducting interferometers"*, Physics Letters A **24** (1967), n. 1, 32–33. [P. 2]

[3] G. Papini, *"Superconducting and normal metals as detectors of gravitational waves"*, Lett. Nuovo Cim. **4S1** (1970) 1027–1032. [P. 2]

[4] F. Rothen, *"Application de la theorie relativiste des phenomenes irreversible a la phenomenologie de la supraconductivite"*, Helv. Phys. Acta **41** (1968) 591. [P. 2]

[5] R.G. Rystephanick, *"On the london moment in rotating superconducting cylinders"*, Canadian Journal of Physics **51** (1973), n. 8, 789–794. [P. 2]

[6] H. Hirakawa, *"Superconductors in gravitational field"*, Physics Letters A **53** (1975), n. 5, 395–396. [P. 2]

[7] I.S. Minasyan, *"Londons equations in riemannian space"*, Doklady Akademii Nauk SSSR **228** (1976), n. 3, 576–578. [P. 2]

[8] J. Anandan, *"Gravitational and rotational effects in quantum interference"*, Phys. Rev. D **15** (1977), n. 6, 1448. [P. 2]

[9] J. Anandan, *"Interference, Gravity and Gauge Fields"*, Nuovo Cim. A **53** (1979) 221. [P. 2]

[10] J. Anandan, *"Relativistic thermoelectromagnetic gravitational effects in normal conductors and superconductors"*, Physics Letters A **105** (1984), n. 6, 280–284. [P. 2]

[11] J. Anandan, *"Relativistic gravitation and superconductors"*, Class. Quantum Grav. **11** (1994), n. 6, A23. [P. 2]

[12] D.K. Ross, *"The london equations for superconductors in a gravitational field"*, Journal of Physics A: Mathematical and General **16** (1983), n. 6, 1331. [P. 2]

[13] S. B. Felch, J. Tate, B. Cabrera and J. T. Anderson, *"Precise determination of h/m_e using a rotating, superconducting ring"*, Phys. Rev. B **31** (Jun, 1985) 7006–7011. [P. 2]

[14] O. Yu Dinariev and A.B. Mosolov, *"A relativistic effect in the theory of superconductivity"*, in *Soviet Physics Doklady*, vol. 32, p. 576, 1987. [P. 2]

[15] H. Peng, *"A new approach to studying local gravitomagnetic effects on a superconductor"*, General Relativity and Gravitation **22** (1990), n. 6, 609–617. [P. 2, 5]

[16] H. Peng, D.G. Torr, E.K. Hu and B. Peng, *"Electrodynamics of moving superconductors and superconductors under the influence of external forces"*, Phys. Rev. B **43** (1991), n. 4, 2700. [P. 2]

[17] H. Peng, G. Lind and Y.S. Chin, *"Interaction between gravity and moving superconductors"*, General relativity and gravitation **23** (1991), n. 11, 1231–1250. [P. 2]

[18] Ning Li and D.G. Torr, *"Effects of a gravitomagnetic field on pure superconductors"*, Physical Review D **43** (1991), n. 2, 457. [P. 2]

[19] Ning Li and Douglas G. Torr, *"Gravitational effects on the magnetic attenuation of superconductors"*, Physical Review B **46** (1992), n. 9, 5489. [P. 2]

[20] Douglas G. Torr and Ning Li, *"Gravitoelectric-electric coupling via superconductivity"*, Foundations of Physics Letters **6** (1993), n. 4, 371–383. [P. 2]

[21] L.C. Garcia de Andrade, *"Torsion, superconductivity, and massive electrodynamics"*, International journal of theoretical physics **31** (1992), n. 7, 1221–1227. [P. 2]

[22] E. Podkletnov and R. Nieminen, *"A possibility of gravitational force shielding by bulk $YBa_2Cu_3O_{7-X}$ superconductor"*, Physica C: Superconductivity **203** (1992), n. 3-4, 441–444. [P. 2]

[23] E. Podkletnov, *"Weak gravitation shielding properties of composite bulk $YBa_2Cu_3O_{7-X}$ superconductor below 70 K under em field"*, cond-mat/9701074. [P. 2]

[24] Giovanni Modanese, *"Theoretical analysis of a reported weak gravitational shielding effect"*, Europhys. Lett. **35** (1996) 413–418, [hep-th/9505094]. [P. 2, 3]

[25] Giovanni Modanese, *"Role of a 'local' cosmological constant in Euclidean quantum gravity"*, Phys. Rev. D **54** (1996) 5002–5009, [hep-th/9601160]. [P. 3]

[26] L.I. Schiff and M.V. Barnhill, *"Gravitation-induced electric field near a metal"*, Physical Review **151** (1966), n. 4, 1067. [P. 3]

[27] A.J. Dessler, F.C. Michel, H.E. Rorschach and G.T. Trammell, *"Gravitationally Induced Electric Fields in Conductors"*, Phys. Rev. **168** (1968) 737–743. [P. 3]

[28] F.C. Witteborn and W.M. Fairbank, *"Experimental comparison of the gravitational force on freely falling electrons and metallic electrons"*, Physical Review Letters **19** (1967), n. 18, 1049. [P. 3]

[29] F.C. Witteborn and W.M. Fairbank, *"Experiments to determine the force of gravity on single electrons and positrons"*, Nature **220** (1968), n. 5166, 436–440. [P. 3]

[30] C. Herring, *"Gravitationally induced electric field near a conductor, and its relation to the surface-stress concept"*, Phys. Rev. **171** (1968), n. 5, 1361. [P. 3]

[31] Giovanni Alberto Ummarino and Antonio Gallerati, *"Superconductor in a weak static gravitational field"*, Eur. Phys. J. **C77** (2017), n. 8, 549, [arXiv:1710.01267]. [P. 3, 4, 5]

[32] Giovanni Alberto Ummarino and Antonio Gallerati, *"Josephson AC effect induced by weak gravitational field"*, Class. Quant. Grav. **37** (2020), n. 21, 217001, [arXiv:2009.04967]. [P. 3, 4, 5, 11]

[33] G. A. Ummarino and A. Gallerati, *"Possible alterations of local gravitational field inside a superconductor"*, Entropy **23** (2021), n. 2, 193, [arXiv:2102.01489]. [P. 3, 4, 5]

[34] Robert M. Wald, *"General Relativity"*; Chicago Univ. Pr., Chicago, USA (1984). [P. 4]

[35] Charles W. Misner, K.S. Thorne and J.A. Wheeler, *"Gravitation"*; W. H. Freeman, San Francisco (1973). [P. 4]

[36] Giovanni Alberto Ummarino and Antonio Gallerati, *"Exploiting weak field gravity-Maxwell symmetry in superconductive fluctuations regime"*, Symmetry **11** (2019), n. 11, 11. [P. 5]

[37] Vladimir B. Braginsky, Carlton M. Caves and Kip S. Thorne, *"Laboratory Experiments to Test Relativistic Gravity"*, Phys. Rev. D **15** (1977) 2047. [P. 5]

[38] H. Peng, *"On calculation of magnetic-type gravitation and experiments"*, General relativity and gravitation **15** (1983), n. 8, 725–735. [P. 5]

[39] M Agop, C Gh Buzea and P Nica, *"Local gravitoelectromagnetic effects on a superconductor"*, Physica C: Superconductivity **339** (2000), n. 2, 120–128. [P. 5]

[40] Antonio Gallerati, *"Interaction between superconductors and weak gravitational field"*, J. Phys. Conf. Ser. **1690** (2020), n. 1, 012141. [P. 5]

[41] Harihar Behera, *"Comments on gravitoelectromagnetism of Ummarino and Gallerati in "Superconductor in a weak static gravitational field" vs other versions"*, Eur. Phys. J. **C77** (2017), n. 12, 822, [arXiv:1709.04352]. [P. 5]

[42] M. Cyrot, *"Ginzburg-landau theory for superconductors"*, Reports on Progress in Physics **36** (1973), n. 2, 103. [P. 6]

[43] JP Hurault, *"Nonlinear effects on the conductivity of a superconductor above its transition temperature"*, Physical Review **179** (1969), n. 2, 494. [P. 6]

[44] Albert Schmid, *"Diamagnetic susceptibility at the transition to the superconducting state"*, Physical Review **180** (1969), n. 2, 527. [P. 6]

[45] Charles K Poole, Horacio A Farach and Richard J Creswick, *"Handbook of superconductivity"*; Academic press (1999). [P. 9, 10]

[46] Juan Martin Maldacena, *"The large N limit of superconformal field theories and supergravity"*, Int. J. Theor. Phys. **38** (1999) 1113–1133. [P. 11]

[47] Edward Witten, *"Anti-de Sitter space and holography"*, Adv. Theor. Math. Phys. **2** (1998) 253–291, [hep-th/9802150]. [P. 11]

[48] W. H. Zurek, *"Cosmological experiments in condensed matter systems"*, Phys. Rept. **276** (1996) 177–221, [cond-mat/9607135]. [P. 11]

[49] G. E. Volovik, *"Superfluid analogies of cosmological phenomena"*, Phys. Rept. **351** (2001) 195–348, [gr-qc/0005091]. [P. 11]

[50] C. Baeuerle, Yu. M. Bunkov, S. N. Fisher, H. Godfrin and G. R. Pickett, *"Laboratory simulation of cosmic string formation in the early Universe using superfluid He-3"*, Nature **382** (1996) 332–334. [P. 11]

[51] V.M.H. Ruutu, V.B. Eltsov, A.J. Gill, T.W.B. Kibble, M. Krusius, Yu.G. Makhlin, B. Placais, G.E. Volovik and Wen Xu, *"Big bang simulation in superfluid He-3-b: Vortex nucleation in neutron irradiated superflow"*, Nature **382** (1996) 334, [cond-mat/9512117]. [P. 11]

[52] A. Vilenkin and E. P. S. Shellard, *"Cosmic Strings and Other Topological Defects"*; Cambridge University Press, Cambridge, UK (2000). [P. 11]

[53] Jeffrey C. Y. Teo and C. L. Kane, *"Topological Defects and Gapless Modes in Insulators and Superconductors"*, Phys. Rev. **B82** (2010) 115120, [arXiv:1006.0690]. [P. 11]

[54] Mario Novello, Matt Visser and Grigory E. Volovik, *"Artificial black holes"*; World Scientific (2002). [P. 11]

[55] Carlos Barcelo, Stefano Liberati and Matt Visser, *"Analogue gravity"*, Living Rev. Rel. **8** (2005) 12. [P. 11]

[56] Antonio Gallerati, *"Graphene properties from curved space Dirac equation"*, Eur. Phys. J. Plus **134** (2019), n. 5, 202, [arXiv:1808.01187]. [P. 11]

[57] L. Andrianopoli, B. L. Cerchiai, R. D'Auria, A. Gallerati, R. Noris, M. Trigiante and J. Zanelli, *"$\mathcal{N}$-extended $D = 4$ supergravity, unconventional SUSY and graphene"*, JHEP **01** (2020) 084, [arXiv:1910.03508]. [P. 11]

[58] Antonio Gallerati, *"Supersymmetric theories and graphene"*, PoS ICHEP 2020 (2020). [P. 11]

[59] Antonio Gallerati, *"Negative-curvature spacetime solutions for graphene"*, J. Phys. Condens. Matter **33** (2021), n. 13, 135501, [arXiv:2101.03010]. [P. 11]

[60] Salvatore Capozziello, Richard Pincak and Emmanuel N. Saridakis, *"Constructing superconductors by graphene Chern-Simons wormholes"*, Annals Phys. **390** (2018) 303–333. [P. 11]

[61] Tommaso Morresi, Daniele Binosi, Stefano Simonucci, Riccardo Piergallini, Stephan Roche, Nicola M Pugno and Taioli Simone, *"Exploring event horizons and hawking radiation through deformed graphene membranes"*, 2D Materials **7** (2020), n. 4, 041006. [P. 11]

[62] James Q. Quach, *"Gravitational Casimir effect"*, Phys. Rev. Lett. **114** (2015), n. 8, 081104, [arXiv:1502.07429]. [Erratum: Phys. Rev. Lett. **118** (2017) 139901]. [P. 11]