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# A first-order lumped parameters model of electrohydraulic actuators for low-inertia rotating systems with dry friction

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**Abstract**. In aerospace engineering, there are several control systems affected by dry friction, which are characterized by low inertia and high working frequencies. For these systems, it is possible to use a downgraded, first-order dynamic model to represent their behaviour properly without run into numerical problems that would be harmful for the solution itself and would require high computational power to be solved, which means more weight, costs, and complexity. Yet, the effect of dry friction is still possible to be accounted for accurately using a new algorithm based on the Coulomb friction model applied to the downgraded, first-order dynamic model. In this paper, the degraded first-order model is applied to an electrohydraulic servomechanism with its PID control unit, hydraulic motor, electrohydraulic servo-valve, and applied load. These components represent a classic airplane actuator system. The downgraded model will be compared to the second-order one focusing on the pros and cons of the reduction process with a focus on the effect of dry friction for reversible and irreversible actuators.

## 1. Introduction

The design of hydraulically powered flight controls for aerospace, typical high position accuracy servomechanisms, involves the in-depth knowledge of their behaviors (generally affected by several nonlinear phenomena such as backlashes, saturations, dry frictions, etc). As reported in [1], the simulation of the dynamic behaviors of these systems may require mathematical models capable of taking into account the said nonlinearities, which affect more or less all working conditions. Usually, these models describe the main characteristics of the simulated on-board devices (e.g. electrohydraulic or electromechanical actuators) with a wealth of details, adopting non-linear mathematical models of a sufficiently high order and with a suitable number of degrees of freedom. Therefore, accurate modelling of a considered mechatronic system generally implies the use of high order dynamic models (typically, of second-order nonlinear or higher) [1]. However, for several applications (preliminary design, monitoring, diagnostics or prognostics), the detailed digital twins mentioned above can be too cumbersome (from a computational point of view); they can slow down the system (compromising real-time) or request data that are difficult to find.

In these specific cases, it is necessary to develop simplified algorithms that allow combining reduced computational burdens with acceptable levels of accuracy and robustness. The optimal methods for developing the simplified model of a given system are still topics of debate; indeed, they are strongly influenced by the function of the model (preliminary design, monitoring, etc.), by the modeled system, and by the quality and quantity of the data available. Under certain conditions (discussed in the following section of this work), it is possible (and advisable) to simplify the mathematical structure of the numerical model, degrading it to a simple first-order. When the necessary conditions are verified (one or more higher-order terms of the differential equations constituting the mathematical model are negligible), the degradation of the model reduces the complexity and computational cost of the entire algorithm. At the same time, however, it guarantees results comparable with those provided by higher-order models. Nowadays, the development of downgraded models for mechanical systems is mandatory to reduce the time and costs of preliminary design analysis and to develop robust and reliable software for onboard monitoring of critical components. A reduced model is usually a lighter solution in terms of weight, and it often needs a more straightforward and less powerful central processing unit (CPU). To develop a first-order downgraded model for the analyses and simulations of an electro-hydraulic actuator (EHA) commonly used to control the position of an aircraft trim-tab, the authors developed a new MATLAB-Simulink numerical model. This model uses a PID control logic to command a flapper-nozzle type electro-hydraulic servo valve (EHSV), which drives a hydraulic motor of given cylinder capacity. The motor output torque is used to control the trim-tab position through a rotary actuator affected by dry friction. The model allows simulating both a reversible and irreversible actuator through aiding and opposing efficiencies. The motion transmission elements are generally influenced by dry friction, which may give rise to reversible or irreversible behavior of the whole system. So, the effect of the dry friction on the dynamic behavior of the mechanical system requires proper simulation models characterized by high computational accuracy, nevertheless compactness and efficiency. The model used to achieve the adequate simulation of dry friction is the Borello's algorithm [1], properly modified to match a first-order model instead of a second-order one.

## 2. Physical-mathematical model

The classic second-order dynamic mass-damper-spring (MCK) model [2-3] can be expressed as:

$$M\ddot{\boldsymbol{x}} + C\dot{\boldsymbol{x}} + K\boldsymbol{x} = \boldsymbol{F} \tag{1}$$

In a nutshell, any mechanical system can be traced back to an elementary structure similar to that expressed in Eq. (1). As for the electrohydraulic rotary actuators commonly used onboard the aircraft, typically to drive the secondary flight controls, it is possible to implement this strategy starting from the detailed model illustrated in [4-5] and to simplify it in analogy to Eq. (1) obtaining:

$$J_{eq}\frac{d\vartheta^2}{dt^2} + CM\frac{d\vartheta}{dt} + TFR + TLD = T12 \tag{2}$$

where $J_{eq}$ is the equivalent moment of inertia of the system, *CM* is the viscous damping coefficient, *TFR* is the friction torque, *TLD* is the aerodynamic load acting on the surface, *T12* is the motor torque and θ is the actuator angular position. It should be noted that, apparently, in the model illustrated in Eq. (2) the elastic term does not appear. In reality, an action similar to that introduced by the spring is performed by the control ring in a position which, based on the comparison between the command input and the actual position of the system, determines the driving torque *T12*. When the inertia of the system is very low, the undamped natural frequency (proportional to $1/\sqrt{J_{eq}}$) increases and its characteristic time decreases [2]; this can lead to some problems during the numerical solution process (numerical instabilities, limit cycles, calculation overflow). These numerical problems can be avoided (or at least reduced) by adopting a very short integration time, but this leads to very high computational burdens and processing times. In the cases of mobile surfaces with low inertia, considering the high viscous damping of a hydraulic system [6], it is possible to neglect the first term of Eq. (2). In this way the equation becomes a first-order downgraded differential equation:

$$C_{eq}\frac{d\vartheta}{dt} + TFR + TLD = T12 \tag{3}$$

where, referring to the symbols shown in [2], the equivalent viscous damping $C_{eq}$ can be expressed as:

$$C_{eq} = CM + \frac{GP}{GQ}\cdot Disp^2 \tag{4}$$

The first term of the second member of Eq. (4) is the motor damping (*CM*) whilst the second one is the EHSV viscous damping (where *GP* and *GQ* are respectively the valve pressure and flow gains [7], and *Disp* is the volumetric displacement of the hydraulic motor [5]). The EHSV viscous damping is generally 2÷3 order of magnitude greater than the motor one, and therefore can't be neglected. It is necessary to introduce this equivalent viscous damping because of the differences between the second-order Simulink model and the first-order one (see Figures 2 and 3). This degradation is possible only if the orders of magnitude of the elastic and viscous terms are greater than the inertial one, which means that the velocity transient is described by the instantaneous equilibrium between viscous forces and applied loads. In Figure 1 it is shown the effect of inertia (or mass) on the system behavior.

## 3. Dry friction model

In the simulation model developed by the authors, the phenomenon of dry friction is modeled according to the discrete parameter formulation proposed by Coulomb. The model proposed by the authors is based on the discrete parameter implementation proposed in [8] and declined for a rotary system, as illustrated in [1]. Hence, the instantaneous value of the friction torque *TFR0* is obtained as:

$$TFR0 = \begin{cases} T_{act} & IF\ \dot{\vartheta} = 0 \wedge |T_{act}| \le TFR_{stat} \\ TFR_{stat}\cdot sign(T_{act}) & IF\ \dot{\vartheta} = 0 \wedge |T_{act}| > TFR_{stat} \\ TFR_{din}\cdot sign(\dot{\vartheta}) & IF\ \dot{\vartheta} \neq 0 \end{cases} \tag{5}$$

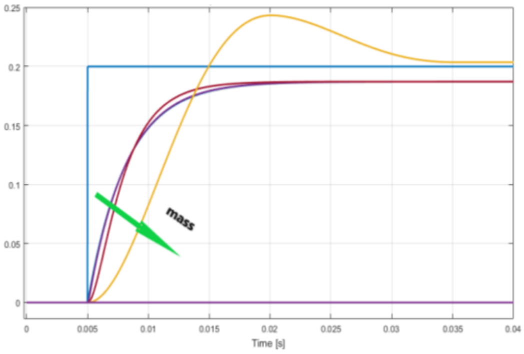



**Figure 1**. EHA Dynamic response for a step command, parameterized in the actuator mass. Comparison between the second-order model and the analogous first-order degraded one. For low inertia systems the first and second-order curves are almost identical, allowing us to run the first-order model with negligible errors.

where $\dot{\vartheta}$ and $T_{act}$ represent respectively the motor angular velocity ($\dot{\vartheta} = \omega$ in rad/s) and the net active torque (which determinates the instantaneous value of the angular acceleration acting on the system). Referring to Eq. (2), the active torque $T_{act}$ should be expressed as $T12 - TFR - TLD - CM\cdot\omega$. However, referring to [1-3], the model is also sensitive to the external load *TLD*, and is able to distinguish between aiding and opposing loads because of the reversible or irreversible behavior of the actuating system. The friction torque *TFR* is composed of the no-load torque (*TFR0*), fundamentally due to preloads on bearings and gaskets, and an additional term, deriving to the external torque *TLD* (generated by the aerodynamic load applied on the control surface) through a proportionality coefficient related to loading conditions. The efficiency of the mechanism depends on the load condition related to the motion (aiding or opposing). Therefore, in slipping conditions, the Coulomb acting on moving elements is characterized by the following two different types of efficiency: *EtOJ* for the opposing case and *EtAJ* for the aiding one. The total friction torque (*TFR*) is computed as:

$$TFR = TFR0 + \left(\frac{1}{EtOJ} - 1\right)\cdot TLD \tag{6}$$

$$TFR = TFR0 + (1 - EtAJ)\cdot TLD \tag{7}$$

where Eq. (6) represents the opposing load whilst Eq. (7) the aiding load conditions.

As regards the dynamic system modeled as a second-order (or higher) model, the abovementioned Borello's algorithm [1] allows keeping into account all the possible scenarios faced by a mechanical actuator, without the complications that other models (e.g. Quinn [9], Karnopp [10], Dahl [11]) may introduce in the solution process [12]. Instead, as regards the friction algorithm to be integrated into the numerical model of the EHA degraded to a first-order nonlinear dynamic system, the approach illustrated in [13] was adopted. In this friction model the calculation of the instantaneous actuation speed supplied as output by the model is performed through a memory block, which allows interrupting the algebraic loop which, otherwise, would be established inside the speed reset loop

## 4. Electrohydraulic actuator numerical models

The EHA reference model was implemented on the base of the mathematical model shown in [14]. As shown in Figure 2, the position error (*Err*) determined by the difference between the command (*Com*) generated by the pilot and the actual position ($\theta$) of the flight command, is converted by means of the PID control logic into a current input (*Cor*). This allows the first valve stage to be activated, resulting in a torque by the electric torque motor (expressed as a function of *Cor* and of the gain in torque *GM*). The generated torque allows the movement of the flapper (*XF*), explaining the dynamics of the valve second stage (modelled as a first-order system). As a consequence of the said torque, a displacement of the spool (*XS*), eventually limited by a saturation block (SV hard-stops), is calculated. The displacement *XS* allows to determine the differential pressure *P12* acting on the hydraulic motor (considering saturation). The two gains fluid dynamic valve model is implemented, by means that the differential pressure *P12* is determined as the difference between *XS* and the flows through the motor (*QJ*) divided by *GQ* (flow gain), multiplied by *GP* (pressure gain). The differential pressure *P12* acting on the motor displacement (*Disp*) determines the pressure torque *T12*. The net active torque $T_{act}$ is obtained by the comparison between *T12* with the dry friction torque (*TFR*), the viscous torque determined by the viscous coefficient (*CM*), and the aerodynamic load (*TLD*). The motor acceleration is obtained as ratio between the active force ($T_{act}$) and the equivalent moment of inertia of the actuation system. Its integration gives the angular velocity, determining the viscous term and the working flow *QJ* above discussed. Finally, the integration of the velocity gives the instantaneous angular position $\theta$ that allows closing the feedback ring (considering the position transducer as ideal). The EHA reference model is represented by the lumped-parameters second-order nonlinear model shown in Figure 2. The numerical model of the two-stage, four-way hydraulic servovalve, based on [15-18], initially implemented in a nonlinear fourth-order model, is simplified to an instantaneous model (zero-order dynamics). The final actuator model is a typical nonlinear second-order type able to simulate the effects due to dry friction acting on moving elements, and mechanical hard-stops limiting the total actuator stroke [19]. The authors evaluated the performance of the first-order degraded numerical model by comparing it with the corresponding complete system (i.e. the higher-level dynamic one). The dynamic response from the degraded model (Figure 3), calculated in some reference cases (defined command and load time-history), is compared with those simulated with the complete second-order model, considering different values of the moment total inertia of the EHA.

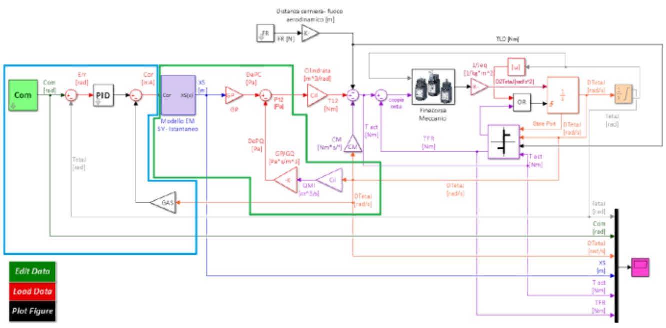


**Figure 2**. Simulink block diagram of the EHA with nonlinear second-order model

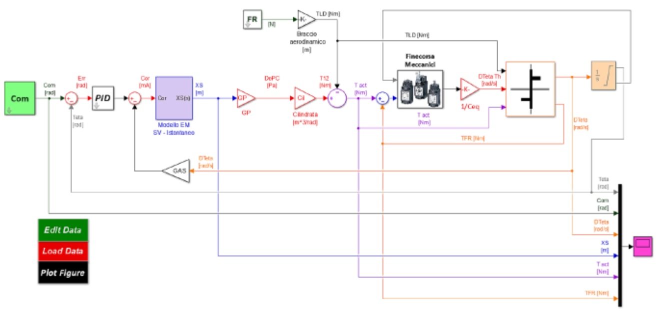
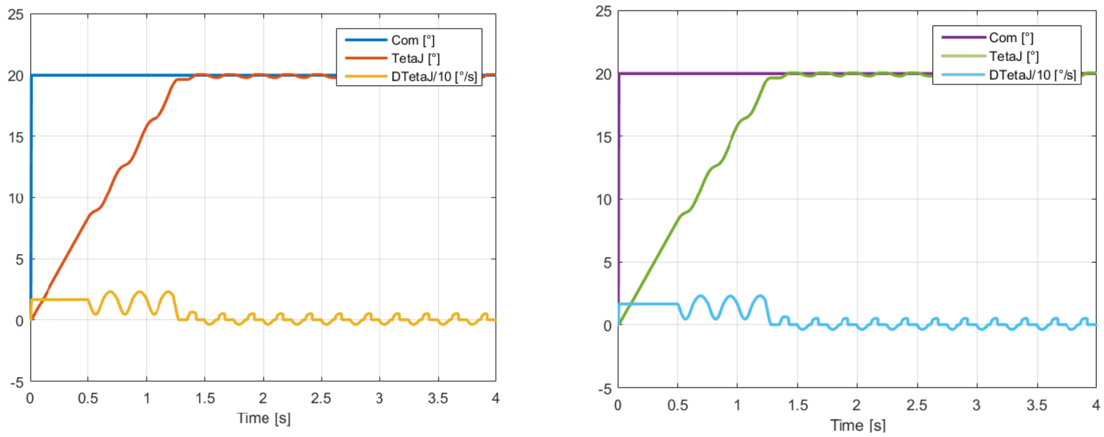


**Figure 3**. Simulink block diagram of the EHA with degraded nonlinear first-order model

## 5. Results

To evaluate the performance of the proposed EHA model, we compared the dynamic responses generated by the detailed numerical algorithm with those obtained with the first order degraded actuator model. Figure 4(a) shows the dynamic response of a reversible actuator with low inertia (negligible compared to the other terms of the equation of equilibrium). The EHA command is a position step input with amplitude equal to 20 °, applied at the initial instant of the simulation. This simulation is performed under the effect of a sinusoidal external torque *TLD*, applied at time 0.5 s, acting as disturbance input. Figure 4(b) shows the response of the corresponding degraded model.




(a) (b)

**Figure 4**. Dynamic response of the considered EHA (a) second-order model (b) first-order model

## 6. Conclusions

The simulations show the accuracy of the proposed algorithm taking into account the effects of the dry friction and the end-of-travel on the behavior of actuators. It must be noted the ability of the proposed model to describe the dynamic/static behavior of both reversible and irreversible types correctly, employing the proper values of the respective efficiencies. The proposed first-order model is still able to simulate with satisfactory accuracy the dynamic response of the servomechanism considered (friction, saturation, limit switch) and is also capable of simulating reversible and irreversible actuators under opposing and aiding loads. The simulations showed that the proposed first-order model is a good choice for those actuating systems characterized by low inertia (i.e. the inertial term of the mathematical model is negligible respect to the viscous or elastic contributions). At the same time, it is not recommended for higher-inertia systems. Moreover, for these dynamic systems, the second-order model can be used with larger time discretization due to the low speed and frequencies at which those work, and therefore the numerical issues shouldn't occur. In conclusion, it should be noted that the approach proposed by the authors in this work can also be extended to the case of linear actuators, without applying substantial changes to the algorithms illustrated here. In fact, without intervening on its mathematical model, it is possible to convert the Matlab-Simulink models proposed from a rotary configuration to a translational one, simply by changing the physical meaning of the parameters implemented in the model (e.g. by replacing the torques with the forces, the moments of inertia with the masses and the volumetric displacement of the hydraulic motor with the active area of the jack).

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