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Analysis of the Stability of Openings Excavated in Anisotropic Rocks

Chiara Deangeli(✉), Marilena Cardu, and Daniele Martinelli

Department of Environment, Land and Infrastructure Engineering (DIATI), Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Turin, Italy
chiara.deangeli@polito.it

Abstract. Openings excavated in rocks with anisotropic strength are often affected by serious instability, related to slip along the weakness planes. The Jaeger criterion, which is a discontinuous approach, is widely used in the mining and oil and gas industry, because is based on well-known rock strength parameters. However, this model cannot capture features related to the stability of openings drilled in some anisotropic rocks with the combined effect of the *in situ* state of stress. The Hoek & Brown criterion, adapted to anisotropic rocks, is a continuous criterion that can describe the complex behavior of different types of anisotropy exhibited by rock material. Here we interpreted the results of triaxial tests carried out on a shale and we defined the parameters of the Jaeger criterion and the modified Hoek & Brown criterion. We investigated the stability of boreholes drilled in this shale by varying the *in situ* state of stress and we compared the results of the two criteria. We found that the Hoek & Brown criterion can appropriately describe the behavior of this shale and can predict more accurately the width of the instability of openings excavated in different conditions.

Keywords: Anisotropic rocks · Hoek & brown criterion · Borehole stability · Anisotropic *in situ* state of stress

1 Introduction

The stability of openings excavated in formations with structural anisotropy depends on the presence of weakness planes that are generally characterized by a reduced strength (Fjaer et al. 2008). At the opening wall, failure can develop as a slip, sometimes coupled with shear failure in the bulk rock. This peculiar condition commonly characterizes deep civil and mining openings such as tunnels and shafts, where the failure often develops along the foliation, while the rock mass is usually considered isotropic for modelling purposes. As a matter of fact, most of these deep excavations are often considered as stable regarding the deformations, and the modelling is usually carried out for assessing possible stress induced damage, such as spalling. This is particularly evident especially in crystalline competent rock masses (Uotinen et al. 2013), while on the contrary, discrete models are mostly applied for shallower cases (Martinelli et al. 2012).

Several studies indicated that the stability of wellbores depends also on the relative orientation between the *in situ* maximum stress and the weakness planes (i.e. Last and McLean 1996; Aadnoy, 1988; Økland and Cook 1998; Willson et al. 1999; Brehm et al. 2006; Narayanasamy et al. 2010; Deangeli and Omwanghe 2018; Parkash and Deangeli 2019). In these studies, failure is investigated with anisotropic strength criteria. The most widely used criterion for the stability analysis is the Jaeger's (1960) weakness plane model (WPM). However, this criterion is discontinuous and the plateau of constant strength is not always in agreement with experimental data. Deangeli and Omwanghe (2018) and Parkash and Deangeli (2019) carried out stability analyses with the Hoek and Brown criterion (1988) adapted to anisotropic strength (H&B). This criterion is continuous and can reproduce a continuous variation of the strength with the inclination of the weakness planes in the range 0°–90°.

Here we carried out a comparative analysis of the stability of boreholes drilled in an anisotropic shale by using the WPM (analytical approach and numerical simulation with FLAC-Ubiquitous Joint Model (ver. 8, Itasca Consulting Group, Minneapolis) and the adapted H&B criterion. In particular, with the H&B we were able to capture a particular feature of the instability which was not predicted by the WPM.

2 Borehole Pressures to Avoid Slip Calculated with WPM and H&B

We selected two strength criteria for rocks with anisotropic strength to analyze the stability of boreholes drilled along a principal direction. We determined the stress distribution around a circular hole with the Kirsch (1898) solution. We adapted this solution to boreholes, by imposing plane strain condition in the direction of the hole axis. We considered the condition $\sigma_\theta > \sigma_{axis} > \sigma_r$, which is very common in several field cases.

The Kirsch (1898) solution assumes a continuous, homogenous, isotropic, linear, elastic material, while the selected strength criteria assume an anisotropy in strength. Meier et al. (2015) and Meier (2016) carried out hollow cylinder tests, true triaxial tests and numerical simulations of wellbores drilled in transverse isotropic Posidonia shale. The results of the numerical analyses carried out with isotropic, linear, elastic material and transverse isotropic elastic material, coupled with an anisotropic strength criterion, showed that the variations of stress fields and failure patterns are negligible. The elastic anisotropy characteristics of Posidonia shale are similar to ones of Tournemire shale, hence, for the sake of simplicity, we decided to use the Kirsch solution in our analysis.

The spacing of weakness planes can play a relevant role on failure pattern scenarios at different scales. The purpose of our study is the determination of failure at the wall of a borehole drilled in a shale. In general, shales are considered thinly laminated at the scale of wellbores. Therefore in our analysis we considered ubiquitous weakness planes in a given direction.

The WPM is discontinuous and considers two independent failure modes: failure along the discontinuity and failure through the intact rock material. The limit condition for sliding along the weakness plane is:

$$(\sigma_1 - \sigma_3)_{slip} = 2\left(c'_w + \sigma'_3 \tan\phi'_w\right)\left[\left(1 - \frac{\tan\phi'_w}{\tan\beta_w}\right)\sin 2\beta_w\right]^{-1} \quad (1)$$

Where c'_w and ϕ'_w are the cohesion and the friction angle of the weakness planes and the β_w is defined in Fig. 1.

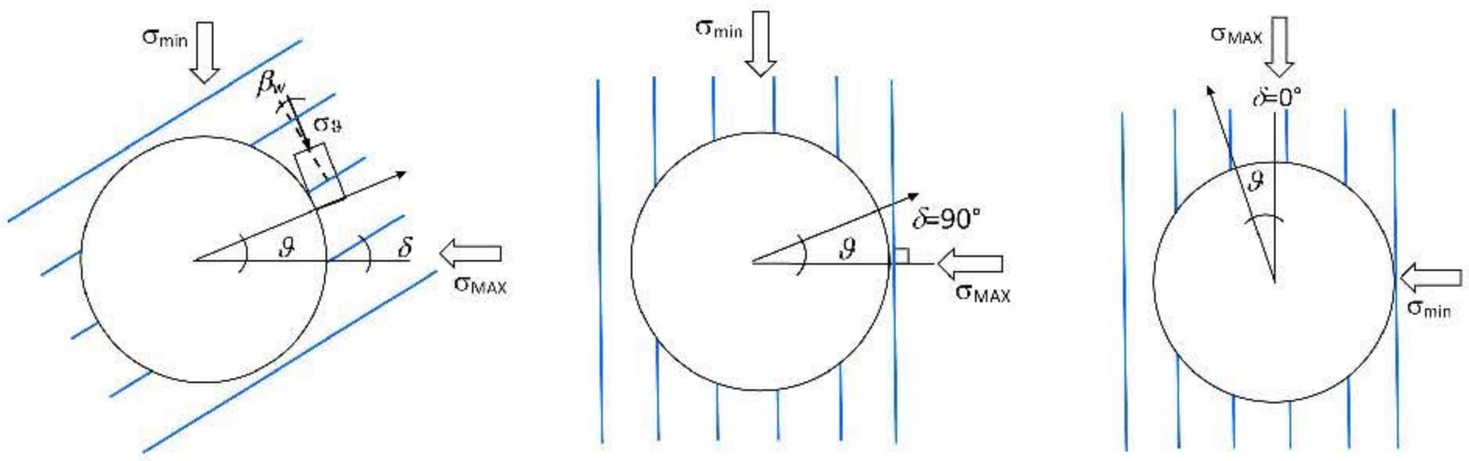

Fig. 1. Definition of the angles δ, β_w and ϑ around the opening. The blue lines are the weakness planes. The dotted line represents the normal to the planes. The angle δ is measured counterclockwise from σ_{MAX}. The angle between the normal to the weakness plane and the maximum principal stress (here $\sigma_\theta = \sigma_1$) is β_w. The azimuth ϑ is measured counterclockwise from σ_{MAX}.

According to Eq. (1) the strength parameters of the weakness planes are constant and the variation of the slip condition is ruled by the angle β_w. For values of β_w approaching 90° and in the range 0° to ϕ'_w the criterion considers a plateau of constant strength, which is not always in agreement with experimental data. We coupled the Kirsch solution with Eq. (1) and we obtained the borehole pressure to avoid slip:

$$P_{WPM} = \frac{S\left(1 - \frac{\tan\phi'_w}{\tan\beta_w}\right)\sin 2\beta_w - 2c'_w + 2\tan\phi'_w P_f}{2\left[\tan\phi'_w + \left(1 - \frac{\tan\phi'_w}{\tan\beta_w}\right)\sin 2\beta_w\right]} \quad (2)$$

Where ϑ is the borehole azimuth, P_f is the *in situ* pore pressure and S is the tangential stress σ_θ at different azimuths ϑ, (Fig. 1):

$$S = [\sigma_{MAX} + \sigma_{\min} - 2(\sigma_{MAX} - \sigma_{\min})\cos 2\vartheta] \quad (3)$$

Where σ_{MAX} and $\sigma_{\min}$ are the maximum and minimum *in situ* stress, respectively.

Recently, Tien and Kuo (2001) and Colak and Unlu (2004) adapted the H&B criterion, which is a continuous function, to transversely isotropic rocks, by assuming

that the rock is intact (s = 1) and instantaneously isotropic at every inclination of the weakness planes. Under these conditions, the H&B criterion becomes:

$$(\sigma_1 - \sigma_3)_{\beta w} = \left(m_{\beta w} C_{o\beta w} \sigma'_3 + C^2_{o\beta w}\right)^{0,5} \quad (4)$$

Where: $C_{o\beta w}$ and $m_{\beta w}$ are the instantaneous uniaxial compressive strength of the rock and the empirical dimensionless constant respectively, which vary with the inclination β_w of the weakness planes.

Deangeli and Omwanghe (2018) coupled the Kirsch solution with Eq. (4) and obtained the borehole pressure to avoid slip:

$$P_{H\&B} = \frac{4S + m_{\beta w} C_{o\beta w} - \left[\left(4S + m_{\beta w} C_{o\beta w}\right)^2 - 16\left(S^2 + m_{\beta w} C_{o\beta w} P_f - C^2_{o\beta w}\right)\right]^{0,5}}{8} \quad (5)$$

We used Eqs. (3) and (5) to carry out comparative stability analyses of wellbores drilled in Tournemire shale. The angle β_w changes at different azimuths ϑ, for a given δ. We calculated β_w at the wall of the borehole according to the solution found by Deangeli and Omwanghe (2018).

3 Interpretation of Triaxial Tests with the WPM and H&B Criteria

The anisotropy of Tournemire shale was investigated by Niandou et al. (1997) with triaxial tests carried out at different confinements (σ_3= 1, 5, 20, 40 and 50 MPa) and different β_w= 0°, 30°, 45° 60° and 90°. We interpreted the results of these tests with the H&B criterion and the WPM. With the H&B criterion, we found at each β_w the correspondent $C_{o\beta w}$ and $m_{\beta w}$ with a linear regression in the diagram [σ_3; $(\sigma_1$-$\sigma_3)^2$]. Figure 2a shows a well-defined trend of the calculated $C_{o\beta w}$ with β_w with minimum uniaxial strength in the range β_w= 30°–60°. The maximum value of $C_{o\beta w}$= 44,85 MPa occurs at β_w= 90° and the minimum value of $C_{o\beta w}$= 24,55 MPa occurs at β_w= 60°. We also noted that $C_{o\beta w}$ at β_w= 45° is very close to the strength at β_w= 60°. We interpolated the $C_{o\beta w}$ data with a polynomial regression (dotted line in Fig. 2a) that is characterized by a minimum close to β_w= 45°. Figure 2a shows that $m_{\beta w}$ varies between 3,6 and 5, without a defined trend. The interpretation of the results of the triaxial tests with the WPM required the calculation of the strength parameters of the weakness planes. We calculated the cohesion c'w and the friction angle ϕ'_w at an inclination β_w* that corresponds to the minimum strength, with a linear regression of the experimental triaxial. We found c'_w= 8,58 MPa and ϕ'_w= 22° at β_w= 60° and very close values at β_w= 45° (c'_w= 9,78 MPa and ϕ'_w= 22°). We adopted the strength parameters at β_w* = 60°, because the WPM constrains the location of the minimum strength at β_{wcrit}= 45° + ϕ'_w/2 = 56°. We also calculated c' and ϕ' at β_w= 0° and β_w= 90°, for the rock matrix, which resulted very close. The average values (c' = 15 MPa and ϕ' = 23,75°) were used to calculate the plateau strength of C_o for the

calculation of P_{matrix} with the Mohr Coulomb criterion. Figures 2b and 2c show the simulation of the triaxial tests with the WPM (c'_w and ϕ'_w at β_w= 60°) and the H&B criterion ($C_{o\beta w}$ and $m_{\beta w}$ in Fig. 2a). The comparison of these two simulations indicates that the experimental data scattering is not well matched by the WPM and is matched very well by the H&B criterion for the majority of inclinations and confinements.

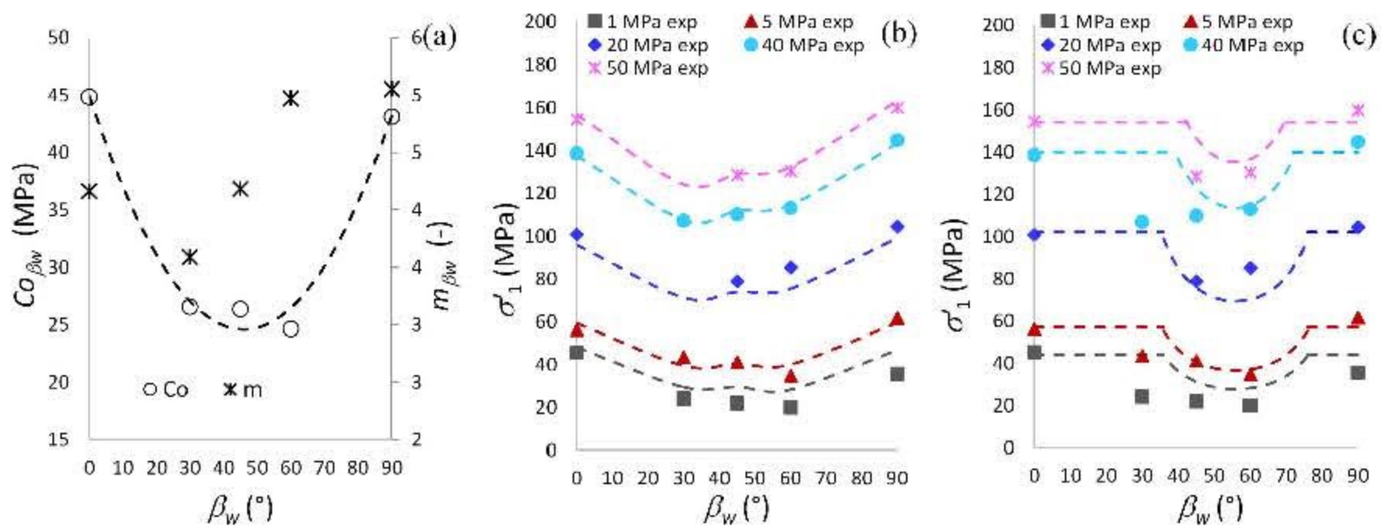

Fig. 2. (a) Variation of the calculated $C_{o\beta w}$ and $m_{\beta w}$ with β_w (symbols) and polynomial regression of calculated $C_{o\beta w}$ (dotted line) (b) Simulation of the triaxial tests (filled symbols) with the H&B criterion (dotted lines) by using $C_{o\beta w}$ and $m_{\beta w}$. (c) Simulation of the triaxial tests (filled symbols) with the WPM (dotted lines) by using c'_w= 8,58 MPa and ϕ'_w= 22° calculated at β_w= 60°.

4 Synthetic Case Studies of Borehole Stability

We analyzed borehole stability in the shale by varying δ, and *in situ* stress anisotropy $K = \sigma_{MAX}/\sigma_{min}$. Based on the findings of Deangeli and Omwanghe (2018) we analyzed three relevant cases: δ = 0°, 45° and 90° with $K = \sigma_{MAX}/\sigma_{min}$= 23/20 = 1,15 and $K = \sigma_{MAX}/\sigma_{min}$= 36/20 = 1,8. The *in situ* pore pressure was maintained constant P_f = 8 MPa. We carried out the stability analyses with the WPM and the H&B criterion by using Eqs. (2) and (5), respectively. We also performed numerical simulations with FLAC -Ubiquitous Joint Model for the case δ = 90°. In the Ubiquitous Joint Model implemented in FLAC (ver. 8, Itasca Consulting Group, Minneapolis) yield can occur in either the solid or along the weakness planes, or both, depending on the stress state, orientation of the planes, and material properties. The criterion for failure in the rock matrix is the Mohr Coulomb model. The criterion for failure on the weakness plane is represented by Eq. (1) (WPM). At first, the code detects general failure and applies plastic corrections. The corrected stresses are then analyzed for failure on the weakness plane, and updated accordingly. This model is based on stress redistribution when plasticity occurs. A radial grid (Fish DONUT, implemented in FLAC) with 60 zones in the radial direction and 60 zones on the circumference represents the model used in the numerical simulations.

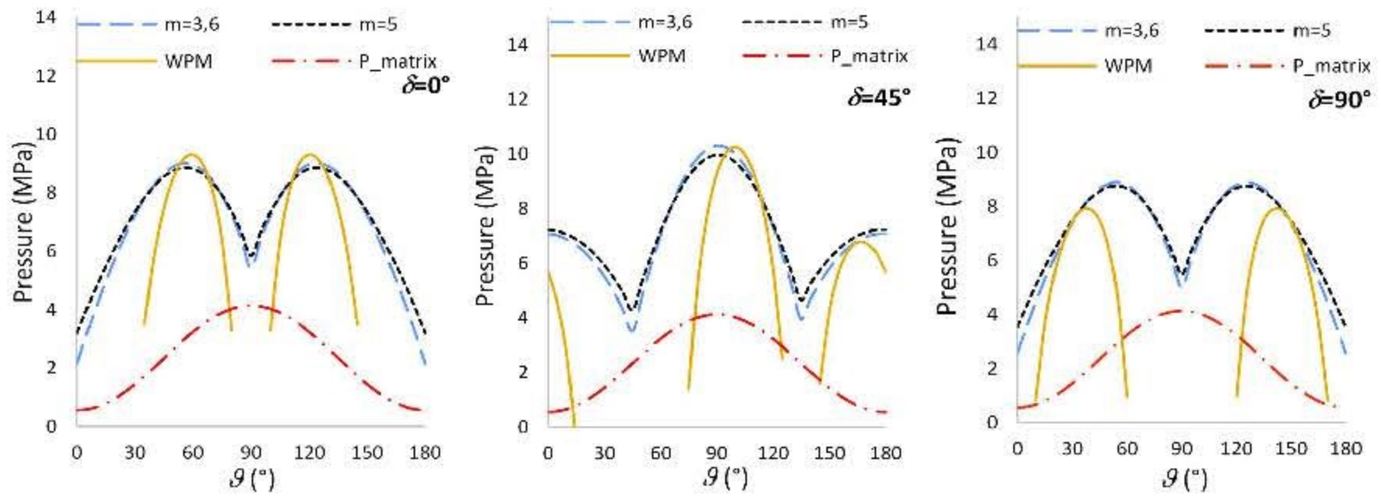

Fig. 3. Borehole pressures calculated with the WPM and H&B criterion. σ_{MAX}= 23 MPa, $K = 1,15$

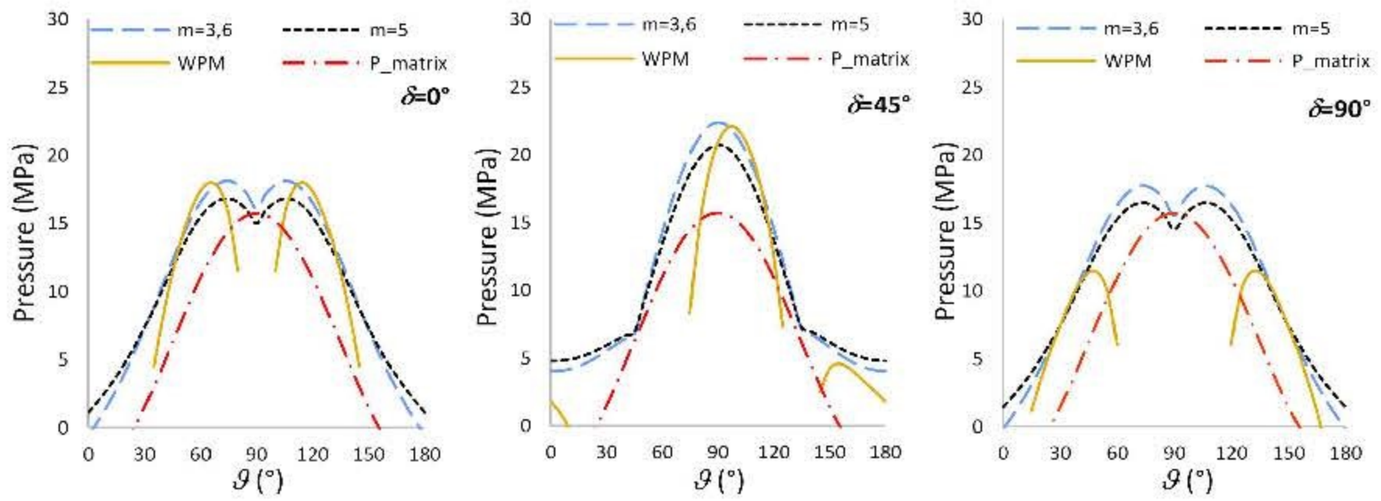

Fig. 4. Borehole pressures calculated with the WPM and H&B criterion. σ_{MAX}= 36 MPa, $K = 1,8$

In all the approaches the stability is evaluated by considering, at a given depth, the peak of the borehole pressure required to avoid slip along the weakness planes or shear failure in the rock matrix (with the Mohr Coulomb criterion C_o= 46 MPa (i.e. c' = 15 MPa) and ϕ'_w= 23,75°).

We observed in the two cases (Figs. 3 and 4) a good agreement between the WPM and the H&B criterion when $\delta = 0°$ and 45°.

The borehole pressure P_{matrix} required for the rock matrix (isotropic material) exclusively depends on the anisotropy of the *in situ* stresses. The borehole pressure P_{matrix} when K = 23/20 = 1,15 is negligible, but is relevant when K = 1,8. The P_{matrix} refers to the strength of the constant plateau in the WPM. When $\delta = 90°$ and K = 1,15 (Fig. 3) the WPM supplies a lower borehole pressure than H&B and the P_{matrix} cannot fulfill the lack of stabilizing pressure. The same scenario is obtained with FLAC (Fig. 5a). When $\delta = 90°$ and K = 1,8 (Fig. 4) the pressure calculated with WPM is very low, compared to the pressure calculated with H&B, even if the P_{matrix} is considered. This result indicates that the plateau of strength of the WPM cannot capture the real behavior of some rocks, because is based on a constant strength, while the H&B approach is characterized by a continuous variation of strength. Furthermore, the H&B criterion predicts a probable larger width of the unstable arc of half of wall

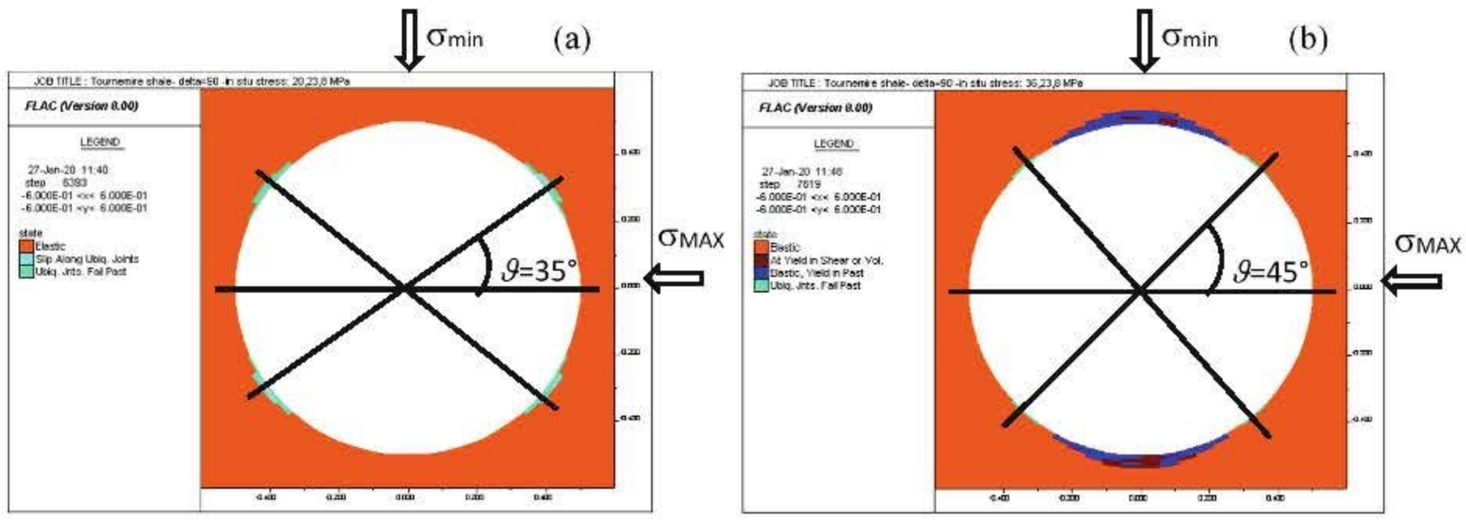

Fig. 5. Simulation of instability with FLAC –Ubiquitous Joint at $\delta = 90°$. Colors: orange is elastic, turquoise is slip, red is shear failure (rock matrix) and blue is at yield in past. P_f= 8 MPa (a) K = 1,15. Instability occurs at a borehole pressure 7 MPa (b) K = 1,8. Instability occurs at a borehole pressure 11 MPa

circumference (from $\vartheta = 75°$ to $\vartheta = 105°$) because the drop in pressure at $\vartheta = 90°$ is low. P_{matrix} predicts just a peak and hence local failure (breakouts) at $\vartheta = 90°$, as usual with the isotropic material. The FLAC simulation (Fig. 5b) shows the occurrence of local plasticity in the rock matrix (also without the weakness planes) and negligible slip, in agreement with the analytical solution.

5 Conclusions

We carried out a comparative analysis to investigate the stability of boreholes drilled in anisotropic rocks. The results show that the H&B criterion, which is characterized by a continuous variation of the strength, can appropriately predict, in different conditions, the width of borehole instability, which is marginally predicted by the WPM.

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